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Decay constants of heavy meson of 0^- state in relativistic Salpeter method

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Abstract

The decay constants of pseudoscalar heavy mesons of 0^- state are computed by means of the relativistic (instantaneous) Salpeter equation. We solved the full Salpeter equation without making any further approximation, such as ignoring the small component wave function. Therefore, our results for the decay constants include the complete relativistic contributions from the light and the heavy quarks. We obtain $F_{D_s} \approx 248 \pm 27$, $F_D \approx 230 \pm 25$ $(D^0, D^\pm)$, $F_{B_s} \approx 216 \pm 32$, $F_B \approx 196 \pm 29$ $(B^0, B^\pm)$, $F_{B_c} \approx 322 \pm 42$ and $F_{\eta_c} \approx 292 \pm 25$ MeV.

1. Introduction

The decay constants of mesons are very important quantities. The study of the decay constants has become an interesting topic in recent years, since they provide a direct source of information on the Cabibbo–Kobayashi–Maskawa matrix elements. In the leptonic or nonleptonic weak decays of B or D mesons, the decay constants play an important role. Further, the decay constant plays an essential role in the neutral D–$\bar{D}$ or B–$\bar{B}$ mixing process.

Up until now, the only experimentally obtained values of the decay constants are those of F_{D^+} and F_{D_s}. The first value is $F_{D^+} = 300^{+180+80}_{-150-40}$ MeV by BES [1], with very large uncertainties. The experimental values of F_{D_s} have been obtained from both $D_s \to \mu\nu_\mu$ and $D_s \to \tau\nu_\tau$ branching fractions by many experimental collaborations (Refs. [2–11]). They are shown in Table 1. The central values from various experiments range from 194 to 430 MeV. The experimental uncertainties in each experiment are large, even in the most

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Table 1
Summary of the experimental determinations of the decay constant F_{D_s}

Ref.	F_{D_s} (MeV)
[2] WA75 1993	$232 \pm 45 \pm 52$
[3] CLEO I 1994	$344 \pm 37 \pm 52 \pm 42$
[4] E653 1996	$194 \pm 35 \pm 20 \pm 14$
[5] L3 1997	$309 \pm 58 \pm 33 \pm 38$
[6] DELPHI 1997	330 ± 95
[7] BES 1998	$430^{+150}_{-130} \pm 40$
[8] CLEO II 1998	$280 \pm 19 \pm 28 \pm 34$
[9] BEATRICE 2000	$323 \pm 44 \pm 12 \pm 34$
[10] OPAL 2001	$286 \pm 44 \pm 41$
[11] ALEPH 2002	$285 \pm 19 \pm 40$

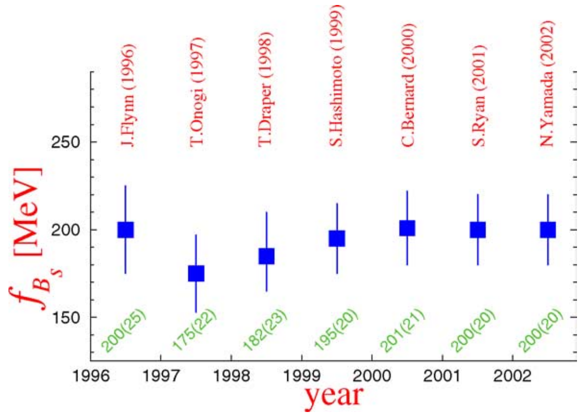

Fig. 1. World average of the quenched lattice estimates of F_{B_s}.

recent measurement, by ALEPH [11] ($F_{D_s} = 285 \pm 19 \pm 40$ MeV), which has the smallest uncertainty. Further, also in ALEPH's measurement, the contribution from the decay $D_s \to \mu\nu_\mu\gamma$ is ignored. Unlike the tree level case which is Helicity-suppressed, this radiative decay does not have the Helicity suppression. Therefore, this radiative decay may contribute several per cent to the branching ratio [12], and may thus cause a sizeable change in the value of the decay constant F_{D_s}. Fortunately, new experiments such as Belle, BaBar, Tevatron Run II and CLEO-c will give us a wealth of precision data for B and D mesons soon, and will determine the decay constants to a few per cent.

Many theoretical groups are working on the calculation of the decay constants, using different methods, for example, lattice quantum chromodynamics (QCD), QCD sum rules, and the potential model. Here we give a few comments especially on the method of lattice QCD, since it is a method that gives a means for performing first principles calculations starting from QCD. In Fig. 1 (taken from Ref. [13]), as an example, the world average of the quenched lattice results for F_{B_s} [14,15] is shown. From this figure, we notice that they give a stable estimate of the decay constant F_{B_s} over several years. Since the quenched calculations have an unavoidable uncertainty due to quenching and they assume approximately the same quenching error, their uncertainties still remain unchanged and not small over the last several years of work. More precise predictions are still not available. Decay constants of other mesons calculated by lattice methods face the same problem as F_{B_s}, the uncertainties are still large. Precise experimental results with uncertainties of only a few per cent will be obtained soon. Therefore, more precise calculations with different models are and will continue to be needed.

