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Renormalization of the Cabibbo–Kobayashi–Maskawa matrix in standard model

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Abstract

We have investigated the present renormalization prescriptions of Cabibbo–Kobayashi–Maskawa (CKM) matrix, and found there is still not an integrated prescription to all loop levels in the on-shell renormalization scheme. In this Letter we attempt proposing a new prescription designed for all loop levels in the present perturbative theory. This new prescription will keep the unitarity of the CKM matrix and make the amplitude of an arbitrary physical process involving quark mixing convergent and gauge independent.

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As an important part of standard model (SM) [1], the renormalization of Cabibbo–Kobayashi–Maskawa (CKM) quark mixing matrix is a matter of great account in theory. At present, along with the development of exact determination of CKM matrix elements [2], the importance of renormalization of CKM matrix becomes more and more apparent. This was realized for the Cabibbo angle with two fermion generations by Marciano and Sirlin [3] and for the CKM matrix of the three-generation SM by Denner and Sack [4] more than a decade ago. Though Denner and Sack's prescription is very delicate and simple, it reduces the physical amplitude involving quark mixing gauge dependent [5]. Recently many authors have discussed this problem [5,6], but because of its complexity all of them are limited to one-loop level and an integrated prescription beyond one-loop level in the on-shell renormalization scheme has been not obtained. So we want to propose a new prescription to solve this problem.

As we know a CKM matrix renormalization prescription must satisfy the three conditions [6]:

(1) In order to keep the transition amplitude of any physical process involving quark mixing ultraviolet finite, the CKM counterterm must cancel out the ultraviolet divergence left in the loop-corrected amplitude.
(2) It must guarantee such transition amplitude gauge parameter independent [7].

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(3) SM requires the bare CKM matrix V^0 is unitary,

$$\sum_k V^0_{ik} V^{0*}_{jk} = \delta_{ij}, \tag{1}$$

with i, j, k the generation index and δ_{ij} the unit matrix element. If we split the bare CKM matrix element into the renormalized one and its counterterm

$$V^0_{ij} = V_{ij} + \delta V_{ij} \tag{2}$$

and keep the unitarity of the renormalized CKM matrix, Eq. (1) requires

$$\sum_k \left(\delta V_{ik} V^*_{jk} + V_{ik} \delta V^*_{jk} + \delta V_{ik} \delta V^*_{jk}\right) = 0. \tag{3}$$

In order to satisfy these conditions we will renormalize the CKM matrix through two steps. First we introduce a CKM counterterm which makes the physical amplitude of $W^+ \to u_i \bar{d}_j$ convergent and gauge independent below certain loop levels. Next we mend it to satisfy the unitary condition of Eq. (3) below the loop levels, and simultaneously keep the divergent and gauge-dependent (if it has) part of it unchanged. Then by recursion we construct the CKM counterterms till infinite loop levels.

In order to elaborate our idea clearly we firstly introduce the n-loop ($n \geqslant 1$) decay amplitude of $W^+ \to u_i \bar{d}_j$ as follows (here all of the contributions of the counterterms lower than n-loop level have been included in the formfactors):

$$\begin{aligned} T_{nij} = A_L \bigg[F_{Lnij} + V_{ij} \left(\frac{\delta g_n}{g} + \frac{1}{2} \delta Z_{Wn} \right) \\ + \frac{1}{2} \delta \bar{Z}^{uL}_{nik} V_{kj} + \frac{1}{2} V_{ik} \delta Z^{dL}_{nkj} + \delta V_{nij} \bigg] \\ + A_R F_{Rnij} + B_L G_{Lnij} + B_R G_{Rnij}, \end{aligned} \tag{4}$$

with g and δg the $SU(2)$ coupling constant and its counterterm, δZ_W the W boson wave-function renormalization constant (WRC), $\delta \bar{Z}^{uL}$ and δZ^{dL} the left-handed up-type and down-type quark's WRC [8]. The added denotation n represents the n-loop result, and

$$\begin{aligned} A_L &= \frac{g}{\sqrt{2}} \bar{u}_i(p_1) \not{\varepsilon} \gamma_L \nu_j(q - p_1), \\ B_L &= \frac{g}{\sqrt{2}} \bar{u}_i(p_1) \frac{\varepsilon \cdot p_1}{M_W} \gamma_L \nu_j(q - p_1), \end{aligned} \tag{5}$$

with ε^μ the W boson polarization vector, γ_L and γ_R the left-handed and right-handed chiral operators, and M_W the W boson mass. Similarly, replacing γ_L with γ_R in Eq. (5) we get A_R and B_R, respectively. $F_{L,R}$ and $G_{L,R}$ are four formfactors. Here we will only care about the coefficient of A_L which contains the n-loop CKM counterterm. The simplest method to make the amplitude T_n convergent and gauge independent is to make the coefficient of A_L equal to zero, i.e.,

$$\begin{aligned} \delta V_{nij} = -F_{Lnij} - V_{ij} \left(\frac{\delta g_n}{g} + \frac{1}{2} \delta Z_{Wn} \right) \\ - \frac{1}{2} \delta \bar{Z}^{uL}_{nik} V_{kj} - \frac{1}{2} V_{ik} \delta Z^{dL}_{nkj}. \end{aligned} \tag{6}$$

