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Geometric Error Modeling of Machine Tools Based on Screw Theory

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Abstract

In order to build an linear, high computational efficiency geometric error model for machine tools, a new error modeling method based on screw theory is presented. Geometrical error sources are expressed as error twists, and the integrated model is calculated as the sum of error twists. The method is demonstrated on a 2-DOF mechanism and a simulation error modeling procedure at 1000 random points for a 5-axis machine tool is conducted. The simulation results show that the computational efficiency increase by 5 times. The feasibility of the proposed method is also verified and the physical influences of the error sources are also analyzed. This modeling method has high computational efficiency and clear physical meanings. It is useful to guide the design of new machine tools.

*Keywords:*routing strategy; Machine tool accuracy; Geometric error; Screw theory; Error model.

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1. Introduction

Geometric error of a machine tool is one of the most significant factors that are responsible for errors during machining process [1,2,7,8]. Homogeneous transform matrix (HTM) is a conventional method in geometric modeling [3,4,6]. However, there are several disadvantages for 5-axis machine tool geometric modeling when HTM is adopted. HTM has disadvantages such as low computational efficiency, being incapable of evaluating contribution of error sources and lacking of physical meaning. Y. Lin and Y. Shen [5] have proposed a new matrix summation method: kinematic equation is converted into six components and each of which has a clear physical meaning. However, the physical meaning of the geometric error source is not clear, and the error source chosen in the modeling procedure is doubtable.

In this paper, a geometric modeling approach based on screw theory for machine tools is presented. This method enables a better choice of geometric error sources with definite physical meanings. A five-

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axis blade milling machine is chosen as an example to demonstrate the feasibility and efficiency of the proposed method.

2. Geometric error modeling based on screw theory

According to the manufacture and assembly of machines, the geometric error sources can be divided into two groups: the position dependent errors and position independent errors. The position dependent errors are the errors caused by the inaccuracy of assembly, such as joint misalignments and angular offsets. These errors are consistent during the movement of the machine. The position dependent errors are the errors caused by the inaccuracy of parts like sliding guides or rotation axes. These errors vary during the movement of the machine. For a single axis V, the position independent errors are δS_{xV}，δS_{yV}，δS_{zV}, and δS_{aV}，δS_{bV}，δS_{cV}. The position dependent errors are δx_V, δy_V, δz_V and δa_V, δb_V, δc_V. These geometric errors are shown in Fig. 1.

Geometric errors can be considered as tiny motions. The geometric error twist at the end of a kinematic chain is the sum of the error twists which belong to the elements in the chain.

In order to explain the geometric error modeling procedure better, a 2-DOF mechanism is chosen as an example to demonstrate the modeling process. This mechanism is also shown in Fig. 1.

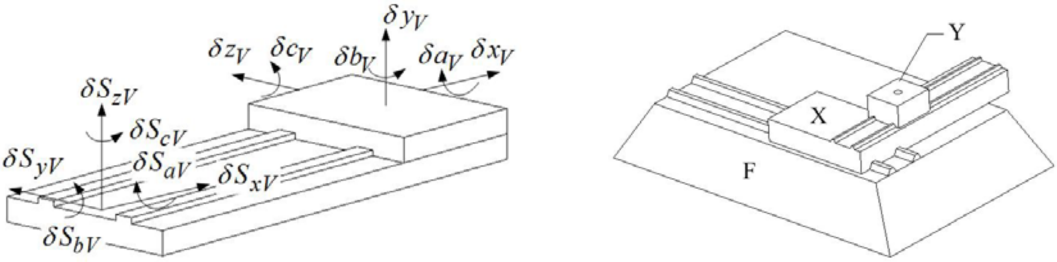

Fig. 1. (a) two types of geometric errors for axis V; (b) a 2-DOF mechanism

The geometric error twist for X axis is:

$$\boldsymbol{\$}_{\Delta u\mathrm{X}}=\delta u_{\mathrm{X}}[\boldsymbol{u}_{\mathrm{X}}\ \ \boldsymbol{O}_{\mathrm{X}}\times\boldsymbol{u}_{\mathrm{X}}]^{\mathrm{T}}. \tag{1}$$

$$\boldsymbol{\$}_{\Delta v\mathrm{X}}=\delta v_{\mathrm{X}}[\mathbf{0}\ \ \boldsymbol{v}_{\mathrm{X}}]^{\mathrm{T}}. \tag{2}$$

$$\boldsymbol{\$}_{\Delta Su\mathrm{X}}=\delta S_{u\mathrm{X}}[\boldsymbol{u}_{\mathrm{F}}\ \boldsymbol{O}_{\mathrm{F}}\times\boldsymbol{u}_{\mathrm{F}}]^{\mathrm{T}}. \tag{3}$$

$$\boldsymbol{\$}_{\Delta Sv\mathrm{X}}=\delta \mathrm{S}_{v\mathrm{X}}[\mathbf{0}\ \ \boldsymbol{v}_{\mathrm{F}}]^{\mathrm{T}}. \tag{4}$$

