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M-theory giant gravitons with C field

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Abstract

We find giant graviton configurations of an M5-brane probe in the $D = 11$ supergravity background generated by a stack of non-threshold (M2, M5) bound states. The M5-brane probe shares three directions with the background and wraps a two-sphere transverse to the bound states. For a particular value of the worldvolume gauge field of the PST formalism, there exist solutions of the equations of motion for which the M5-brane probe behaves as a wave propagating in the (M2, M5) background. We have checked that the probe breaks the supersymmetry of the background exactly as a massless particle moving along the trajectory of its center of mass.

1. Introduction

The so-called giant gravitons are configurations of branes which behave as an expanded massless particle. They were introduced in Ref. [1] for branes moving in a spacetime of the type $AdS_m \times S^{p+2}$ and generalized in Refs. [2–4] for more general near-horizon brane geometries. The supersymmetry of the giant graviton configurations in $AdS_m \times S^{p+2}$ spacetimes was studied in Refs. [5,6], where it was proved that they preserve the same supersymmetries as the point-like graviton in the same spacetime (see also [7–12]).

The general mechanism underlying the construction of Ref. [1] is the coupling of the brane probe to the background gauge field. The flux of this gauge field captured by the wrapped brane probe stabilizes it against shrinking, which allows the existence of stable solutions behaving as massless particles. This situation was generalized in Ref. [4], where giant gravitons for a type II background created by a stack of non-threshold (D$(p-2)$, Dp) bound states were found. In this case, the probe is a D$(8-p)$-brane which wraps the S^{6-p} sphere transverse to the background, and is extended along two directions parallel to it. This configuration is such that the probe captures both the Ramond–Ramond flux and the flux of the Kalb–Ramond B field.

In this Letter we extend the analysis of Ref. [4] to M-theory backgrounds generated by a stack of non-threshold bound states of the type (M2, M5). The corresponding solution of the $D = 11$ supergravity equations was given in Ref. [13] and used as supergravity dual of a non-commutative field theory in Ref. [14]. As suggested in [4], the probe we will consider is an M5-brane wrapped on an S^2 transverse sphere and extended along three directions parallel to the background. We will show that, after switching on a particular value of the worldvolume gauge field,

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one can find giant graviton solutions of the corresponding worldvolume equations of motion. We will also analyze the supersymmetry of the problem, and we will show that our M5-brane configurations break supersymmetry exactly in the same way as a wave which propagates with the velocity of the center of mass of the M5-brane probe.

2. The supergravity background

The metric for the eleven-dimensional supergravity solution of the background we will consider is [13]:

$$\begin{aligned} ds^2 = {} & f^{-1/3}h^{-1/3}\Big[-\big(dx^0\big)^2 + \big(dx^1\big)^2 + \big(dx^2\big)^2 + h\Big(\big(dx^3\big)^2 + \big(dx^4\big)^2 + \big(dx^5\big)^2\Big)\Big] \\ & + f^{2/3}h^{-1/3}\big[dr^2 + r^2\, d\Omega_4^2\big], \end{aligned} \tag{2.1}$$

where $d\Omega_4^2$ is the line element of a unit 4-sphere and the functions f and h are given by:

$$\begin{aligned} f &= 1 + \frac{R^3}{r^3}, \\ h^{-1} &= \sin^2\varphi f^{-1} + \cos^2\varphi. \end{aligned} \tag{2.2}$$

The metric (2.1) is the one generated by a stack of parallel non-threshold $(\mathrm{M2}, \mathrm{M5})$ bound states. The M5-brane component of this bound state is extended along the directions $x^0, \ldots, x^5$, whereas the M2-brane lies along x^0, x^1, x^2. The angle φ in Eq. (2.2) determines the mixing of the M2- and M5-branes in the bound state and the "radius" R is given by $R^3\cos\varphi = \pi N l_p^3$, where l_p is the Planck length in eleven dimensions and N is the number of bound states of the stack. The solution of $D = 11$ supergravity is also characterized [13] by a non-vanishing value of the four-form field strength $F^{(4)}$, namely:

$$\begin{aligned} F^{(4)} = {} & \sin\varphi\,\partial_r\big(f^{-1}\big)\, dx^0 \wedge dx^1 \wedge dx^2 \wedge dr - 3R^3\cos\varphi\,\epsilon_{(4)} \\ & - \tan\varphi\,\partial_r\big(hf^{-1}\big)\, dx^3 \wedge dx^4 \wedge dx^5 \wedge dr, \end{aligned} \tag{2.3}$$