Obviously such CKM counterterm cannot be guaranteed to satisfy the unitary condition of Eq. (3), so needs to be mended. Here we introduce a new set of denotation: $\delta \bar{V}_n$, to denote the amended CKM counterterm which satisfies the unitary condition. Our method is to construct $\delta \bar{V}_n$ through $\delta V_n, \delta \bar{V}_{n-1}, \ldots, \delta \bar{V}_1$. Here we state that δV_n is obtained by using $\delta \bar{V}_{n-1}, \ldots, \delta \bar{V}_1$ as the lower-loop CKM counterterms in Eq. (6). Now the unitary condition of Eq. (3) becomes

$$\begin{aligned} &\delta \bar{V}_1 V^\dagger + V \delta \bar{V}_1^\dagger = 0, \\ &\delta \bar{V}_2 V^\dagger + V \delta \bar{V}_2^\dagger = -\delta \bar{V}_1 \delta \bar{V}_1^\dagger, \\ &\delta \bar{V}_3 V^\dagger + V \delta \bar{V}_3^\dagger = -\delta \bar{V}_1 \delta \bar{V}_2^\dagger - \delta \bar{V}_2 \delta \bar{V}_1^\dagger, \\ &\vdots \\ &\delta \bar{V}_n V^\dagger + V \delta \bar{V}_n^\dagger = -\delta \bar{V}_1 \delta \bar{V}_{n-1}^\dagger - \delta \bar{V}_2 \delta \bar{V}_{n-2}^\dagger \cdots \\ &\qquad - \delta \bar{V}_{n-2} \delta \bar{V}_2^\dagger - \delta \bar{V}_{n-1} \delta \bar{V}_1^\dagger, \\ &\vdots \end{aligned} \tag{7}$$

In order to solve these equations, we introduce a set of symbols B_n

$$\begin{aligned} B_1 &= 0, \\ B_n &= \sum_{i=1}^{n-1} -\delta \bar{V}_i \delta \bar{V}_{n-i}^\dagger. \end{aligned} \tag{8}$$

Obviously B_n satisfies

$$B_n = B_n^\dagger. \tag{9}$$

Assuming the CKM counterterms $\delta \bar{V}_1, \delta \bar{V}_2, \ldots, \delta \bar{V}_{n-1}$ and δV_n have been obtained, the n-loop amended

CKM counterterm $\delta\bar{V}_n$ can be determined as follows:

$$\delta\bar{V}_n = \frac{1}{2}\big(\delta V_n - V\delta V_n^{\dagger} V + B_n V\big). \tag{10}$$

This method takes example by Diener and Kniehl's prescription [6]. It is easy to check that such CKM counterterm satisfies Eqs. (7) at n-loop level. Since the counterterms δg and δZ_W are both real in the on-shell renormalization scheme, we can obtain the following result from Eq. (10) and Eq. (6)

$$\begin{aligned}\delta\bar{V}_{nij} = {}& \frac{1}{2}\left(\sum_{kl} V_{ik} F^*_{Lnlk} V_{lj} - F_{Lnij}\right) \\ & + \frac{1}{4}\sum_k \big(\delta\bar{Z}^{uL*}_{nki} - \delta\bar{Z}^{uL}_{nik}\big) V_{kj} \\ & + \frac{1}{4}\sum_k V_{ik}\big(\delta Z^{dL*}_{njk} - \delta Z^{dL}_{nkj}\big) \\ & + \frac{1}{2}\sum_k B_{nik} V_{kj}.\end{aligned} \tag{11}$$

The remaining problem is to test whether the amended CKM counterterm $\delta\bar{V}_n$ has the same divergent and gauge-dependent part as δV_n, which is the requirement of making the physical amplitude involving quark mixing finite and gauge independent. Based on the renormalizability and predictability of SM, we can predict that the divergent and gauge-dependent part of δV_n (if it has) must satisfy the unitary condition of Eq. (3) at n-loop level

$$\delta V_n^{\mathrm{DG}} V^{\dagger} + V\delta V_n^{\mathrm{DG}\dagger} = B_n^{\mathrm{DG}}, \tag{12}$$

where the superscript DG denotes the divergent or gauge-dependent part of the quantity. This is because if not so the unitary condition of Eq. (3) will require the divergent or gauge-dependent part of the right CKM counterterm different from δV_n, thus will reduce the physical amplitude of $W^+ \to u_i\bar{d}_j$ divergent or gauge dependent (see Eq. (4)). In fact Eq. (12) is satisfied at one-loop level [4,6]. From Eqs. (10) and (12), it is easy to obtain

$$\begin{aligned}&\big(\delta\bar{V}_n^{\mathrm{DG}} - \delta V_n^{\mathrm{DG}}\big)V^{\dagger} \\ &\quad = \frac{1}{2}\big(B_n^{\mathrm{DG}} - \delta V_n^{\mathrm{DG}} V^{\dagger} - V\delta V_n^{\mathrm{DG}\dagger}\big) = 0.\end{aligned} \tag{13}$$