In which:

$$\boldsymbol{u}=\begin{cases}\boldsymbol{i}, & u=a\\ \boldsymbol{j}, & u=b\\ \boldsymbol{k}, & u=c\end{cases}. \tag{5}$$

$$\boldsymbol{v}=\begin{cases}\boldsymbol{i}, & v=x\\ \boldsymbol{j}, & v=y\\ \boldsymbol{k}, & v=z\end{cases}. \tag{6}$$

The subscript of $\boldsymbol{u}$ and $\boldsymbol{v}$ denotes the coordinate frame they belong to. While the geometric error twist for Y axis is:

$$\boldsymbol{\$}_{\Delta u\mathrm{Y}}=\delta u_{\mathrm{Y}}[\boldsymbol{u}_{\mathrm{Y}}\ \ \boldsymbol{O}_{\mathrm{Y}}\times\boldsymbol{u}_{\mathrm{Y}}]^{\mathrm{T}}. \tag{7}$$

$$\boldsymbol{\$}_{\Delta v\mathrm{Y}}=\delta v_{\mathrm{Y}}[\mathbf{0}\ \ \boldsymbol{v}_{\mathrm{Y}}]^{\mathrm{T}}. \tag{8}$$

$$\boldsymbol{\$}_{\Delta Su\mathrm{Y}}=\delta S_{u\mathrm{Y}}[\boldsymbol{u}_{\mathrm{X}}\ \boldsymbol{O}_{\mathrm{X}}\times\boldsymbol{u}_{\mathrm{X}}]^{\mathrm{T}}. \tag{9}$$

$$\boldsymbol{\$}_{\Delta Sv\mathrm{Y}}=\delta \mathrm{S}_{v\mathrm{Y}}[\mathbf{0}\ \ \boldsymbol{v}_{\mathrm{X}}]^{\mathrm{T}}. \tag{10}$$

The integrated geometric error of this mechanism can be denoted as follows:

$$\boldsymbol{\$}_{\Delta e}=\sum\boldsymbol{\$}_{\Delta ui}+\sum\boldsymbol{\$}_{\Delta Sui}\ . \qquad (11)$$

According to the analytical expression of the error twist and its physical meaning, the position error and orientation error are as follows:

$$\Delta x=\delta x_X+\delta S_{xX}+\delta x_Y+\delta S_{xY}-(y+Y_t)(\delta c_X+\delta S_{cY}+\delta S_{cX})-Y_t\delta c_Y. \qquad (12)$$
$$\Delta y=\delta y_X+\delta S_{yX}+\delta y_Y+\delta S_{yY}+X_t(\delta c_X+\delta S_{cY}+\delta S_{cX}+\delta c_Y)+x\delta S_{cX}. \qquad (13)$$
$$\Delta z=\delta z_X+\delta S_{zX}+\delta z_Y+\delta S_{zY}+(y+Y_t)(\delta a_X+\delta S_{aX}+\delta S_{aY})-X_t(\delta b_X+\delta b_Y+\delta S_{bY})-(x+X_t)\delta S_{bX}+Y_t\delta a_Y. \qquad (14)$$
$$\Delta a=\delta a_X+\delta S_{aX}+\delta S_{aY}+\delta a_Y. \qquad (15)$$
$$\Delta b=\delta b_X+\delta b_Y+\delta S_{bY}+\delta S_{bX}. \qquad (16)$$
$$\Delta c=\delta c_X+\delta S_{cY}+\delta S_{cX}+\delta c_Y. \qquad (17)$$

This result gives the relationship between the integrated error and error sources.

3. Simulation experiment

In order to evaluate the feasibility of the modeling method, a simulation experiment of a 5-axis blade milling machine is conducted. This machine tool is shown in Fig. 2.

