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Design of fractional order differentiator using type-III and type-IV discrete cosine transform

Manjeet Kumar *, Tarun Kumar Rawat

Department of Electronics and Communication Engineering, Bennett University, Greater Noida, Uttar Pradesh 201310, India
Department of Electronics and Communication Engineering, Netaji Subhas Institute of Technology, Sector-3, Dwarka, New Delhi 110078, India

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A B S T R A C T

In this paper, an interpolation method based on discrete cosine transform (DCT) is employed for digital finite impulse response-fractional order differentiator (FIR-FOD) design. Here, a fractional order digital differentiator is modeled as finite impulse response (FIR) system to get an optimized frequency response that approximates the ideal response of a fractional order differentiator. Next, DCT-III and DCT-IV are utilized to determine the filter coefficients of FIR filter that compute the Fractional derivative of a given signal. To improve the frequency response of the proposed FIR-FOD, the filter coefficients are further modified using windows. Several design examples are presented to demonstrate the superiority of the proposed method. The simulation results have also been compared with the existing FIR-FOD design methods such as DFT interpolation, radial basis function (RBF) interpolation, DCT-II interpolation and DST interpolation methods. The result reveals that the proposed FIR-FOD design technique using DCT-III and DCT-IV outperforms DFT interpolation, RBF interpolation, DCT-II interpolation and DST interpolation methods in terms of magnitude error.

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1. Introduction

Fractional calculus was invented by Leibnitz in 1695 [1,2], but have only recently focused on the theory and applications, particularly in the areas of science and engineering. The field of fractional calculus has maintained tremendous vitality over the past few decades and there is a clear indication that this trend will continue. The fractional calculus has been used to describe many phenomena in almost all fields such as fluid dynamics [2], physics [3], automatic control [2], image processing [1], electromagnetism [4], signal processing [5] and chaotic systems [6]. The computational complexity involved with fractional calculus make the early research difficult. Advances in computational capability offer the practical implementation of fractional calculus. New applications continue to be found and the existing applications continue to be expend in diverse area of science and engineering. Some existing applications of fractional derivative are described below. The fractional derivative can be applied in linear prediction of speech signal [7], image denoising, signature verification, edge detection and texture enhancement of image signal [8,9], R wave detection of ECG signal [10], one dimensional (1-D) linear phase filter design [11] and two dimensional (2-D) linear phase filter design [12], design of multiplier-less filter [13], quadrature mirror filter bank design [14], and FIR filter design [15].

Digital differentiators are used to find the time derivative of the input signal. For real time applications it is mandatory that a differentiator should have smaller order for easier implementation. Fractional order digital differentiator is an extended version of integer order differentiator which provide more flexibility in real applications [1]. Fractional order calculus is a generalization of our traditional integral and differential equations. The integer order derivative $D^n f(x) = \frac{d^n f(x)}{dx^n}$ (nth order derivative of the function $f(x)$) has been generalized to fractional order derivative $D^\alpha f(x) = \frac{d^\alpha f(x)}{dx^\alpha}$, where n is an integer and α is a real number [2].

From last few decades, many techniques have been put forward for the design of fractional order differentiators [16]. Samadi et al. presented a FOD for polynomial signal using Newton series expansion. This method gives exact fractional derivative of polynomial signal [17]. Tseng applied the logarithm and Taylor series expansion to approximate the variable fractional order integrator and differentiator [18]. Tseng also presented a FOD design using fractional sample delay and design accuracy was improved in the high frequency region [19]. Tseng and Lee investigated the design of FOD using interpolation techniques such as radial basis function [20], DFT interpolation [21], DCT interpolation [22], and DST

* Corresponding author.
E-mail addresses: manjeetchhillar@gmail.com (M. Kumar), tarundsp@gmail.com (T.K. Rawat).

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interpolation [23] to approximate the fractional derivative of a signal. Chen et al. reported a fractional order Savitzky-Golay differentiator for estimating the fractional order derivative of a contaminated signal [24]. Khare et al. applied DCT-III interpolation to design FOD [25]. Kumar and Rawat reported the windowing function based approximation of fractional order differentiator with radial basis function [26]. Kumar and Rawat also introduced the design of fractional order differentiator using power series and least squares method where FOD is modelled as an FIR system that gives fractional order derivative of the Grunwald-Letnikov type for a power function [27]. Further, DCT, DST, DFT and DHT have been applied for the design of matrix fractional order differentiators [28,29].

In recent years, there has been an increasing interest with a class of orthogonal transforms in general area of digital signal processing. The discrete cosine transform (DCT) [30] is a sinusoidal unitary transform which has been applied to many applications of signal processing such as filter design and multi-rate digital signal processing. Some of the popular applications of DCT for the designing of adaptive filter [31], ECG compression, speech enhancement and channel prediction [32], signal compression, image coding and compression [33], very large scale (VLSI) implementation [34] etc. DCT is robust approximation which can be implemented very efficiently and comparable with that of the Karhunen-Loeve transform (KLT), which is considered to be optimal [35].

This paper is dedicated to the implementation of discrete cosine transform for the designing of fractional order differentiator. Grünwald-Letnikov definition of fractional derivative along with DCT interpolation is used to approximate the impulse response of an ideal fractional order differentiator. The rationale behind this work is to improve the design accuracy and performance of the designed FIR-FOD. This is achieved by applying DCT-III and DCT-IV to compute the fractional derivative of the given input signal. Further, the performance of the designed FIR-FOD is improved using window methods. The main contribution of this paper is to implement transform method for the design of FIR-FOD. The performance of the proposed FIR-FOD is compared with existing interpolation based FIR-FOD design methods namely DFT interpolation, radial basis function (RBF) interpolation, DCT-II interpolation and DST interpolation methods. Furthermore, analysis of magnitude error and phase error is carried out to justify the superiority of the proposed FIR-FOD.