where $\epsilon_{(4)}$ represents the volume form of the unit S^4. The field strength $F^{(4)}$ can be represented as the exterior derivative of a three-form potential $C^{(3)}$, i.e., as $F^{(4)} = dC^{(3)}$. In order to obtain the explicit form of $C^{(3)}$, let us introduce a particular set of coordinates for the transverse S^4 [1]. Let ρ and ϕ take values in the range $0 \leqslant \rho \leqslant 1$ and $0 \leqslant \phi \leqslant 2\pi$, respectively. Then, the line element $d\Omega_4^2$ can be written as:

$$d\Omega_4^2 = \frac{1}{1-\rho^2}d\rho^2 + \big(1-\rho^2\big)\, d\phi^2 + \rho^2\, d\Omega_2^2, \tag{2.4}$$

where $d\Omega_2^2$ is the metric of a unit S^2 (which we will parametrize by means of two angles θ^1 and θ^2). In these coordinates one can take $C^{(3)}$ as:

$$\begin{aligned} C^{(3)} = {} & -\sin\varphi f^{-1}\, dx^0 \wedge dx^1 \wedge dx^2 - R^3\cos\varphi\rho^3\, d\phi \wedge \epsilon_{(2)} \\ & + \tan\varphi h f^{-1}\, dx^3 \wedge dx^4 \wedge dx^5, \end{aligned} \tag{2.5}$$

where $\epsilon_{(2)}$ is the volume form of the S^2. It is not difficult to verify from Eq. (2.3) that ${}^*F^{(4)}$ satisfies:

$$d\,{}^*F^{(4)} = -\frac{1}{2}F^{(4)} \wedge F^{(4)}, \tag{2.6}$$

where the seven-form ${}^*F^{(4)}$ is the Hodge dual of $F^{(4)}$ with respect to the metric (2.1). Eq. (2.6) implies that ${}^*F^{(4)}$ can be represented in terms of a six-form potential $C^{(6)}$ as follows:

$${}^*F^{(4)} = dC^{(6)} - \frac{1}{2}C^{(3)} \wedge dC^{(3)}. \tag{2.7}$$

By taking the exterior derivative of both sides of (2.7), one immediately verifies Eq. (2.6). Moreover, it is not difficult to find the potential $C^{(6)}$ in our coordinate system. Actually, one can easily check that one can take $C^{(6)}$ as:

$$\begin{aligned} C^{(6)} = {} & \frac{1}{2}\sin\varphi\cos\varphi f^{-1}R^3\rho^3\, dx^0\wedge dx^1\wedge dx^2\wedge d\phi\wedge\epsilon_{(2)} \\ & - \frac{1}{2}\frac{1+h\cos^2\varphi}{\cos\varphi} f^{-1}\, dx^0\wedge dx^1\wedge dx^2\wedge dx^3\wedge dx^4\wedge dx^5 \\ & - \frac{1}{2}\sin\varphi R^3\rho^3 h f^{-1}\, dx^3\wedge dx^4\wedge dx^5\wedge d\phi\wedge\epsilon_{(2)}. \end{aligned} \tag{2.8}$$

3. The M5-brane probe

We shall now consider the near-horizon region of the $(\text{M2}, \text{M5})$ geometry. In this region the radial coordinate r is small and one can approximate the function f appearing in the supergravity solution as $f \approx R^3/r^3$. Following the analysis of Ref. [4], we place an M5-brane probe in this geometry in such a way that it shares three directions (x^3, x^4, x^5) with the branes of the background and wraps the S^2 transverse sphere parametrized by the angles θ^1 and θ^2. The dynamics of the M5-brane probe is determined by its worldvolume action, i.e., by the so-called PST action [15]. In the PST formalism the worldvolume fields are a three-form field strength F and a scalar field a (the PST scalar). The action is the sum of three terms:

$$S = T_{M5}\int d^6\xi\, [\mathcal{L}_{\text{DBI}} + \mathcal{L}_{H\widetilde{H}} + \mathcal{L}_{\text{WZ}}], \tag{3.1}$$

where the tension of the M5-brane is $T_{M5} = 1/(2\pi)^5 l_p^6$. In the action (3.1) the field strength F is combined with the pullback $P[C^{(3)}]$ of the background potential $C^{(3)}$ to form the field H:

$$H = F - P\big[C^{(3)}\big]. \tag{3.2}$$

Let us now define the field $\widetilde{H}$ as follows:

$$\widetilde{H}^{ij} = \frac{1}{3!\sqrt{-\det g}}\frac{1}{\sqrt{-(\partial a)^2}}\epsilon^{ijklmn}\partial_k a H_{lmn}, \tag{3.3}$$