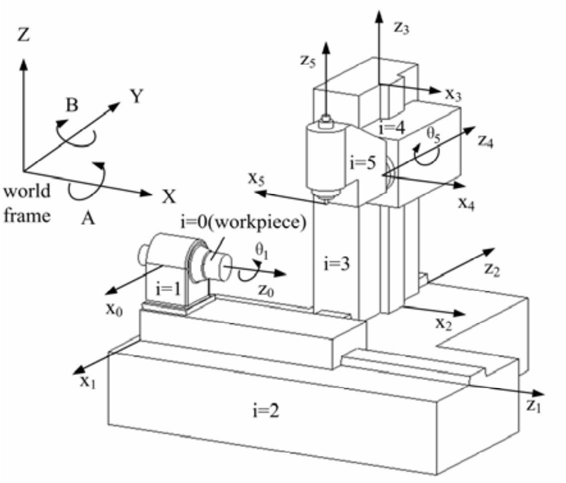

Fig. 2. 5-axis blade milling machine.

The error model of this machine can be conducted with the proposed error modeling method. The error model has a form as:

$$\boldsymbol{\$}_{\Delta e}=\sum_{i=1}^{60}\boldsymbol{\$}_i\ . \qquad (18)$$

The simulation experiment consists of two parts.

First of all, as the proposed model is a linear model, the correctness should be validated. The actual position and orientation errors at 1000 random position are calculated by actual kinematic model. The error is also calculated by the proposed method and the results are compared.

Secondly, to evaluate the contribution of error sources, the error model is simplified. According to the expression of the error model, the influence of geometric errors can be evaluated.

4. Results and discussion

The simulations are carried out in MATLAB. The parameters are given before the simulation. 1000 random points are chosen as shown in Fig. 3.

The rate of the errors calculated by the proposed method and the actual error are shown in Fig. 5.

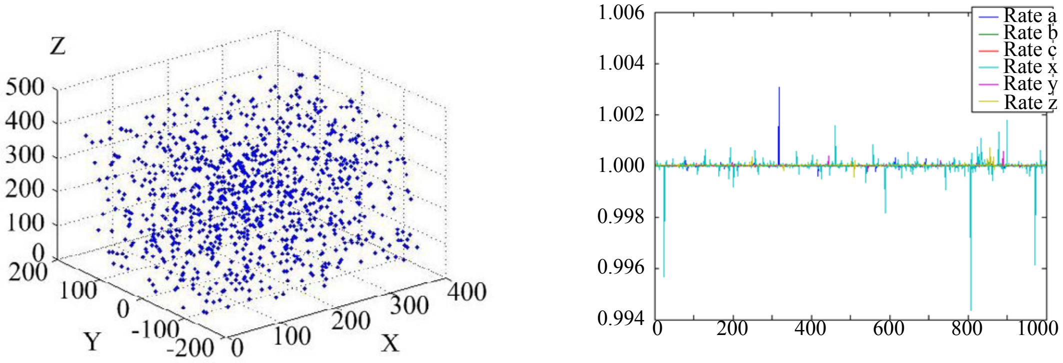

Fig. 3. (a) 1000 random points; (b) rate of the calculated errors and actual errors.

The results show the validity of the proposed model. According to the result, the nonlinear effect in the error model can be ignored. The proposed model shows higher computational efficiency in the simulation. The computational time cost is shown in Fig. 4.

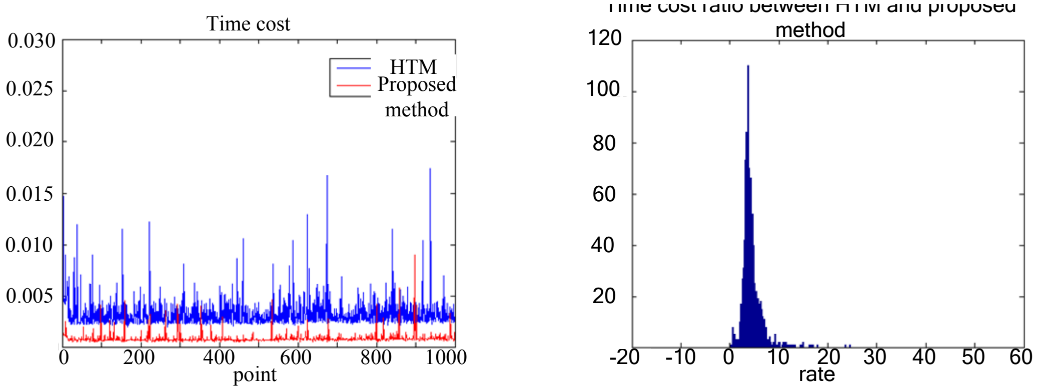

Fig. 4. (a) time costs of HTM and proposed method; (b) time cost ratio of HTM and proposed method.