In this Letter, we present results of a relativistic calculation of decay constants in the framework of full Salpeter equation. The full Salpeter equation is a relativistic equation describing a bound state. Since this method has a very solid basis in quantum field theory, it is very good in describing a bound state which is a relativistic system. In a previous paper [16], we solved the instantaneous Bethe–Salpeter equation [17], which is also called full Salpeter equation [18]. After we solved the full Salpeter equation, we obtained the relativistic wave function of the bound state. We used this wave function to calculate the average kinetic energy of the heavy quark inside a heavy meson in 0^- state, and obtained values which agree very well with recent experiments. We also found there that the relativistic corrections are quite large and cannot be ignored [16]. In this Letter we use this method to predict the values of decay constants of heavy mesons in 0^- state.

2. Decay constants of 0^- state

In this section, we will calculate the decay constants of heavy mesons in 0^- state by using the full Salpeter method. In the previous paper [16], we wrote the relativistic wave function of 0^- state as:

$$\varphi_{^1S_0}(\vec{q}) = \Bigg[\not{P}\varphi_1(\vec{q}) + M_H\varphi_2(\vec{q}) - \not{q}_\perp\varphi_2(\vec{q})\frac{M_H(\omega_Q-\omega_q)}{(m_q\omega_Q+m_Q\omega_q)} + \not{q}_\perp\not{P}\varphi_1(\vec{q})\frac{(\omega_Q+\omega_q)}{(m_q\omega_Q+m_Q\omega_q)}\Bigg]\gamma_5, \tag{1}$$

where m_Q, m_q and M_H are the masses of the heavy quark, light quark, and the corresponding heavy meson; p_Q and p_q are the momenta of the constituent quarks, and P the total momentum of the heavy meson. q is the relative momentum of the meson defined as

$$q \equiv p_Q - \alpha_1 P \equiv \alpha_2 P - p_q,$$

where

$$\alpha_1 \equiv \frac{m_Q}{m_Q+m_q}, \qquad \alpha_2 \equiv \frac{m_q}{m_Q+m_q},$$

the ω_Q and ω_q are defined as

$$\omega_Q \equiv \sqrt{m_Q^2 - q_\perp^2}, \qquad \omega_q \equiv \sqrt{m_q^2 - q_\perp^2},$$

where the orthogonal part $q_\perp$ of momentum q is defined as

$$q^\mu = q_\parallel^\mu + q_\perp^\mu,$$
$$q_\parallel^\mu \equiv \left(P\cdot q/M_H^2\right)P^\mu, \quad q_\perp^\mu \equiv q^\mu - q_\parallel^\mu.$$

In the center-of-mass system of the heavy meson, $q_\parallel$ and $q_\perp$ turn out to be the usual components $(q_0, \vec{0})$ and $(0, \vec{q})$, and $\omega_Q = (m_Q^2 + \vec{q}^{\,2})^{1/2}$ and $\omega_q = (m_q^2 + \vec{q}^{\,2})^{1/2}$. Wave functions $\varphi_1(\vec{q})$ and $\varphi_2(\vec{q})$ will fulfill the normalization condition

$$\int \frac{d\vec{q}}{(2\pi)^3} 4\varphi_1(\vec{q})\varphi_2(\vec{q})M_H^2\left\{\frac{\omega_Q-\omega_q}{m_Q-m_q} + \frac{m_Q-m_q}{\omega_Q-\omega_q} + \frac{2\vec{q}^{\,2}(\omega_Q m_Q+\omega_q m_q)}{(\omega_Q m_q+\omega_q m_Q)^2}\right\} = 2M_H, \tag{2}$$

and they are the eigenfunctions of the heavy meson obtained by solving the full Salpeter equation, which is the instantaneous approximation of the Bethe–Salpeter equation:

$$(\not{p}_Q - m_Q)\chi(q)(\not{p}_q + m_q) = i\int\frac{d^4k}{(2\pi)^4}V(p,k,q)\chi(k). \tag{3}$$