The paper is organized as follows: Following a detail survey in Section 1, Section 2 presents the brief review of the definitions of fractional derivatives. In Section 3, mathematical articulation to compute the fractional derivative using DCT-III has been presented. Also, the design of FIR-FOD using DCT-III and DCT-IV is described. The design examples to illustrate the effectiveness of the proposed FIR-FOD using DCT-III and DCT-IV are given in Section 4. Finally, the paper is concluded in Section 5.

2. Review of fractional derivative

In this section, some basic concepts and commonly used definitions of the fractional order differentiator are reviewed briefly. Let us start with some basic concepts of fractional calculus commonly used in fractional order differentiator. The unique feature of fractional calculus is its ability to generalize the integral and differential operators to noninteger order. The generalized continuous integral-differential Davis operator is given by [1,2]

$$ {}_aD_t^\alpha = \begin{cases} \frac{d^\alpha}{dt^\alpha}, & \alpha > 0 \\ 1, & \alpha = 0 \\ \int_a^t (d\tau)^\alpha, & \alpha < 0 \end{cases} \tag{1} $$

where ${}_aD_t^\alpha$ denotes integral-differential operator to calculate the αth order fractional differentiation and integration of the input signal with respect to t and a is the initial condition of the operation. The result of fractional derivatives depends on the bounds a and t. Most commonly used definitions for fractional order differentiation and integral are Riemann–Liouville (R–L), Grünwald–Letnikov (G–L) and the Caputo definitions [1,2]. In this paper, the Grünwald–Letnikov definition of a fractional derivative is used to compute the fractional order differentiation of $f(t)$, which is given by

$$ {}_aD_t^\alpha f(t) = \lim_{\Delta \to 0} \sum_{k=0}^{[(t-a)/\Delta]} \frac{(-1)^k C_k^\alpha}{\Delta^\alpha} f(t - k\Delta) \tag{2} $$

where C_k^α is the binomial coefficient. The value of C_k^α is given by using the relation between Euler's Gamma function and factorial, defined as

$$ C_k^\alpha = \binom{\alpha}{k} = \frac{\Gamma(\alpha+1)}{\Gamma(k+1)\Gamma(\alpha-k+1)} \tag{3} $$

$$ = \begin{cases} 1 & k = 1 \\ \frac{\alpha(\alpha-1)(\alpha-2)\cdots(\alpha-k+1)}{1\cdot 2\cdot 3\cdots k} & k \geqslant 1 \end{cases} \tag{4} $$

where $\Gamma(\cdot)$ is the gamma function. From the above definition, the fractional derivative of a trigonometric function [36] is given by

$$ D^\alpha A\cos(\omega t + \phi) = A\omega^\alpha \cos\left(\omega t + \phi + \frac{\pi}{2}\alpha\right) \tag{5} $$

This fractional derivative of trignometric functions is used to compute the fractional derivative of a given digital signal using DCT.

3. Design of fractional order differentiator

In this section, DCT method is presented to compute fractional derivative $D^\alpha f(t)$ of a continuous time signal $f(t)$. There are four types of DCT, DCT-I through DCT-IV, which differ in the boundary conditions at the ends of the interval. Here, we have implemented the DCT-III which is same as discrete symmetric cosine transform (DSCT) with a specific preprocessing of input data with less computational complexity than DCT in terms of multiplication and slightly more additions.

3.1. Fractional derivative using DCT-III

Given the discrete-time sequence $f(0), f(1), \ldots, f(N-1)$ which are sampled from continuous-time signal $f(t)$. The DCT-III is defined as [37]

$$ F(k) = \sqrt{\frac{2}{N}} \sum_{m=0}^{N-1} c_k f(m) \cos\left(\frac{\pi(2k+1)m}{2N}\right) \tag{6} $$

$$ f(n) = \sqrt{\frac{2}{N}} \sum_{k=0}^{N-1} c_k F(k) \cos\left(\frac{\pi(2k+1)n}{2N}\right) \tag{7} $$

where

$$ c_k = \begin{cases} \frac{1}{\sqrt{2}} & k = 0 \\ 1 & \text{otherwise} \end{cases} \tag{8} $$

Substituting the $F(k)$ from Eq. (6) into Eq. (7), we obtain

$$ \begin{aligned} f(n) &= \sqrt{\frac{2}{N}} \sum_{k=0}^{N-1} c_k \left[\sqrt{\frac{2}{N}} \sum_{m=0}^{N-1} c_k f(m) \cos\left(\frac{\pi(2k+1)m}{2N}\right) \right] \cos\left(\frac{\pi(2k+1)n}{2N}\right) \\ &= \sum_{m=0}^{N-1} f(m) \frac{2}{N} \sum_{k=0}^{N-1} c_k^2 \cos\left(\frac{\pi(2k+1)m}{2N}\right) \cos\left(\frac{\pi(2k+1)n}{2N}\right) \end{aligned} \tag{9} $$

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Replaceing discrete-time variable n with continuous-time variable t, to obtain $f(t)$ which is given by

$$f(t) = \sum_{m=0}^{N-1} f(m) \frac{2}{N} \sum_{k=0}^{N-1} c_k^2 \cos\left(\frac{\pi(2k+1)m}{2N}\right) \cos\left(\frac{\pi(2k+1)t}{2N}\right)$$

$$f(t) = \sum_{m=0}^{N-1} f(m) b(m,t) \tag{10}$$

where interpolation basis is given by

$$b(m,t) = \frac{2}{N} \sum_{k=0}^{N-1} c_k^2 \cos\left(\frac{\pi(2k+1)m}{2N}\right) \cos\left(\frac{\pi(2k+1)t}{2N}\right) \tag{11}$$

Apply the αth order fractional derivative on both sides of Eq. (10), we get

$$D^\alpha f(t) = \sum_{m=0}^{N-1} f(m)[D^\alpha b(m,t)] \tag{12}$$

Using linear property of the fractional differentiation along with Eqs. (5) and (12), we get

$$D^\alpha b(m,t) = \frac{2}{N} \sum_{k=0}^{N-1} c_k^2 \left(\frac{\pi(2k+1)}{2N}\right)^\alpha \cos\left(\frac{\pi m(2k+1)}{2N}\right) \cos\left(\frac{\pi t(2k+1)}{2N} + \frac{\pi}{2}\alpha\right) \tag{13}$$