with g being the induced metric on the M5-brane worldvolume. The explicit form of the three terms of the action is:

$$\begin{aligned} \mathcal{L}_{\text{DBI}} &= -\sqrt{-\det(g_{ij} + \widetilde{H}_{ij})}, \\ \mathcal{L}_{H\widetilde{H}} &= \frac{1}{24(\partial a)^2}\epsilon^{ijkmnr} H_{mnr} H_{jkl} g^{ls}\partial_i a\partial_s a, \\ \mathcal{L}_{\text{WZ}} &= \frac{1}{6!}\epsilon^{ijklmn}\big[P\big[C^{(6)}\big]_{ijklmn} + 10 H_{ijk} P\big[C^{(3)}\big]_{lmn}\big]. \end{aligned} \tag{3.4}$$

The worldvolume coordinates ξ^i $(i = 0, \ldots, 5)$ will be taken as $\xi^i = (x^0, x^3, x^4, x^5, \theta^1, \theta^2)$. In this system of coordinates the configurations we are interested in are described by functions of the type $r = r(t)$, $\rho = \rho(t)$ and $\phi = \phi(t)$, where $t \equiv x^0$. Moreover, we will assume that the only non-vanishing components of H are those of $P[C^{(3)}]$, i.e., $H_{x^3x^4x^5} \equiv H_{345}$ and $H_{x^0\theta^1\theta^2} \equiv H_{0*}$. As discussed in Ref. [15], the scalar field a is an auxiliary field which, by fixing its gauge symmetry, can be eliminated from the action at the expense of loosing manifest covariance. In this Letter we will work in the gauge $a = x^0$. In this gauge the only non-zero component of $\widetilde{H}$ is:

$$\widetilde{H}_{\theta^1\theta^2} = f^{7/6}h^{-4/3}r^2\rho^2\sqrt{\hat{g}^{(2)}}\, H_{345}, \tag{3.5}$$

where $\hat{g}^{(2)}$ is the determinant of the metric of the two-sphere. By using (3.5) one can easily obtain $\mathcal{L}_{\mathrm{DBI}}$ for our configurations. Indeed, after a short calculation one gets:

$$\mathcal{L}_{\mathrm{DBI}} = -R^3\rho^2\sqrt{\hat{g}^{(2)}}\,\lambda_1\sqrt{r^{-2}f^{-1} - r^{-2}\dot{r}^2 - \frac{\dot{\rho}^2}{1-\rho^2} - (1-\rho^2)\dot{\phi}^2}, \tag{3.6}$$

where the dot denotes time derivative and λ_1 is defined as:

$$\lambda_1 \equiv \sqrt{hf^{-1} + (H_{345})^2 h^{-1}}. \tag{3.7}$$

It is also very easy to prove that the remaining terms of the action are:

$$\begin{aligned}\mathcal{L}_{H\tilde{H}} + \mathcal{L}_{\mathrm{WZ}} = {} & \frac{1}{2}F_{345}F_{0*} - F_{345}P\big[C^{(3)}\big]_{0*} \\ & + P\big[C^{(6)}\big]_{0345*} + \frac{1}{2}P\big[C^{(3)}\big]_{345}P\big[C^{(3)}\big]_{0*},\end{aligned} \tag{3.8}$$

with $F_{0*} \equiv F_{x^0\theta^1\theta^2}$ and similarly for the pullbacks of $C^{(6)}$ and $C^{(3)}$. From Eqs. (2.5) and (2.8) it follows that:

$$\begin{aligned}&P\big[C^{(6)}\big]_{0345*} = \frac{1}{2}R^3\rho^3\sin\varphi\, hf^{-1}\sqrt{\hat{g}^{(2)}}\,\dot{\phi}, \\ &P\big[C^{(3)}\big]_{345} = \tan\varphi\, hf^{-1}, \\ &P\big[C^{(3)}\big]_{0*} = -R^3\rho^3\cos\varphi\sqrt{\hat{g}^{(2)}}\,\dot{\phi}.\end{aligned} \tag{3.9}$$

By using Eq. (3.9) it is straightforward to demonstrate that the sum of the last two terms in $\mathcal{L}_{H\tilde{H}} + \mathcal{L}_{\mathrm{WZ}}$ vanishes and, thus, we can write:

$$\mathcal{L}_{H\tilde{H}} + \mathcal{L}_{\mathrm{WZ}} = R^3\rho^3F_{345}\cos\varphi\sqrt{\hat{g}^{(2)}}\,\dot{\phi} + \frac{1}{2}F_{345}F_{0*}. \tag{3.10}$$