The relation between the instantaneous wave function $\varphi_{^1S_0}(\vec{q})$ and the Bethe–Salpeter wave function $\chi(q)$ is $\varphi_{^1S_0}(\vec{q}) \equiv i\int\frac{dq_0}{2\pi}\chi(q)$. In our calculation, Cornell potential, a linear scalar interaction plus a vector interaction is chosen as the instantaneous interaction kernel V:

$$V(\vec{q}) = V_s(\vec{q}) + \gamma_0\otimes\gamma^0 V_v(\vec{q}),$$
$$V_s(\vec{q}) = -\left(\frac{\lambda}{\alpha}+V_0\right)\delta^3(\vec{q}) + \frac{\lambda}{\pi^2}\frac{1}{(\vec{q}^{\,2}+\alpha^2)^2},$$
$$V_v(\vec{q}) = -\frac{2}{3\pi^2}\frac{\alpha_s(\vec{q})}{(\vec{q}^{\,2}+\alpha^2)}. \tag{4}$$

The coupling constant $\alpha_s(\vec{q})$ is running:

$$\alpha_s(\vec{q}) = \frac{12\pi}{27}\frac{1}{\log(a+\frac{\vec{q}^{\,2}}{\Lambda_{\mathrm{QCD}}^2})}.$$

Here the constants λ, α, a, V_0 and Λ_{QCD} are the parameters that characterize the potential. In Ref. [16], we obtained the following best fit values of the input parameters by fitting the mass spectra for heavy mesons of 0^- states:

$$a = e = 2.7183, \qquad \alpha = 0.06\ \mathrm{GeV},$$
$$V_0 = -0.60\ \mathrm{GeV}, \qquad \lambda = 0.2\ \mathrm{GeV}^2,$$
$$\Lambda_{\mathrm{QCD}} = 0.26\ \mathrm{GeV} \quad \text{and}$$
$$m_b = 5.224\ \mathrm{GeV}, \qquad m_c = 1.7553\ \mathrm{GeV},$$
$$m_s = 0.487\ \mathrm{GeV}, \qquad m_d = 0.311\ \mathrm{GeV},$$
$$m_u = 0.305\ \mathrm{GeV}.$$

With this parameter set, we solved the full Salpeter equation and obtained the eigenvalues and the eigenfunction of the ground heavy 0^- states. We will not show here how the full Salpeter equation is solved and what the calculated mass spectra are, interested reader can find them in Ref. [16]. We can use the obtained eigenfunction of heavy mesons to calculate the decay constant F_P. The decay constant is defined as

$$\langle 0|\bar{q}_1\gamma_\mu\gamma_5 q_2|P\rangle \equiv iF_P P_\mu, \tag{5}$$

which can be written in the language of the Salpeter

Table 2
Decay constants of heavy 0^- meson (in GeV) as predicted by the relativistic Salpeter method

F_{B_c}	F_{B_s}	F_{B_d}	F_{B_u}	F_{η_c}	F_{D_s}	F_{D_d}	F_{D_u}
322	216	197	196	292	248	230	230

Table 3
The theoretical relative uncertainties, obtained as explained in the text, in per cents (%)

	B_c	B_s	B_d	B_u	η_c	D_s	D_d	D_u
$\Delta F_P/F_P$	± 13	± 15	± 15	± 15	± 8.6	± 11	± 11	± 11

Table 4
Recent calculations by other methods. Here PM means potential model, BS means Bethe–Salpeter method, QL means quenched lattice, AQL means average quenched lattice, UL means unquenched lattice, AUL means averaged unquenched lattice, QSR means QCD sum rules. In Ref. [21], the uncertainties are statistical, systematic within the $N_f = 2$ partially quenched approximation, the systematic errors due to partial quenching and the missing virtual strange quark, and an estimate of the effect of chiral logarithms, respectively. In Ref. [23], the uncertainties are from statistics, chiral extrapolation and systematics

Ref.	F_{B_s}	F_{B_d} or F_{B_u}	F_{D_s}	F_{D_d} or F_{D_u}
PM [19]	196 ± 20	178 ± 15	266 ± 25	243 ± 25
BS [20]		192		
QL [21]	$217(6)(^{+32}_{-28})(^{+9}_{-3})(^{+17}_{-0})$	$190(7)(^{+24}_{-17})(^{+11}_{-2})(^{+8}_{-0})$	$241(5)(^{+27}_{-26})(^{+9}_{-4})(^{+5}_{-0})$	$215(6)(^{+16}_{-15})(^{+8}_{-3})(^{+4}_{-0})$
QL [22]			252 ± 9	
AQL [15]	200 ± 20	173 ± 23	230 ± 14	203 ± 14
UL [23]		190(14)(07)(19)		
AUL [15]	230 ± 30	198 ± 30	250 ± 30	226 ± 15
QSR [24]	236 ± 30	203 ± 23	235 ± 24	204 ± 20
QSR [25]		206 ± 20		195 ± 20

wave functions as:

$$\langle 0|\bar{q}\gamma_\mu\gamma_5 Q|P\rangle = i\sqrt{N_c}\int \mathrm{Tr}\left[\gamma_\mu(1-\gamma_5)\varphi_{{}^1S_0}(\vec{q})\frac{d\vec{q}}{(2\pi)^3}\right] = i4\sqrt{N_c}P_\mu\int\frac{d\vec{q}}{(2\pi)^3}\varphi_1(\vec{q}). \tag{6}$$

Therefore, we have

$$F_P = 4\sqrt{N_c}\int\frac{d\vec{q}}{(2\pi)^3}\varphi_1(\vec{q}), \tag{7}$$

and the calculated values of decay constants are displayed in Table 2.