From Eqs. (12) and (13), we get

$$D^\alpha f(t) = \sum_{m=0}^{N-1} f(m) q_m(t) \tag{14}$$

where

$$q_m(t) = \frac{2}{N} \sum_{k=0}^{N-1} c_k^2 \left(\frac{\pi(2k+1)}{2N}\right)^\alpha \cos\left(\frac{\pi m(2k+1)}{2N}\right) \cos\left(\frac{\pi t(2k+1)}{2N} + \frac{\pi}{2}\alpha\right) \tag{15}$$

The result obtained in Eq. (14) is applied to design fractional order differentiator in the following section.

3.2. Fractional order differentiator design

The frequency response of an ideal fractional order differentiator is given by

$$H_{id}(\omega) = (j\omega)^\alpha e^{-j\omega I} \tag{16}$$

where I is a prescribed delay. In this section, the aim is to obtain the transfer function of fractional order differentiator by using the results of Eq. (14) whose frequency response approximates the ideal response. The transfer function of a FIR filter is given by

$$H(z) = \sum_{r=0}^{N-1} h(r) z^{-r} \tag{17}$$

If a signal $g(n)$ is applied at the input of this FIR filter then its output is the weighted average of the integer delayed samples $g(n), g(n-1), g(n-2), \ldots, g(n-N+1)$, which is given as

$$y(n) = \sum_{r=0}^{N-1} h(r) g(n-r) \tag{18}$$

Now, the objective is to compute the filter coefficients $h(r)$ such that the filter output $y(n)$ matches the delayed fractional derivative $D^\alpha g(n-I)$, that is

$$y(n) \simeq D^\alpha g(n-I) = \sum_{r=0}^{N-1} h(r) g(n-r) \tag{19}$$

To solve this problem index mapping technique is applied by choosing

$$\begin{aligned} g(n) &= f(N-1) \\ g(n-1) &= f(N-2) \\ &\vdots \\ g(n-N+1) &= f(0) \end{aligned} \tag{20}$$

In general, it can be written as

$$f(m) = g(n-(N-1)+m) \quad 0 \leqslant m \leqslant N-1 \tag{21}$$

By using index mapping technique, Eqs. (14) and (18) are related with each other.

Substituting Eq. (21) and $f(t) = g(n-(N-1)+t)$ into Eq. (14), we get

$$D^\alpha g(n-(N-1)+t) = \sum_{m=0}^{N-1} g(n-(N-1)-m) q_m(t) \tag{22}$$

Let $r = (N-1)-m$, Eq. (22) becomes

$$D^\alpha g(n-(N-1)+t) = \sum_{r=0}^{N-1} g(n-r) q_{N-1-r}(t) \tag{23}$$

Furthermore, let $I = (N-1)-t$, Eq. (23) reduces to

$$D^\alpha g(n-I) = \sum_{r=0}^{N-1} g(n-r) q_{N-1-r}(N-1-I) \tag{24}$$

Comparing Eq. (24) with Eq. (18), we get

$$D^\alpha g(n-I) = \sum_{r=0}^{N-1} h(r) g(n-r) = \sum_{r=0}^{N-1} g(n-r) q_{N-1-r}(N-1-I) \tag{25}$$

where

$$h(r) = q_{N-1-r}(N-1-I) \tag{26}$$

Substituting Eq. (15) into Eq. (26), the filter coefficients are given by

$$h(r) = \frac{2}{N} \sum_{k=0}^{N-1} c_k^2 \left(\frac{\pi(2k+1)}{2N}\right)^\alpha \cos\left(\frac{\pi(N-1-r)(2k+1)}{2N}\right) \times \cos\left(\frac{\pi(N-1-I)(2k+1)}{2N} + \frac{\pi}{2}\alpha\right), \quad 0 \leqslant r \leqslant N-1 \tag{27}$$

From the above equation filter coefficients are easily computed without applying optimization techniques. Here, the length-N filter coefficients $h(r)$ can be viewed as obtained by using rectangular window. To improve the performance of the designed filter, the filter coefficients are further modified using tapered windows. For simplicity, we are using Hann window which is given as

$$w(r) = 0.5\left(1 - \cos\left(\frac{2\pi r}{N}\right)\right) \quad 0 \leqslant r \leqslant N-1 \tag{28}$$

The modified filter coefficients can be obtained by

$$h_w(r) = h(r) w(r) \tag{29}$$

3.3. Design of fractional differentiator using DCT-IV

The FIR fractional order differentiator can also be designed using other variants of DCT, one being DCT-IV which finds its application in multi-carrier modulator in frequency offset channels and image processing. In this section, DCT-IV is applied to obtain the filter coefficients using the similar approach as used in the previous section.

Given the discrete-time sequence $f(0), f(1), \ldots, f(N-1)$ which are sampled from continuous-time signal $f(t)$. The DCT-IV is defined as [37]

$$F(k) = \sqrt{\frac{2}{N}}\sum_{n=0}^{N-1} f(n)\cos\left(\frac{\pi(2k+1)(2n+1)}{4N}\right) \quad (30)$$

$$f(n) = \sqrt{\frac{2}{N}}\sum_{k=0}^{N-1} F(k)\cos\left(\frac{\pi(2k+1)(2n+1)}{4N}\right) \quad (31)$$

Using the same approach as that of DCT-III, we obtain the filter coefficients for DCT-IV. They are given by

$$h(r) = \frac{2}{N}\sum_{k=0}^{N-1}\left(\frac{\pi(2k+1)}{2N}\right)^{\alpha}\cos\left(\frac{\pi(2(N-r)-1)(2k+1)}{4N}\right) \times \cos\left(\frac{\pi(2(N-I)-1)(2k+1)}{4N} + \frac{\pi}{2}\alpha\right) \quad (32)$$

4. Design examples

In this section, the design of FIR fractional order differentiator using DCT-III and DCT-IV is presented. In order to evaluate the performance of proposed FIR-FOD using DCT-III and DCT-IV, two design examples are presented. The performance of the proposed method is measured by magnitude error, phase error and integral square error. In addition, the effect of delay value I and fractional order α on the performance of designed FIR-FOD are investigated in this section. All the simulations have been done in MATLAB 7.11 version on Intel core (TM) i5 processor, 3.20 GHz with 4 GB RAM.