Let us assume that $F_{0*} = \sqrt{\hat{g}^{(2)}}\,f_{0*}$ with f_{0*} independent of the angles of the S^2. With this ansatz for the electric component of F, the action can be written as:

$$S = \int dt\,dx^3\,dx^4\,dx^5\,\mathcal{L}, \tag{3.11}$$

with the lagrangian density $\mathcal{L}$ given by:

$$\begin{aligned}\mathcal{L} = 4\pi R^3T_{M5}\bigg[& -\rho^2\lambda_1\sqrt{r^{-2}f^{-1} - r^{-2}\dot{r}^2 - \frac{\dot{\rho}^2}{1-\rho^2} - (1-\rho^2)\dot{\phi}^2} \\ & + \lambda_2\rho^3\dot{\phi} + \frac{1}{2R^3}F_{345}f_{0*}\bigg].\end{aligned} \tag{3.12}$$

In Eq. (3.12) we have defined λ_2 as:

$$\lambda_2 \equiv F_{345}\cos\varphi. \tag{3.13}$$

As in Ref. [4], it is interesting to characterize the spreading of the M5-brane in the $x^3x^4x^5$ directions by means of the flux of the worldvolume gauge field F. We shall parametrize this flux as follows:

$$\int dx^3\,dx^4\,dx^5\,F = \frac{2\pi}{T_{M2}}N', \tag{3.14}$$

where $T_{M2} = 1/(2\pi)^2 l_p^3$ in the tension of the M2-brane. Notice that, when the coordinates $x^3x^4x^5$ are compact, the condition (3.14) is just the M-theory flux quantization condition found in Ref. [16], with the flux number N' being an integer for topological reasons.

In order to perform a canonical hamiltonian analysis of this system, let us introduce the density of momenta:

$$\mathcal{P}_r = \frac{\partial \mathcal{L}}{\partial \dot{r}} \equiv 4\pi R^3 T_{M5} \lambda_1 \pi_r,$$
$$\mathcal{P}_\rho = \frac{\partial \mathcal{L}}{\partial \dot{\rho}} \equiv 4\pi R^3 T_{M5} \lambda_1 \pi_\rho,$$
$$\mathcal{P}_\phi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} \equiv 4\pi R^3 T_{M5} \lambda_1 \pi_\phi, \tag{3.15}$$

where we have defined the reduced momenta π_r, π_ρ and π_ϕ. From the explicit value of $\mathcal{L}$ (Eq. (3.12)), we get:

$$\pi_r = \frac{\rho^2}{r^2} \frac{\dot{r}}{\sqrt{r^{-2} f^{-1} - r^{-2}\dot{r}^2 - \frac{\dot{\rho}^2}{1-\rho^2} - (1-\rho^2)\dot{\phi}^2}},$$
$$\pi_\rho = \frac{\rho^2}{1-\rho^2} \frac{\dot{\rho}}{\sqrt{r^{-2} f^{-1} - r^{-2}\dot{r}^2 - \frac{\dot{\rho}^2}{1-\rho^2} - (1-\rho^2)\dot{\phi}^2}},$$
$$\pi_\phi = (1-\rho^2)\rho^2 \frac{\dot{\phi}}{\sqrt{r^{-2} f^{-1} - r^{-2}\dot{r}^2 - \frac{\dot{\rho}^2}{1-\rho^2} - (1-\rho^2)\dot{\phi}^2}} + \Lambda\rho^3, \tag{3.16}$$

where we have introduced the quantity $\Lambda \equiv \lambda_2/\lambda_1$. The hamiltonian density of the system is:

$$\mathcal{H} = \dot{r}\mathcal{P}_r + \dot{\rho}\mathcal{P}_\rho + \dot{\phi}\mathcal{P}_\phi + F_{0*}\frac{\partial \mathcal{L}}{\partial F_{0*}} - \mathcal{L}. \tag{3.17}$$

After a short calculation one can prove that $\mathcal{H}$ is given by:

$$\mathcal{H} = 4\pi R^3 T_{M5} \lambda_1 r^{-1} f^{-\frac{1}{2}} \left[r^2\pi_r^2 + \rho^4 + (1-\rho^2)\pi_\rho^2 + \frac{(\pi_\phi - \Lambda\rho^3)^2}{1-\rho^2} \right]^{\frac{1}{2}}. \tag{3.18}$$