Fig. 4. (b) shows the statistical time cost ratio of HTM and the proposed method. From the simulation results, we can see that the most possible ratio is 5. Thus the computational efficiency of the proposed method is roughly 5 times as high as HTM.

The error model can be simplified into 20 terms. The form of the 20 twists indicates that there are 20 minor movement caused by geometric errors including translations and rotations. The rotation angles/translation lengths and movement directions of these movements induced by geometric errors are shown in table 1.

Table 1. Value and direction characteristic of error movements

value	Direction	value	direction
Δ_1	translation along X axis of S_F	Δ_{11}	rotation along X axis of S_Z
Δ_2	translation along Y axis of S_F	Δ_{12}	rotation along Y axis of S_Z
Δ_3	translation along Z axis of S_F	Δ_{13}	rotation along Y axis of S_A
Δ_4	rotation along X axis of S_X	Δ_{14}	rotation along Z axis of S_A
Δ_5	rotation along Y axis of S_X	Δ_{15}	translation along Y axis of S_A
Δ_6	rotation along Z axis of S_X	Δ_{16}	translation along Z axis of S_A
Δ_7	rotation along Y axis of S_F	Δ_{17}	rotation along X axis of S_B
Δ_8	rotation along Z axis of S_F	Δ_{18}	rotation along Z axis of S_B
Δ_9	rotation along X axis of S_Y	Δ_{19}	translation along X axis of S_B
Δ_{10}	rotation along Z axis of S_Y	Δ_{20}	translation along Z axis of S_B

This table gives the physical effluence caused by the geometric error sources. The result can be used to guide the design of a new machine tool.

5. Conclusion

In this paper, a novel geometric error model based on screw theory is presented. The method can be used to build a linear error model efficiently and accurately. A 2-DOF mechanism is chosen to demonstrate the modeling method and a 5-axis blade milling machine is chosen to conduct a simulation experiment. The results show the feasibility and efficiency of this method. The error model of the 5-axis machine tool is also analyzed to evaluate the physical effluence of the geometric error sources.

Acknowlegement

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References

[1] E. L. J. Bohez*, et al.*, "Systematic geometric rigid body error identification of 5-axis milling machines,"C*omputer-Aided Design,* vol. 39, pp. 229-244, Apr 2007.

[2] B. K. Jha and A. Kumar, "Analysis of geometric errors associated with five-axis machining centre in improving the quality of cam profile," *International Journal of Machine Tools & Manufacture,* vol. 43, pp. 629-636, May 2003.

[3] A. W. Khan and W. Y. Chen, "Systematic Geometric Error Modeling for Workspace Volumetric Calibration of a 5-axis Turbine Blade Grinding Machine," *Chinese Journal of Aeronautics,* vol. 23, pp. 604-615, Oct 2010.

[4] A. Lamikiz*, et al.*, "The Denavit and Hartenberg approach applied to evaluate the consequences in the tool tip position of geometrical errors in five-axis milling centres," *International Journal of Advanced Manufacturing Technology,* vol. 37, pp. 122-139, Apr 2008.

[5] Y. Lin and Y. Shen, "Modelling of Five-Axis Machine Tool Metrology Models Using the Matrix Summation Approach," *The International Journal of Advanced Manufacturing Technology,* vol. 21, pp. 243-248, 2003.

[6] A. C. Okafor and Y. M. Ertekin, "Derivation of machine tool error models and error compensation procedure for three axes vertical machining center using rigid body kinematics," *International Journal of Machine Tools & Manufacture,* vol. 40, pp. 1199-1213, Jun 2000.

[7] C. Raksiri and M. Parnichkun, "Geometric and force errors compensation in a 3-axis CNC milling machine," *International Journal of Machine Tools & Manufacture,* vol. 44, pp. 1283-1291, Oct 2004.

[8] M. S. Uddin, et al., "Prediction and compensation of machining geometric errors of five-axis machining centers with kinematic errors," Precision Engineering-Journal of the International Societies for Precision Engineering and Nanotechnology, vol. 33, pp. 194-201, Apr 2009.