Example 1. In this example, we implement transform methods, namely, DCT-III and DCT-IV to design the fractional order differentiator. The design parameters are chosen as filter length $N = 60$, filter order $\alpha = 0.5$ and delay $I = 30$. The ideal magnitude and phase response of FIR-FOD are given by ω^{α} and 90α, respectively.

4.1. DCT-III based design

Here, FIR fractional order differentiator is designed using DCT-III. First, the filter coefficients of FIR-FOD are computed using Eq. (27). Then, Hann window defined in Eq. (28) is employed to modify the computed filter coefficients. Fig. 1 depicts the magnitude response of the proposed FIR-FOD using DCT-III. The normalized phase response $90[\angle(H(e^{jw})) + \omega I]/0.5\pi$ in degree of the designed FIR-FOD using DCT-III is shown in Fig. 2.

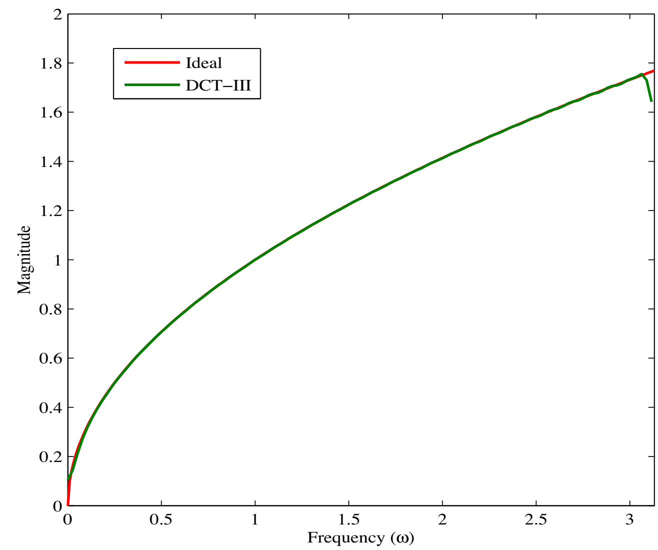

Fig. 1. Magnitude response of the designed FIR-FOD using DCT-III with $N = 60, I = 30$ and $\alpha = 0.5$.

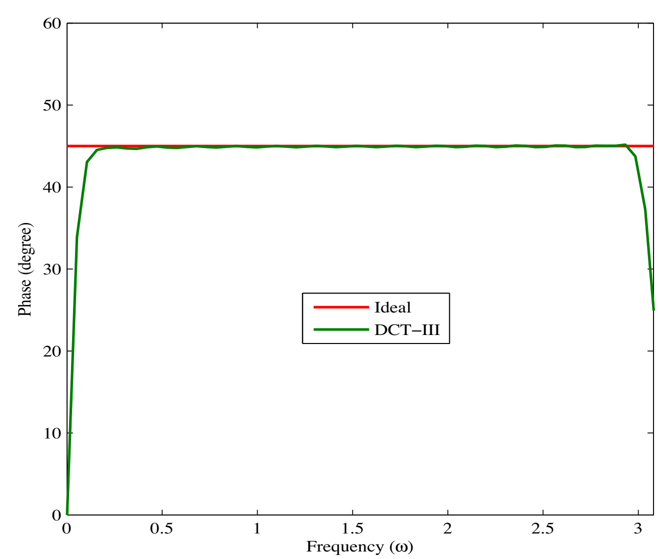

Fig. 2. Phase response of the designed FIR-FOD using DCT-III with $N = 60, I = 30$ and $\alpha = 0.5$.

4.2. DCT-IV based design

Here, FIR fractional order differentiator is designed using DCT-IV. First, the filter coefficients of FIR-FOD are computed using Eq. (32). Then Hann window defined in Eq. (28) is employed to modify the computed filter coefficients. The magnitude response of the proposed FIR-FOD using DCT-IV is demonstrated in Fig. 3. The normalized phase response of the designed FIR-FOD using DCT-IV is plotted in Fig. 4.

The magnitude and phase response of the proposed FIR-FOD using DCT-II [22] with above mentioned parameter have been plotted in Figs. 5 and 6, respectively. From the graphical results shown in Figs. 1–6, it can concluded that the proposed FIR-FOD using DCT-III outperforms the other type of DCTs in terms of magnitude error. Whereas, the comparison of the phase response reveals that DCT-IV gives slightly better results compared to others. Finally, it can be concluded that the DCT-III based FIR-FOD yields superior result in terms of magnitude response and comparable for phase response.

