Simulation of ductile crack initiation and propagation by means of a non-local Gurson-model

Gerald Hütter a,*, Thomas Linse b, Uwe Mühlich a, Meinhard Kuna a

a TU Bergakademie Freiberg, Institute for Mechanics and Fluid Dynamics, Lampadius Str. 4, 09596 Freiberg, Germany
b TU Dresden, Institute for Solid Mechanics, 01062 Dresden, Germany

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A B S T R A C T

In the present study a non-local GTN-model in an implicit gradient-enriched formulation is employed to simulate ductile crack growth under small-scale yielding conditions numerically. The problem in handling full material failure in a non-local damage model is pointed out. An approach is presented to deal with this problem and a numerical implementation is derived. This technique allows to simulate large amounts of crack growth. The results show that the model captures correctly the states of initial crack tip blunting, crack initiation and crack propagation if a sufficiently fine mesh is used. A systematic parameter study is performed. On this basis, a procedure to determine the model parameters from experiments is proposed.

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1. Introduction

Today, the local approach to fracture is used in many applications. Within this methodology, the material degradation or damage is computed as field quantity simultaneous with stresses and deformations. For this purpose inner state variables for damage are introduced in the constitutive approach which describe the continuous softening of the material. However, if this is realized within the classical framework of simple materials the deformation localizes within an infinitely thin layer at the initiation of softening and the boundary value problem gets ill-posed. In finite element implementations, the smallest possible localization width is controlled by the spatial discretization. As a consequence, there is no convergence with respect to the element size anymore.

In order to overcome these problems, so-called non-local material models were developed where the softening state of a material point does not depend on the deformation history of this point only. Rather, a non-local state variable is introduced as a weighted average of the local states of the surrounding points within a certain distance. The mathematical formulations of this idea can be divided into non-local integral type, explicit gradient-enriched and implicit gradient-enriched approaches which can be correlated to each other under certain circumstances. Non-local models have been employed for many failure mechanisms. For an overview about the formulations and non-local material models, see Bazant and Jirásek (2002), Jirásek (1998), Jirásek and Rolshoven (2009), Jirásek and Rolshoven (2003) and Peerlings et al. (2001).

In an implicit gradient-enriched formulation a further partial differential equation (PDE) of Helmholtz-type is introduced for the non-local field variable or variables wherein the corresponding local counterpart forms the source term. This approach leads to a well-posed boundary value problem in all stages of localization. In addition, it allows a straightforward implementation in finite element codes why it has been established. Further boundary conditions have to be formulated which is not trivial due to the purely heuristic nature of the non-local PDE. There are reasonable arguments to prescribe zero NEUMANN conditions at boundaries in sufficient distance to a localization zone (e.g., Peerlings et al., 2001, 2002). Implicit gradient-enriched non-local damage model were successfully applied for an adequate description of localization phenomena (e.g., Engelen et al., 2003; Reusch et al., 2003; Geers et al., 2000; Geers, 2004; Jirásek and Rolshoven, 2009). According to Forest (2009) implicit non-local formulations fit in the generalized framework of micro-morphic media.

Problems arise if the evolution of the localization zone is to be simulated up to the complete loss of stress carrying capacity. In a fully failed region all stresses are zero and the equilibrium condition is fulfilled identically. However, this does not apply to the non-local PDE. This has the consequence that in FE-simulations fully failed GAUSS-points still contribute to the right-hand side terms as pointed out by Geers et al. (1998), Simone et al. (2003), and Peerlings et al. (2002). To the authors’ knowledge there is no generally accepted way in handling this problem. In many studies (e.g., Geers et al., 1998; Samal et al., 2008a,b; Seidenfuss et al.,
2011; Reusch et al., 2008) the local values get fixed at the moment of failure. In this case the non-local PDE (or its discretized weak form, respectively) still has to be solved in the fully failed region. Peerlings et al. (2002) argued that a new surface with trivial NEUMAN boundary condition should be introduced where failure occurs as done, e.g., in Simone et al. (2003) and Mediavilla et al. (2006). However, Linse et al. (2012) encountered severe numerical problems when applying this approach in a large-displacement analysis. In phase-field damage models, which can be classified as generalized micro-morphic model as well (Forest et al., 2011), DIRICHLET boundary conditions are prescribed at the crack surface (Miehe et al., 2010; Kuhn and Müller, 2010).