4. Giant graviton configurations

By inspecting the line element displayed in Eq. (2.4) one easily concludes that the coordinate ρ plays the role of the size of the system on the S^2 sphere. We are interested in finding configurations of fixed size, i.e., those solutions of the equations of motion with constant ρ. By comparing the hamiltonian density written in (3.18) with the one studied in Ref. [4], it is not difficult to realize that these fixed size solutions exist if the quantity Λ takes the value $\Lambda = 1$. Indeed, if this condition holds, the hamiltonian density $\mathcal{H}$ can be put as:

$$\mathcal{H} = 4\pi R^3 T_{M5} \lambda_1 r^{-1} f^{-\frac{1}{2}} \left[\pi_\phi^2 + r^2\pi_r^2 + (1-\rho^2)\pi_\rho^2 + \frac{(\pi_\phi\rho - \rho^2)^2}{1-\rho^2} \right]^{\frac{1}{2}}, \tag{4.1}$$

and, as we will verify soon, one can easily find constant ρ solutions of the equations of motion for the hamiltonian (4.1). Moreover, by using the value of $P[C^{(3)}]_{345}$ given in Eq. (3.9), one can write λ_1 as:

$$\lambda_1^2 = \cos^2\varphi F_{345}^2 + f^{-1}\left(F_{345}\sin\varphi - \frac{1}{\cos\varphi} \right)^2. \tag{4.2}$$

Taking into account the definition of λ_2 (Eq. (3.13)), it follows that the condition $\Lambda = 1$ (or $\lambda_1 = \lambda_2$) is equivalent to have the following constant value of the worldvolume gauge field:

$$F_{345} = \frac{1}{\sin\varphi\cos\varphi} = 2\csc(2\varphi). \tag{4.3}$$

It follows from Eq. (3.16) that for a configuration with $\dot\rho = 0$ the momentum π_ρ necessarily vanishes and, in particular, one must require that $\dot\pi_\rho = 0$. Then, the hamiltonian equations of motion imply that $\partial\mathcal{H}/\partial\rho$ must be zero, which happens if the last term inside the square root of the right-hand side of Eq. (4.1) vanishes, i.e., when $\pi_\phi\rho - \rho^2 = 0$. This occurs either when $\rho = 0$ or else when the angular momentum π_ϕ is:

$$\pi_\phi = \rho. \tag{4.4}$$

In order to clarify the nature of these solutions, let us invert the relation between π_ϕ and $\dot\phi$ (Eq. (3.16)). After a simple calculation one gets:

$$\dot\phi = \frac{\pi_\phi - \rho^3}{1-\rho^2}\,\frac{\left[r^{-2}(f^{-1} - \dot r^2) - \frac{\dot\rho^2}{1-\rho^2}\right]^{\frac{1}{2}}}{\left[\pi_\phi^2 + \frac{(\pi_\phi\rho-\rho^2)^2}{1-\rho^2}\right]^{\frac{1}{2}}}. \tag{4.5}$$

By taking $\dot\rho = \pi_\phi\rho - \rho^2 = 0$ on the right-hand side of Eq. (4.5), one finds the following relation between $\dot\phi$ and $\dot r$:

$$f\left[r^2\dot\phi^2 + \dot r^2\right] = 1. \tag{4.6}$$

Remarkably, Eq. (4.6) is the condition satisfied by a particle which moves in the (r,ϕ) plane at $\rho = 0$ along a null trajectory (i.e., with $ds^2 = 0$) in the metric (2.1). Therefore, our brane probe configurations behave as a massless particle: the so-called giant graviton. The point $\rho = 0$ can be interpreted as the "center of mass" of the expanded brane. Actually, if one defines the velocity vector $\mathbf{v}$ as $\mathbf{v} = (v^{\underline{r}}, v^{\underline{\phi}}) \equiv f^{\frac{1}{2}}(\dot r, r\dot\phi)$, Eq. (4.6) is equivalent to the condition $(v^{\underline{r}})^2 + (v^{\underline{\phi}})^2 = 1$ and, thus, the center of mass of the giant graviton moves at the speed of light. On the other hand, the angular momentum density $\mathcal{P}_\phi$ for the $\pi_\phi = \rho$ solution can be obtained from Eq. (3.15), namely:

$$\mathcal{P}_\phi = \frac{T_{M2}}{2\pi}F_{345}N\rho. \tag{4.7}$$

Moreover, by integrating the densities $\mathcal{P}_\phi$ and $\mathcal{P}_r$ along the $x^3x^4x^5$ directions, one gets the values of the momenta p_ϕ and p_r:

$$p_\phi = \int dx^3\,dx^4\,dx^5\,\mathcal{P}_\phi, \qquad p_r = \int dx^3\,dx^4\,dx^5\,\mathcal{P}_r. \tag{4.8}$$

By using the value of the momentum density $\mathcal{P}_\phi$ displayed in Eq. (4.7), together with the flux quantization condition (3.14), one gets the following value of p_ϕ:

$$p_\phi = NN'\rho, \tag{4.9}$$

which implies that the size ρ of the wrapped brane increases with its angular momentum p_ϕ. As $0 \leqslant \rho \leqslant 1$, the momentum p_ϕ has a maximum given by $p_\phi^{\max} = NN'$. This maximum is reached when $\rho = 1$ and its existence is a manifestation of the stringy exclusion principle.