4.3. Comparison of the proposed FOD with the other reported works

We evaluate the performance of the proposed FIR-FOD with the existing FOD design methods, namely, frequency response approximation method [38], fractional sample delay method [19], DFT interpolation method [21], radial basis function interpolation method [20], DCT-II interpolation method [22] and DST interpolation method [23]. Here, a comparative analysis of the frequency response of FIR-FOD using different types of DCTs is demonstrated taking design parameters as $N = 60, I = 30$ and $\alpha = 0.5$. Fig. 7 depicts the comparison of the magnitude response of designed FIR-FOD based on different types of DCTs interpolation method. Comparison of the phase response of the proposed FIR-FOD based on different types of the DCTs interpolation method is shown in

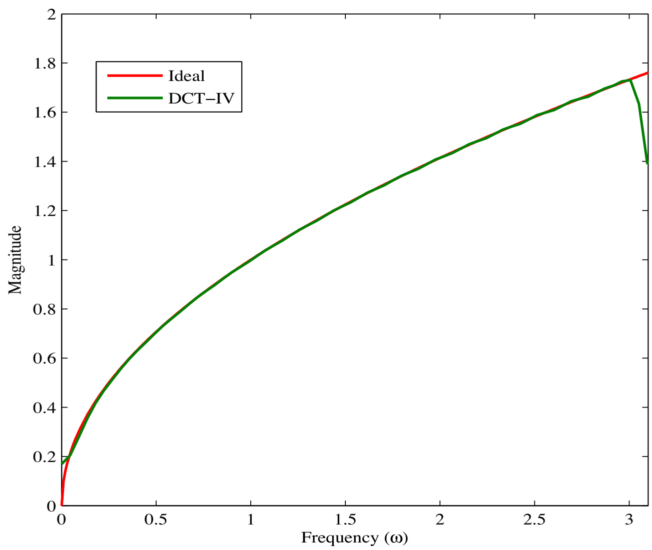

Fig. 3. Magnitude response of the designed FIR-FOD using DCT-IV with $N = 60, I = 30$ and $\alpha = 0.5$.

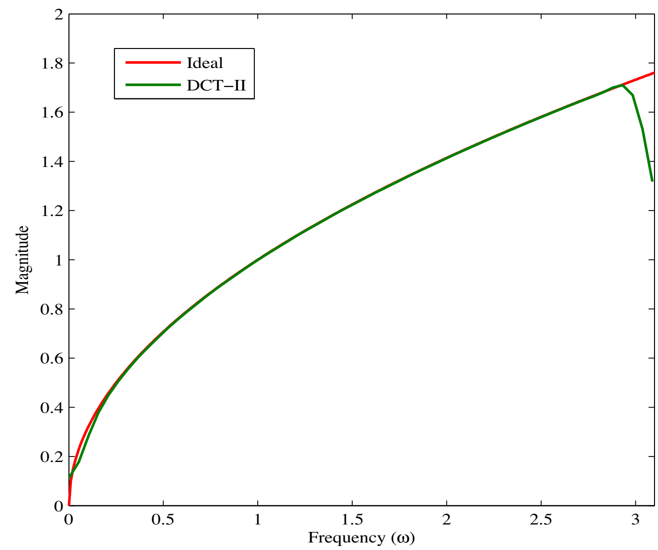

Fig. 5. Magnitude response of the designed FIR-FOD using DCT-II [22] with $N = 60, I = 30$ and $\alpha = 0.5$.

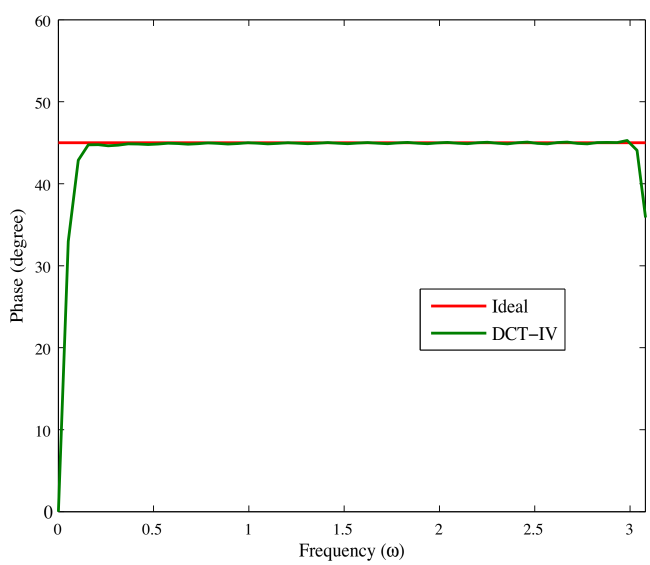

Fig. 4. Phase response of the designed FIR-FOD using DCT-IV with $N = 60, I = 30$ and $\alpha = 0.5$.

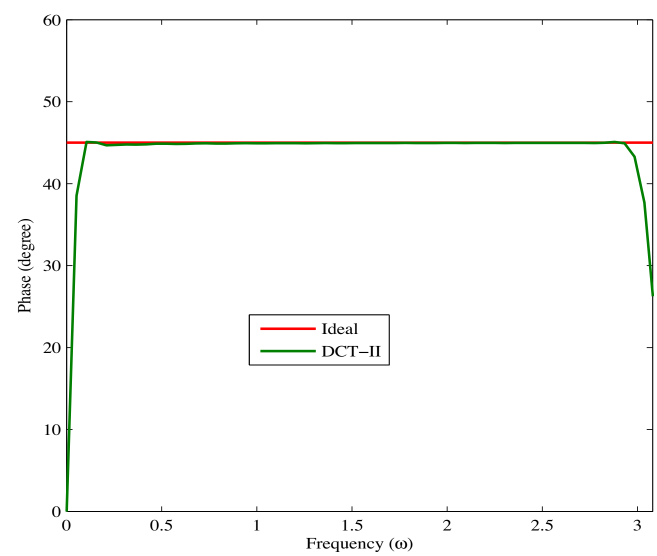

Fig. 6. Phase response of the designed FIR-FOD using DCT-II [22] with $N = 60, I = 30$ and $\alpha = 0.5$.

Fig. 8. Based on the graphical results in Fig. 7, it is evident that the proposed FIR-FOD design method using DCT-III produces smaller magnitude error in comparison to that of the other DCT interpolation based methods. It is observed from Fig. 8 that the proposed FIR-FOD design method using DCT-IV gives the best approximation to the phase response compared to other type of DCTs methods.