Times CKM matrix V at the right-hand side of Eq. (13), we have

$$\delta\bar{V}_n^{\mathrm{DG}} = \delta V_n^{\mathrm{DG}}. \tag{14}$$

Now we have obtained the proper CKM counterterm at n-loop level. We can construct CKM counterterms till infinite loop levels by recursion, which will satisfy the unitary condition of Eq. (3) and make the physical amplitude involving quark mixing convergent and gauge independent. Since the renormalization of CKM matrix is a very complex problem (one can see it from the fact that at present an integrated prescription applicable to all loop levels has not been obtained in the on-shell renormalization scheme), our solution is quite simple and practical (see Eq. (11)). On the other hand, we suppose our prescription will not break the present symmetries of SM, e.g., Ward–Takahashi identity, because it only changes the values of CKM matrix elements from V^0_{ij} to $V_{ij} + \delta\bar{V}_{ij}$.

Lastly we want to point out that the problem of infrared divergence is unclear in our renormalization prescription. As we know, the correction of Eq. (4) to the amplitude of $W^+ \to u_i\bar{d}_j$ has the infrared divergence coming from the Feynman diagrams including photons. This divergence should be cancelled in the inclusive decay width $W^+ \to (u_i\bar{d}_j, u_i\bar{d}_j\gamma, u_i\bar{d}_j\gamma\gamma, \ldots)$ by the corresponding divergence of the real photon emission processes. However, there are no a priori reasons for the cancellation of this divergence in the proposed CKM matrix counterterm, Eqs. (10), (11). Since this problem looks very difficult to be solved, we want to leave it for the next work.

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Appendix A

In this appendix we give the explicit result of $\delta\bar{V}_1$. From Eqs. (11) and (8), we obtain

$$\delta\bar{V}_{1ij} = \frac{1}{2}\left(\sum_{kl} V_{ik} F^*_{L1lk} V_{lj} - F_{L1ij}\right) + \frac{1}{4}\sum_k \left(\delta\bar{Z}^{uL*}_{1ki} - \delta\bar{Z}^{uL}_{1ik}\right) V_{kj} + \frac{1}{4}\sum_k V_{ik}\left(\delta Z^{dL*}_{1jk} - \delta Z^{dL}_{1kj}\right), \tag{A.1}$$

which is gauge independent since δV_{1ij} is gauge independent in Eq. (6) [5] and Eq. (10). Eq. (A.1) is similar as Eq. (12) of Ref. [6]. The ultraviolet divergence of $\delta\bar{V}_{1ij}$ is

$$\begin{aligned}\delta\bar{V}_{1ij}\big|_{\mathrm{UV}} &= \frac{3\alpha\Delta}{64\pi M_W^2 s_W^2} \times \Bigg[-\frac{2\sum_{k,l\neq j} m_{d,j} m^2_{u,k} V_{il} V^*_{kl} V_{kj}}{m_{d,l} - m_{d,j}} + \frac{2\sum_{k,l} m_{d,j} m^2_{u,k} V_{il} V^*_{kl} V_{kj}}{m_{d,l} + m_{d,j}} \\ &\quad - \frac{2\sum_{k\neq i,l} m_{u,i} m^2_{d,l} V_{il} V^*_{kl} V_{kj}}{m_{u,k} - m_{u,i}} + \frac{2\sum_{k,l} m_{u,i} m^2_{d,l} V_{il} V^*_{kl} V_{kj}}{m_{u,k} + m_{u,i}} \\ &\quad + V_{ij}\left(\sum_k V_{ik} V^*_{ik} m^2_{d,k} + \sum_k V_{kj} V^*_{kj} m^2_{u,k} - 2m^2_{d,j} - 2m^2_{u,i}\right)\Bigg],\end{aligned} \tag{A.2}$$

with α the fine structure constant, s_W the sine of the weak mixing angle θ_W, and $\Delta = 2/(D-4) + \gamma_E - \ln(4\pi) + \ln(M_W^2/\mu^2)$ (D is the space–time dimensionality, γ_E is the Euler's constant, and μ is an arbitrary energy scale). $m_{u,i}$ and $m_{d,j}$, etc. are up-type and down-type quark's masses. The R_ξ-gauge and the dimensional regularization have been used. This result agrees with the results of Refs. [4,5] and [6].

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