In order to analyze the energy of the giant graviton solution, let $\mathcal{G}_{MN}$ be the metric elements of Eqs. (2.1) and (2.4) at the point $\rho = 0$. Then, it is straightforward to verify that the hamiltonian H_{GG} of the giant graviton configurations is:

$$H_{GG} = \sqrt{-\mathcal{G}_{tt}}\left[\frac{p_\phi^2}{\mathcal{G}_{\phi\phi}} + \frac{p_r^2}{\mathcal{G}_{rr}}\right]^{\frac{1}{2}}, \tag{4.10}$$

which is exactly the one corresponding to a massless particle which moves in the (r, ϕ) plane under the action of the metric $\mathcal{G}_{MN}$. By substituting in Eq. (4.10) the explicit values of the $\mathcal{G}_{MN}$'s, one can write H_{GG} as:

$$H_{GG} = R^{-\frac{3}{2}}\left[r^3 p_r^2 + r p_\phi^2\right]^{\frac{1}{2}}. \tag{4.11}$$

By using the conservation of energy, one can integrate the equations of motion and get the functions $r(t)$ and $\phi(t)$. It turns out that the corresponding equations coincide with one of the cases studied in Ref. [4]. Therefore, we simply write the results of this integration and refer to [4] for the details of the calculation. One gets:

$$\begin{aligned} r &= \frac{r_*}{1 + \frac{r_*}{4R^3}(t - t_*)^2}, \\ \tan\left[\frac{\phi - \phi_*}{2}\right] &= \frac{1}{2R}\left(\frac{r_*}{R}\right)^{\frac{1}{2}}(t - t_*), \end{aligned} \tag{4.12}$$

where r_*, ϕ_* and t_* are constants. Notice that $r \leqslant r_*$ and that $r \to 0$ as $t \to \infty$, which means that the giant graviton always falls asymptotically to the center of the potential.

It is also interesting to study the volume occupied by the M5-brane probe along the $x^3x^4x^5$ directions. By plugging the value (4.3) of the worldvolume gauge field into the flux quantization condition (3.14), one gets that this volume is:

$$\int dx^3\, dx^4\, dx^5 = \frac{\pi N'}{T_{M2}} \sin(2\varphi). \tag{4.13}$$

When $\varphi \to 0$, the M2-brane component of the background bound state disappears and we are left with a M5-brane background. For fixed N', it follows from Eq. (4.13) that the three directions of the M5-brane probe which are parallel to the background collapse and, therefore, the M5-brane probe is effectively converted into a M2-brane, in agreement with the results of Ref. [1].

The gauge field F of the PST action satisfies a generalized self-duality condition which relates its electric and magnetic components. In order to get this self-duality constraint one must use both the equations of motion and the symmetries of the PST action [15]. In our case, this condition reduces to:

$$\frac{\partial \mathcal{L}}{\partial F_{345} = 0.} \tag{4.14}$$

Indeed, after using the explicit expression of $\mathcal{L}$ (Eq. (3.12)), and solving Eq. (4.14) for f_{0*}, one gets:

$$f_{0*} = 2R^3\left[\rho^2 \frac{H_{345}}{\lambda_1 h}\sqrt{r^{-2} f^{-1} - r^{-2}\dot{r}^2 - \frac{\dot{\rho}^2}{1-\rho^2} - \left(1-\rho^2\right)\dot{\phi}^2} - \cos\varphi \rho^3 \dot{\phi}\right]. \tag{4.15}$$

By substituting on the right-hand side of Eq. (4.15) the values corresponding to our giant graviton configurations, one gets a vanishing result, i.e.:

$$f_{0*}|_{GG} = 0. \tag{4.16}$$

Thus, our expanded graviton solutions have zero electric field on the M5-brane worldvolume.