A number of non-local models for ductile failure has been proposed in the literature as modifications of the damage models of GURSON (Tvergaard and Needleman, 1995; Leblond et al., 1994; Jackiewicz and Kuna, 2003; Reusch et al., 2003; Linse et al., 2012) and Rousselier (Samal et al., 2008b; Seidenfuss et al., 2011) or other plasticity models (Engelen et al., 2003; Geers, 2004; Aslan et al., 2011; Bargellini et al., 2009; Mediavilla et al., 2006). Linse et al. (2012) pointed out that models of this type which incorporate pre-damage plasticity in a large displacement formulation predict a transition from the stages of stable crack tip blunting, crack initiation and propagation correctly. A systematic study on the influence of the material parameters on the predicted behavior is performed. Considering the limit case of small-scale yielding, effects of the geometry of a particular specimen are excluded. A systematic procedure is proposed to identify the material parameters from experiments one by one.

2. Non-local GTN-model

2.1. GTN-model

In the present study the non-local modification of the GTN-model (Tvergaard, 1981; Tvergaard and Needleman, 1984) by Linse et al. (2012) is employed where the non-local volumetric plastic strain is introduced in an implicitly gradient-enriched formulation. In the non-local modification the yield condition

$$\Phi = \left( \frac{\sigma_{0\text{v}}}{\sigma} \right)^2 + 2q_0 f^* \cosh \left( \frac{3q_2 \sigma_{0\text{v}}}{2 \sigma} \right) - 1 - (q_1 f^*)^2 \leq 0, \quad \lambda^\text{pl} \geq 0, \quad \Phi^\text{pl} = 0 \quad (1)$$

of the original GTN model is employed. Therein, $\sigma_{0\text{v}}$, $\sigma_{0\text{b}}$, and $\sigma$ denote the Mises, the hydrostatic stress and the effective yield stress of the matrix material, respectively. The symbol $\lambda^\text{pl}$ stands for the plastic multiplier. The parameters $q_1$ and $q_2$ were introduced by Tvergaard (1981) for fitting the results of cell model computations. The effective void volume fraction $f^*$ coincides with the actual one $f$ in the range of stable void growth but takes a larger value in the void coalescence stage:

$$f^* = \begin{cases} f, & f \leq f_c \\ \frac{f - f_c}{f - f_c}K, & f_c < f \leq f_t \end{cases} \quad \text{with} \quad K = \frac{f_u - f_c}{f_t - f_c}, \quad f_u = \frac{1}{q_1} \quad (2)$$

A hyper-elastic approach and an associative flow rule are employed:

$$\dot{\sigma} = C^{el} : (d - d^\text{pl}), \quad d^\text{pl} = \lambda^\text{pl} \frac{\partial \Phi}{\partial \sigma} \quad (3)$$

The symbols $d$ and $d^\text{pl}$ denote the rate of deformation and its plastic part, $\dot{\sigma}$ is the Jaumann-rate of the stress tensor and $C^{el}$ is the isotropic stiffness tensor. In the present study no void nucleation is considered but only void growth beginning at the initial value $f_0$. Requiring volume preserving plastic flow of the matrix material leads to the evolution equation of the void volume fraction

$$f = (1 - f) \dot{\epsilon}_v \quad (4)$$

where the volumetric plastic strain rate $\dot{\epsilon}_v$ is the trace of the plastic part of the rate of deformation tensor:

$$\dot{\epsilon}_v = d^\text{pl} : I \quad (5)$$

The matrix yield stress $\bar{\sigma}$ depends on the equivalent plastic strain $\bar{\varepsilon}$ whose evolution equation is

$$\dot{\bar{\varepsilon}} = \frac{\sigma - \bar{\sigma}}{(1 - f) \bar{\sigma}} \quad (6)$$