Table 1 summarizes the magnitude error reported by the already existing methods such as frequency response approximation method [38], fraction sample delay method [19], DFT interpolation method [21], radial basis function interpolation method [20], DCT interpolation method [22], and DST interpolation method [23]. Zhao et al. applied the frequency response approximation method for the designing of 10th order FIR-FOD and reported the minimum magnitude error of 0.623 [38]. Whereas, the proposed FIR-FOD method using DCT-III and DCT-IV observed much smaller magnitude errors. Tseng described the design of 80th order FOD by applying fractional sample delay and magnitude error of 0.2587 was attained [19]. The magnitude error achieved with the proposed method is smaller with lower order FOD design. Tseng and Lee utilized the DFT interpolation method to design 100th order FOD and error of 0.0225 is obtained [21]. Tseng and Lee reported minimum magnitude error of 0.0550, 0.0529, 0.0371 and 0.0356 for 10th, 60th, 80th and 100th order FOD, respectively, when Gaussian radial basis function interpolation is applied [20]. Tseng and Lee employed DCT interpolation for the design of 100th order FOD and minimum magnitude error reported is 0.0122 [22]. Tseng and Lee also implemented DST interpolation technique for the design of FOD of the order 100 and the error achieved is 0.0169 [23]. It is noticed from Table 1 that the magnitude error of 0.0095 and 0.0120 prevailed for DCT-III and DCT-IV, respectively, for 60th order proposed FIR-FOD design. From Table 1 and Figs. 7 and 8, it can be inferred that the proposed FOD using DCT-III is better than the other existing FODs.

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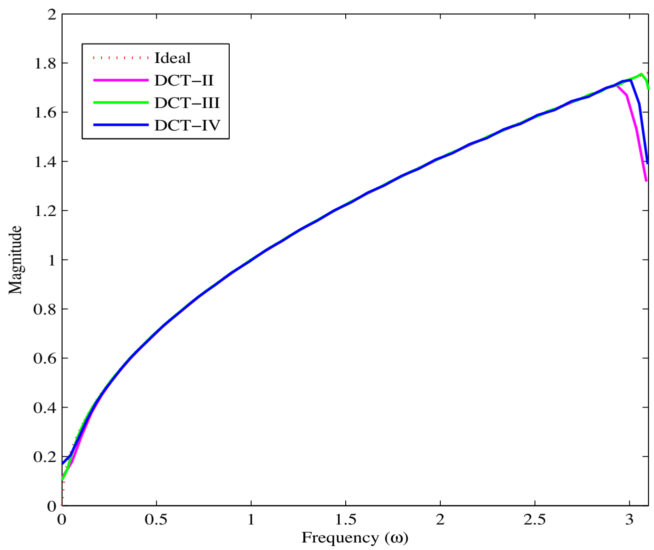

Fig. 7. Comparison of Magnitude response of FIR-FOD designed using DCT-II [22], DCT-III and DCT-IV with $N = 60, I = 30$ and $\alpha = 0.5$.

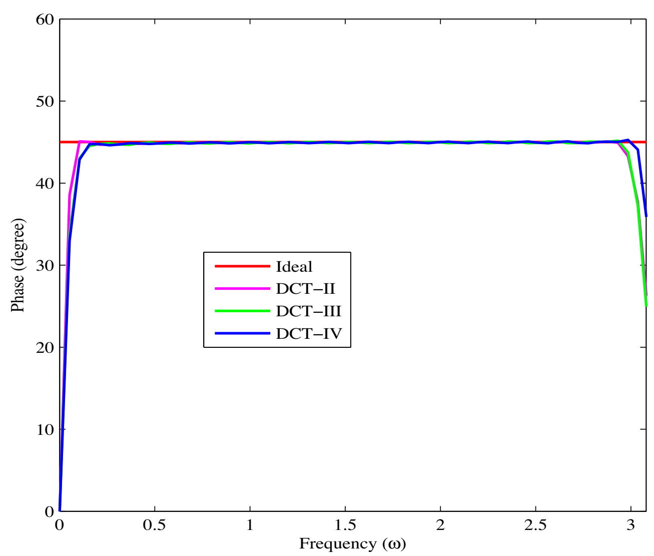

Fig. 8. Comparison of Phase response of FIR-FOD designed using DCT-II [22], DCT-III and DCT-IV with $N = 60, I = 30$ and $\alpha = 0.5$.

Example 2. In this example, we assess the performance of the proposed FIR-FOD with delay values $(30 - 50)$ and order $(0 - 1)$. Performance of the designed FIR-FOD is measured in terms of integral square root error of the frequency response which is given by

$$\epsilon = \sqrt{\int_0^{\pi} |H(\omega) - H_d(\omega)| d\omega} \tag{33}$$

First, performance of the proposed FIR-FOD is measured with different values of delay I. For this purpose, order is fixed to $\alpha = 0.5$ and $N = 80$, and the integral square root error is computed for different delay values. Fig. 9 depicts the integral square root error of the designed FIR-FOD using DCT-III with different delay values. Integral square root error of the designed FIR-FOD using DCT-IV with different delay values is shown in Fig. 10. We can observe that the error decreases for a particular range of delay, in this case the minimum error is obtained at $I = 40$. Comparison of the integral square root error of the designed FIR-FOD with DCT-II, DCT-III and DCT-IV for different delay values are shown in Figs. 11 and 12. One can infer that DCT-III based FIR-FOD design method is the best among the reported literature.