5. Supersymmetry

Let us now examine the supersymmetry of our configurations. First of all, we consider the supersymmetry preserved by the background. As the solution of $D = 11$ supergravity we are dealing with is purely bosonic, it is only invariant under those supersymmetry transformations which do no change the gravitino field ψ_M. This field transforms as:

$$\delta\psi_M = D_M\epsilon + \frac{1}{288}\left(\Gamma_M^{N_1\cdots N_4} - 8\delta_M^{N_1}\Gamma^{N_2\cdots N_4}\right)\epsilon F^{(4)}_{N_1\cdots N_4}. \tag{5.1}$$

The spinors ϵ for which the right-hand side of Eq. (5.1) vanish are the Killing spinors of the background. It is not difficult to find them in our case. Actually, if we define the matrix $\Upsilon = \Gamma_{\underline{\phi}}\Gamma_*$ with $\Gamma_* \equiv \Gamma_{\underline{\theta^1\theta^2}}$, they can be parametrized as follows:

$$\epsilon = e^{\frac{\alpha}{2}\Gamma_{\underline{x^3x^4x^5}}}\, e^{-\frac{\beta}{2}\Upsilon}\,\hat{\epsilon}, \tag{5.2}$$

where α and β are:

$$\begin{aligned} &\sin\alpha = f^{-\frac{1}{2}}h^{\frac{1}{2}}\sin\varphi, \qquad && \cos\alpha = h^{\frac{1}{2}}\cos\varphi, \\ &\sin\beta = \rho, && \cos\beta = \sqrt{1-\rho^2}, \end{aligned} \tag{5.3}$$

and $\hat{\epsilon}$ is independent of ρ and satisfies:

$$\Gamma_{\underline{x^0\ldots x^5}}\hat{\epsilon} = \hat{\epsilon}. \tag{5.4}$$

By working out the condition $\delta\psi_M = 0$ one can determine $\hat{\epsilon}$ completely. We will not reproduce this calculation here since the representation (5.2) is enough for our purposes. Let us however mention that that it follows from this analysis that the (M2, M5) background is $1/2$ supersymmetric.

The number of supersymmetries preserved by the M5-brane probe is the number independent solutions of the equation $\Gamma_\kappa\epsilon = \epsilon$, where ϵ is one of the Killing spinors (5.2) and Γ_κ is the κ-symmetry matrix of the PST formalism [15,17]. In order to write the expression of this matrix, let us define the following quantities:

$$\nu_p \equiv \frac{\partial_p a}{\sqrt{-(\partial a)^2}}, \qquad t^m \equiv \frac{1}{8}\epsilon^{mn_1n_2p_1p_2q}\,\widetilde{H}_{n_1n_2}\widetilde{H}_{p_1p_2}\nu_q. \tag{5.5}$$

Then, the κ-symmetry matrix is:

$$\Gamma_\kappa = -\frac{\nu_m\gamma^m}{\sqrt{-\det(g+\widetilde{H})}}\left[\gamma_n t^n + \frac{\sqrt{-g}}{2}\gamma^{np}\widetilde{H}_{np} + \frac{1}{5!}\gamma_{i_1\cdots i_5}\epsilon^{i_1\cdots i_5 n}\nu_n\right]. \tag{5.6}$$

In Eq. (5.6) $\gamma_{i_1i_2\cdots}$ are antisymmetrized products of the worldvolume Dirac matrices $\gamma_i = \partial_i X^M E_M^{\underline{M}}\Gamma_{\underline{M}}$. In our case the vector t^m is zero and the only non-zero component of ν_m is: $\nu_0 = \sqrt{-G_{tt}}$. Using these facts, after some calculation, one can represent Γ_κ as:

$$\begin{aligned} \Gamma_\kappa = {}& \frac{1}{\sqrt{-G_{tt} - G_{\phi\phi}\dot{\phi}^2 - G_{rr}\dot{r}^2}} \\ & \times\left[\sqrt{-G_{tt}}\,\Gamma_{\underline{x^0}} + \dot{\phi}\sqrt{G_{\phi\phi}}\,\Gamma_{\underline{\phi}} + \dot{r}\sqrt{G_{rr}}\,\Gamma_{\underline{r}}\right]\Gamma_* e^{-\eta\Gamma_{\underline{x^3x^4x^5}}}, \end{aligned} \tag{5.7}$$

with η given by:

$$\sin\eta = \frac{f^{-\frac{1}{2}}h^{\frac{1}{2}}}{\lambda_1}, \qquad \cos\eta = \frac{H_{345}h^{-\frac{1}{2}}}{\lambda_1}. \tag{5.8}$$