A one-parametric power law $\dot{\bar{\varepsilon}} = \bar{\varepsilon}(\bar{\varepsilon})$ is utilized for the hardening of the matrix material which is given implicitly through

$$\bar{\sigma} = \left( \frac{\sigma}{\sigma_0} + \frac{E}{\sigma_0} \right)^N \bar{\varepsilon} \quad (7)$$

The symbols $E$, $\sigma_0$ and $N$ denote Young’s modulus, initial yield stress and the hardening exponent, respectively. This approach results in a uniaxial response

$$\varepsilon = \begin{cases} \frac{\sigma}{E}, & \sigma < \sigma_0 \\ \sigma_0/E(\sigma/\sigma_0)^N, & \text{else} \end{cases} \quad (8)$$

between true stress $\sigma$ and logarithmic strain $\varepsilon$ in absence of voids $f_0 = 0$.

The parameters $E$, $\sigma_0$ and $N$ and the Poisson ratio $\nu$ are set to $\sigma_0 = 0.003E$, $N = 0.1$ and $\nu = 0.3$. A one-parametric power hardening law with these parameters is used in many fundamental studies in the literature (e.g., Gao et al., 2005; McMeeking, 1977; Tvergaard and Needleman, 1984; Tvergaard and Hutchinson, 2002) and corresponds to the behavior of typical medium strength engineering metals. The employed values of the parameters describing the coalescence stage are orientated at the results of cell model computations (Koplik and Needleman, 1988; Kuna and Sun, 1996; Steglich and Brooks, 1998). In addition, the parameter set used in the original work of Tvergaard and Needleman (1984) is incorporated. All used values are given in Table 1.

2.2. Non-local modification

The non-local modification of the GTN-model applies to the evolution equation (4) of the void volume fraction $f$. There, the volumetric plastic strain $\dot{\epsilon}_v$ is replaced by its non-local counterpart $\dot{\epsilon}_{nl}:

$$f = (1 - f) \dot{\epsilon}_{nl} \quad (9)$$

### Table 1

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$f_0$</th>
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<tr>
<td>6</td>
<td>0.1</td>
<td>0.3</td>
<td>0.44</td>
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</table>
The non-local volumetric plastic strain $\varepsilon_{nl}$ is introduced in an implicit way with a further partial differential equation of Helmholtz-type as

$$\varepsilon_{nl} - \nabla \cdot \left( \frac{C_{nl}}{C_{0}} \nabla \varepsilon_{nl} \right) = \varepsilon_{v}$$  \hspace{1cm} (10)$$

wherein $\nabla$ is the Nabla-operator with respect to the current configuration. Eq. (10) can be interpreted such that the non-local volumetric plastic strain $\varepsilon_{nl}$ is determined as an average of its local counterpart $\varepsilon_{v}$ around a material point. Exactly, the Green-function of the Helmholtz-operator in (10) forms the weighting function whose radius of action is characterized by the additional material parameter $l_{nl}$. From another point of view Eq. (10) can be seen as conservation law for the difference between local and non-local volumetric plastic strain. The intrinsic length $l_{nl}$ weights the term of the “flow of non-locality” $\frac{C_{nl}}{C_{0}} \nabla \varepsilon_{nl}$. The trivial NEUMANN boundary condition

$$\frac{C_{nl}}{C_{0}} \nabla \varepsilon_{nl} \cdot \mathbf{n} \big|_{\partial \Omega} = 0.$$  \hspace{1cm} (11)$$
is prescribed at free surfaces $\partial \Omega_{0}$ ensuring that the overall mean values of local and non-local volumetric plastic strains are equal. The balances of momentum and moment of momentum

$$\nabla \cdot \mathbf{\sigma} = \rho \mathbf{u} \quad \mathbf{\sigma} = \mathbf{\sigma}'$$  \hspace{1cm} (12)$$
are not affected by the non-local modifications. The boundary value problem is well-posed as shown by Linse et al. (2012).