Second, performance of the proposed FIR-FOD is measured with different values of fractional order. For this purpose, order is fixed to $N = 80$ and delay $I = 40$, and the integral square root error is

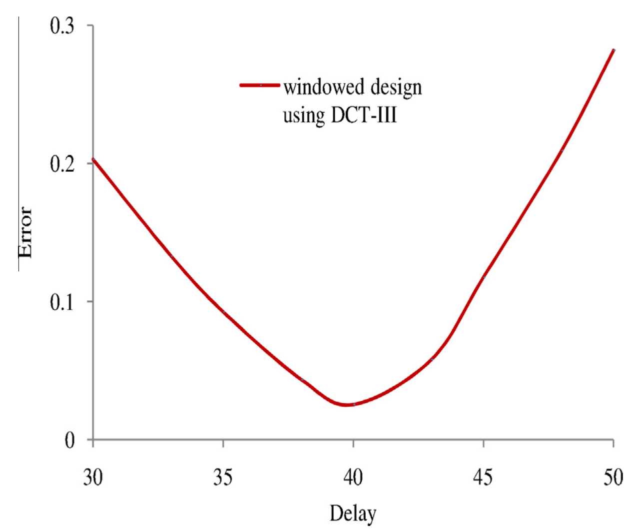

Fig. 9. The error curve ϵ of the proposed FIR-FOD using DCT-III for different delay values I.

Table 1
Comparison of magnitude error attained by other reported work.

Reference	Method	Type	Filter order	Magnitude error
Zhao et al. (2005) [38]	Frequency response approximation	–	10	0.623
Tseng (2006) [19]	Fractional sample delay	–	80	0.2587
Tseng and Lee (2010) [21]	DFT	–	100	0.0225
Tseng and Lee (2010) [20]	RBF	Gaussian	10	0.0550
	RBF	Gaussian	60	0.0529
	RBF	Inverse multiquadric	60	0.0583
	RBF	Gaussian	80	0.0371
	RBF	Gaussian	100	0.0356
Tseng and Lee (2013) [22]	DCT	Type-II	100	0.0122
Tseng and Lee (2013) [23]	DST	Type-I	100	0.0169
Present study	DCT	Type-III	60	0.0095
	DCT	Type-IV	60	0.0120

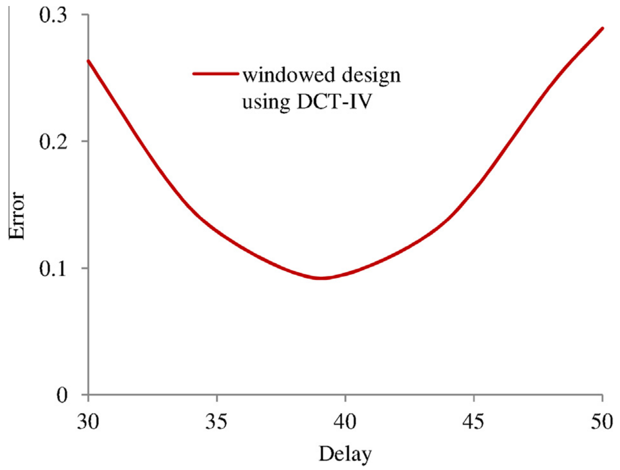

Fig. 10. The error curve ϵ of the proposed FIR-FOD using DCT-IV for different delay values *I*.

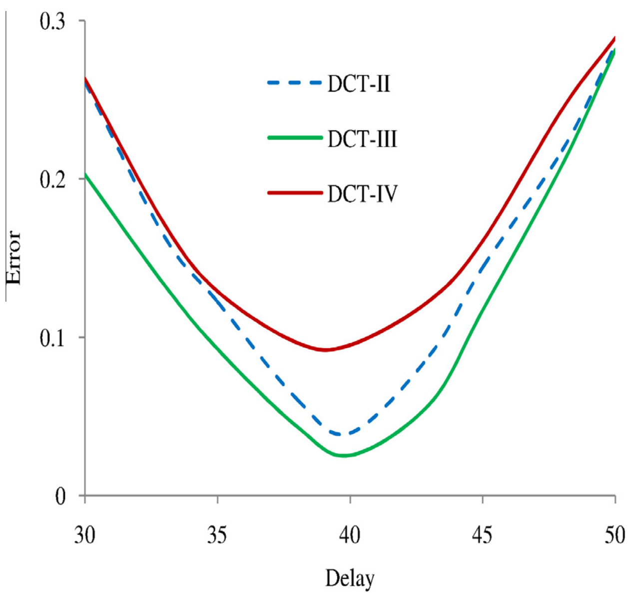

Fig. 11. Error ϵ comparison of designed FIR-FOD using DCT-III and DCT-IV with DCT-II [22] for different delay values *I*.

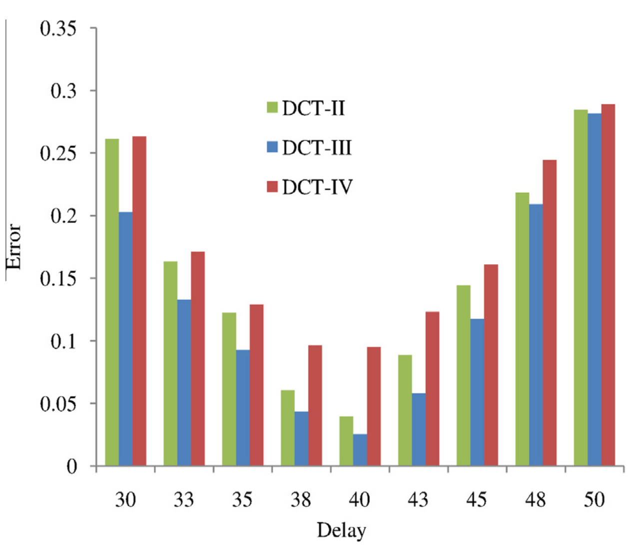

Fig. 12. Comparison of designed FIR-FOD using DCT-III and DCT-IV with DCT-II [22] in term of error ϵ for different delay values *I*.