In Table 3, we show the theoretical uncertainties of our results for the decay constants. These uncertainties are obtained by varying all the input parameters simultaneously within ±10% of the central values, and taking the largest variation of the decay constant.

In Table 4, for comparison, we show recent theoretical predictions for the decay constants as obtained by other methods. For example, we display the recent values from relativistic potential model (PM) [19] based on the quasi-potential approach; most recent value of F_B from another version of using Bethe–Salpeter method (BS) [20], which is also a relativistic result; recent values from the averaged lattice results both in quenched (AQL) and unquenched (AUL) approximation [15]; most recent values from quenched lattice (QL) QCD [21,22] and unquenched lattice (UL) QCD [23]; and values from QCD sum rules (QSR) [24,25]. As can be seen from Tables 2 and 4, our values of the decay constants by solving the Salpeter equation, agree with these recent results by other methods. In particular, they agree very well with the recent average of the unquenched lattice QCD (AUL) [15]. Our value $F_{D_s} \approx 248$ GeV is smaller than the most recent experimental central value, the ALEPH's value

Table 5
Ratios F_{B_s}/F_{B_d}, F_{D_s}/F_{D_d} and the Grinstein ratio by this work and by other methods. In Ref. [26], the first and second uncertainty are the statistical and the systematic errors

Ref.	F_{B_s}/F_{B_d}	F_{D_s}/F_{D_d}	R_1
This work	1.10 ± 0.01	1.08 ± 0.01	1.02 ± 0.02
PM [19]	1.10 ± 0.21	1.09 ± 0.22	1.01 ± 0.40
QL [21]	$1.16(1)(2)(2)(^{+4}_{-0})$	$1.14(1)(^{+2}_{-3})(3)(1)$	$1.02(2)(4)(4)(^{+4}_{-1})$
UL [26]			$1.018 \pm 0.006 \pm 0.010$
AQL [15]	1.15 ± 0.03	1.12 ± 0.02	1.03 ± 0.05
AUL [15]	1.16 ± 0.05	1.12 ± 0.04	1.04 ± 0.08
QSR [27]	1.16 ± 0.05	1.15 ± 0.04	1.01 ± 0.08

$F_{D_s} \approx 285 \pm 19 \pm 40$, but still within the experimental uncertainties.

There are other interesting quantities such as the ratios of decay constants F_{B_s}/F_{B_d}, F_{D_s}/F_{D_d}, and the Grinstein ratio [28] defined as

$$R_1 = \left(\frac{F_{B_s}}{F_{B_d}}\right) \Big/ \left(\frac{F_{D_s}}{F_{D_d}}\right), \tag{8}$$

which is a quantity sensitive to the light quark mass m_s. In Table 5 we show our values of these ratios and some values obtained by other methods in recent literature. Our uncertainties come from the aforementioned $\pm 10\%$ changes of the parameters. The uncertainties of the ratios of decay constants of Ref. [19] are large. This is so because the authors of Ref. [19] did not give the uncertainties for these ratios. We estimated the uncertainties of these ratios on the basis of their given uncertainties of the decay constants. In the same way we estimated the uncertainties of the Grinstein ratio of other references shown in Table 5, with the exception of those of Ref. [26]. From Table 5 one can see that our values of ratios F_{B_s}/F_{B_d} and F_{D_s}/F_{D_d} agree with these recent theoretical results. In particular, our central values are very close to those of the relativistic potential model [19], and our central value of the Grinstein ratio $R_1 = 1.02$ agrees well with the values estimated by other methods.

In conclusion, we calculated the decay constants of heavy 0^- mesons by means of the relativistic Salpeter method. We obtained $F_{D_s} \approx 248 \pm 27$, $F_D \approx 230 \pm 25$ $(D^0, D^\pm)$, $F_{B_s} \approx 216 \pm 32$, $F_B \approx 196 \pm 29$ $(B^0, B^\pm)$, $F_{B_c} \approx 322 \pm 42$ and $F_{\eta_c} \approx 292 \pm 25$ MeV.

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