### 2.3. Material failure

Problems arise if a material region $\Omega_{f}$ fails completely at $f = f_{c}$. Then, the GTN yield condition (1) implies a vanishing stress-carrying capacity $\mathbf{\sigma} = 0$ so that the (static) equilibrium condition (12) is fulfilled identically in $\Omega_{f}$. Since the displacement field is not determined uniquely in $\Omega_{f}$, a new surface $\partial \Omega_{f}$ has to be introduced around $\Omega_{f}$ with vanishing tractions (Peerlings et al., 2002). However, the non-local differential equation (10) is not fulfilled trivially in $\Omega_{f}$ and in general there still non-local flow through $\partial \Omega_{f}$. So the non-local field has still to be considered in $\Omega_{f}$ (or on $\partial \Omega_{f}$, respectively) despite the undefined displacement field, a problem which is physically questionable and numerically difficult to implement. Introducing a trivial NEUMANN boundary condition (11) for the non-local field at $\partial \Omega_{f}$ corresponds in general to a jump-like relaxation of the possible finite flow through $\partial \Omega_{f}$ which is problematic, too.

In an infinitesimal-displacement analysis some authors (Mediavilla et al., 2006; Peerlings et al., 2002; Simone et al., 2003) formulated a trivial NEUMANN boundary condition (11) for the non-local field at $\partial \Omega_{f}$. Under pure mode-I, Eq. (11) holds then before and after failure due to symmetry allowing a steady simulation of crack propagation. However, in a large-displacement approach, as designated in the present study, the failed region $\Omega_{f}$ takes a finite volume in the current configuration and the normal $\mathbf{n}$ to the surface $\partial \Omega_{f}$ changes with the deformation. Thus, the fulfillment of the NEUMANN condition in the intact material in the symmetry plane does not ensure necessarily the fulfillment of this condition at the newly created surface behind an advancing crack tip. The trivial NEUMANN boundary condition for the non-local terms could only be fulfilled during the whole loading history if the flow vector $\frac{C_{nl}}{C_{0}} \nabla \varepsilon_{nl}$ vanishes completely which is difficult to achieve. So Linse et al. (2012) employed (11) in the failed region in a mode-I analysis incorporating large displacements but faced numerical problems which prevented the continuation of their FE-simulation after the failure of the first few elements. The trivial NEUMANN boundary condition for the tractions (free surface boundary condition) is fulfilled in any case since all components of the stress tensor are zero.

In the present study a DIRICHLET boundary condition

$$\varepsilon_{nl} \big|_{\partial \Omega} = \varepsilon_{nl}^{0}$$  \hspace{1cm} (13)$$
is introduced for the non-local strain at locations where material fails completely. The condition of a vanishing traction at $\partial \Omega_{f}$ is retained (and fulfilled automatically as discussed above). The non-local strain at failure $\varepsilon_{nl}^{0}$ is related to the void volume fraction at failure $f_{c}$. The evolution equation (9) of $f$ can be integrated analytically yielding

$$\varepsilon_{nl}^{0} = \ln \frac{1 - f_{0}}{1 - f_{c}}.$$  \hspace{1cm} (14)$$
The DIRICHLET boundary condition (13) can be interpreted such that the non-local volumetric plastic strain $\varepsilon_{nl}$ and thus the directly related void volume fraction $f_{c}$ cannot change anymore at a fracture surface. This would not be the case if NEUMANN or mixed boundary conditions were applied since those would not exclude further non-local flow to the fracture surface from inside the domain and along the crack surface. This would allow for further void growth at the fracture surface $\partial \Omega_{f}$ which is physically not plausible. The different boundary conditions on the initially existing and newly created crack surfaces, Eqs. (11) and (13), respectively, are a matter of the different failure mechanisms. In reality the initial crack is typically not formed by ductile failure but is attributed to fatigue or material imperfections in structural components or the crack is machined or eroded in specimens for fracture testing. In any case, the initial spatial distribution of the void volume fraction $f_{0}$ and thus of the directly related non-local volumetric plastic strain $\varepsilon_{nl}$ has to be known. This distribution needs not inevitably to be homogeneous if the material has already experienced relevant plastic deformations or even ductile crack growth. In the latter case a DIRICHLET condition (13) would have to be formulated from the beginning on for the respective surface.