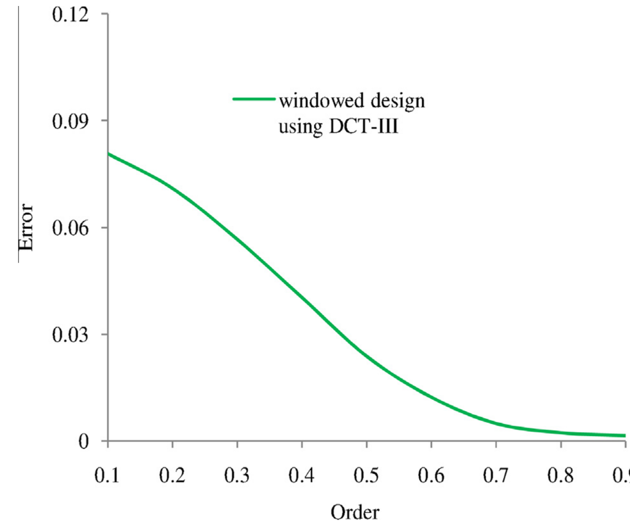

Fig. 13. The error curve ϵ of the proposed FIR-FOD using DCT-III for different fractional order values α.

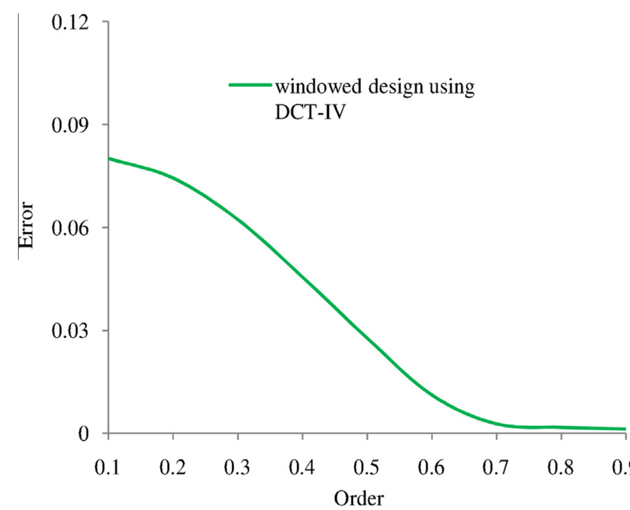

Fig. 14. The error curve ϵ of the proposed FIR-FOD using DCT-IV for different fractional order values α.

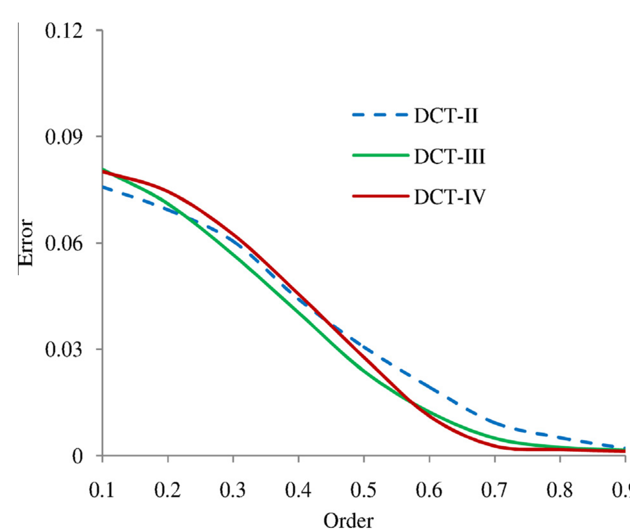

Fig. 15. Error ϵ comparison of designed FIR-FOD using DCT-III and DCT-IV with DCT-II [22] for different fractional order values α.

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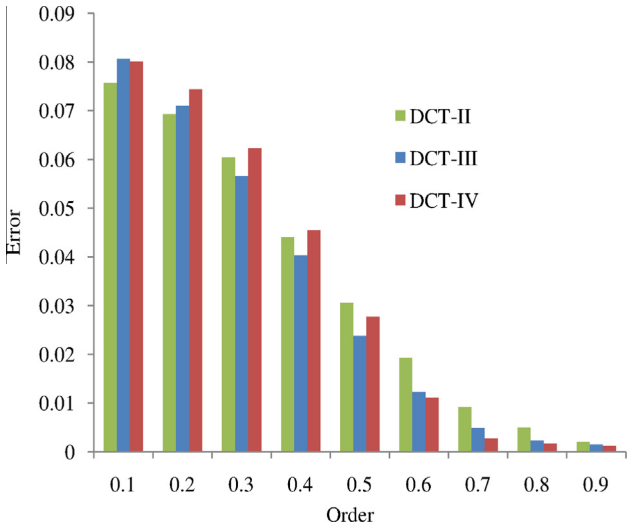

Fig. 16. Comparison of designed FIR-FOD using DCT-III and DCT-IV with DCT-II [22] in term of error ϵ for different fractional order values α.

computed with different fractional order values. Fig. 13 depicts the integral square root error of designed FIR-FOD using DCT-III with different fractional order values. The integral square root error of designed FIR-FOD using DCT-IV with different fractional order values is shown in Fig. 14. From results, it can be concluded that as the order of the filter increases the error decreases. The comparison of the integral square root error of the designed FIR-FOD with DCT-II, DCT-III and DCT-IV for different fractional order values are shown in Figs. 15 and 16.

5. Conclusions

In this work, the discrete cosine transform is applied for the design of FIR fractional order differentiator. Here, DCT-III and DCT-IV are incorporated to model the fractional order system such that the output closely approximates the actual system output. In order to evaluate the performance of the proposed FIR-FOD, an integral squared error function is considered. Comparative study has been carried out for the proposed FIR-FOD with different values of delay and fractional orders. Another study has been performed for the proposed FIR-FOD with existing FIR-FOD design methods such as DFT interpolation, radial basis function (RBF) interpolation, DCT-II interpolation, and DST interpolation methods. Simulation results affirm that the proposed FIR-FOD using DCT-III gives best results in terms of magnitude error compared to other type of DCTs. Whereas, proposed FIR-FOD using DCT-IV outperforms other type of DCTs in terms of phase error. It is clear from the graphical result that the change in the value of delay reflect change in the performance of the designed FIR-FOD.

Further, the proposed method needs to be explored as a future scope, for the designing of 2-D FIR fractional order differentiator. The future research may also focus on the designing of fractional Hilbert transformer and fractional order integrator.

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