By using Eqs. (5.7) and (5.2), the equation $\Gamma_\kappa\epsilon = \epsilon$ takes the form:

$$\begin{aligned} &\frac{1}{\sqrt{-G_{tt} - G_{\phi\phi}\dot{\phi}^2 - G_{rr}\dot{r}^2}}\left[\sqrt{-G_{tt}}\,\Gamma_{\underline{x^0}} + \dot{\phi}\sqrt{G_{\phi\phi}}\,\Gamma_{\underline{\phi}} + \dot{r}\sqrt{G_{rr}}\,\Gamma_{\underline{r}}\right]\Gamma_* e^{-\frac{\beta}{2}\Upsilon}\hat{\epsilon} \\ &\quad = e^{(\alpha-\eta)\Gamma_{\underline{x^3x^4x^5}}}\, e^{-\frac{\beta}{2}\Upsilon}\,\hat{\epsilon}. \end{aligned} \tag{5.9}$$

Let us now evaluate Eq. (5.9) for our solution. First of all, one can verify that, when the worldvolume gauge field F_{345} takes the value (4.3), the angles α and η are equal and, thus, the dependence on $\Gamma_{\underline{x^3 x^4 x^5}}$ of the right-hand side of (5.9) disappears. Moreover, using the condition (4.6), and performing some simple manipulations, one can convert Eq. (5.9) into:

$$\left[f^{-\frac{1}{4}} e^{\beta\Upsilon} \Gamma_{\underline{x^0 \phi}} - \dot{\phi} r f^{\frac{1}{4}} \sqrt{1-\rho^2} + \dot{r} f^{\frac{1}{4}} e^{\beta\Upsilon} \Gamma_{\underline{r\phi}}\right]\hat{\epsilon} = \rho r f^{\frac{1}{4}} \dot{\phi} \Upsilon \hat{\epsilon}. \tag{5.10}$$

If, in particular, we take $\rho = 0$ in Eq. (5.10), one arrives at:

$$\left[f^{-\frac{1}{4}} \Gamma_{\underline{x^0 \phi}} - \dot{\phi} r f^{\frac{1}{4}} + \dot{r} f^{\frac{1}{4}} \Gamma_{\underline{r\phi}}\right]\hat{\epsilon} = 0. \tag{5.11}$$

Remarkably, if Eq. (5.11) holds, then Eq. (5.10) is satisfied for an arbitrary value of ρ. Thus, Eq. (5.11) is equivalent to the κ-symmetry condition $\Gamma_\kappa \epsilon = \epsilon$. In order to interpret (5.11), let us define the matrix $\Gamma_v \equiv v^{\underline{r}} \Gamma_{\underline{r}} + v^{\underline{\phi}} \Gamma_{\underline{\phi}}$, where $v^{\underline{r}}$ and $v^{\underline{\phi}}$ are the components of the center of mass velocity vector $\mathbf{v}$ defined above. This matrix is such that $(\Gamma_v)^2 = 1$, and one can prove that Eq. (5.11) can be written as:

$$\Gamma_{\underline{x^0}} \Gamma_v \hat{\epsilon} = \hat{\epsilon}. \tag{5.12}$$

Taking into account the relation (5.2) between $\hat{\epsilon}$ and ϵ, and using the fact that $\Gamma_{\underline{x^0}} \Gamma_v$ commutes with $\Gamma_{\underline{x^3 x^4 x^5}}$, one can recast Eq. (5.12) as:

$$\Gamma_{\underline{x^0}} \Gamma_v \epsilon|_{\rho=0} = \epsilon|_{\rho=0}, \tag{5.13}$$

which is the supersymmetry projection induced by a massless particle moving in the direction of $\mathbf{v}$ at $\rho = 0$. Notice that, however, the background projector $\Gamma_{\underline{x^0 \dots x^5}}$ does not commute with $\Gamma_{\underline{x^0}} \Gamma_v$ and, therefore, Eqs. (5.4) and (5.12) cannot be imposed at the same time. Thus, the M5-brane probe breaks completely the supersymmetry of the background. The interesting point in this result is that this supersymmetry breaking is just identical to the one corresponding to a massless particle, which constitutes a confirmation of our interpretation of the giant graviton configurations.

6. Summary and conclusions

In this Letter we have found giant graviton configurations of an M5-brane probe in the $D = 11$ supergravity background created by a stack of (M2, M5) bound states. We have solved the probe equations of motion and we have checked that the corresponding solution behaves as an expanded massless particle propagating in the (M2, M5) background. We have also checked that the probe breaks the supersymmetry of the background exactly in the same way as a massless particle moving along the trajectory of the center of mass of the probe. Our results generalize those of Refs. [1–4] and, hopefully, could be useful to shed light on the nature of the blown up graviton systems.

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