### 2.4. Numerical implementation

The non-local GTN-model is implemented in the commercial FE-program Abaqus as a user subroutine. For this purpose the weak formulations of the non-local differential equation (10) and of the equilibrium condition (12) are discretized. For the latter this is well-known and not repeated here. The discretized weak form of non-local differential equation (10) reads

$$\mathbf{R}^{{nl}} = \sum_{i}^{\mathcal{N}_{p}} \mathbf{r}_{i} = 0 \quad \text{with } \mathbf{r}_{i} = J_{W} \int_{W} \mathbf{N}^{{nl}} \left( \varepsilon - \mathbf{N}^{{nl}} \mathbf{e}^{{nl}} \right) \mathbf{B}^{{nl}} \mathbf{B}^{{nl}} \mathbf{e}^{{nl}} \left. \right|_{x-x'}$$  \hspace{1cm} (15)$$
where $\mathbf{N}^{{nl}}, \mathbf{e}^{{nl}}$ and $\mathbf{B}^{{nl}}$ are the vectors of the shape functions and the nodal values of the non-local volumetric strain and the matrix of the derivatives of the shape functions, respectively. The index $i$ counts over all GAUSS-points of which each lies at $x'$ and has the weight $w_{i}$. The JACOBI-determinant is denoted $J$. Quadratic shape functions are used for the displacements and linear ones for the non-local volumetric plastic strain. Reduced integration is employed. The hypoelastic–plastic material law (3) is integrated by means of an incrementally objective algorithm. The algorithmic consistent stiffness matrix is used for the NEWTON–RAPHSON iteration. Details are not given here but can be found in Linse et al. (2012).

The handling of completely failed material is not trivial. If the crack path is known in advance due to pure mode-I loading the additional boundary condition (14) to be introduced at some point of the FE-simulation could be handled the classical way, i.e. by inserting the value directly into the system of equations for the nodal values. The reordering of the system matrices necessary in this
case could be seen as something like “inverse node release”. If the crack path is not known in advance, this technique would require remeshing.

In order to avoid the big effort accompanying a global handling of the possible material failure, we decided to implement the further boundary condition (13) by means of a penalty technique. This has the advance that it can be managed at the element level. The technique works as follows: If a GAUSS-point \(j\) fails, its contribution to the right hand side is stored at this moment as \(r_j\). In the following steps the penalty is introduced as

\[
r_j = r_j^f + \beta r_{\text{fail}} \left( N^{\text{nl}} \cdot \epsilon^{\text{nl}} - r_{\text{fail}}^f \right) N^{\text{nl}}
\]

(16)

and the stress remains zero. The term \(r_j^f\) assures that there is no jump in the right hand side values at the transition point. Since the shape function \(N^{\text{nl}}\) is incorporated, all nodes connected to the particular element are affected. The problem of penalty formulations is that if the penalty factor \(\beta\) is chosen too large, the convergence rate decreases and if it is too small the condition to be enforced is not met. The elements in the regions where failure occur are considerably smaller than \(l_{\text{nl}}\). The value \(\beta = 0.3\) turned out to be an adequate choice and is used in the following simulations.

3. Model for crack propagation

3.1. Model and general considerations

Mode-I crack propagation for a semi-infinite crack under plane strain conditions is investigated for quasi-static loading. The far-field is uniquely determined by the stress intensity factor \(K_I\) with the corresponding energy release rate

\[
J = \frac{K_I^2}{E} (1 - v^2).
\]

(17)

In the \(K_I\)-dominated region the material behaves elastically. As the local volumetric plastic strain \(\epsilon_v\) misses, there is no source to generate non-local strains in this region. Thus, the latter vanishes in a distance to the plastic zone which is large compared to the intrinsic length \(l_{\text{nl}}\). Consequently, the corresponding gradient decays as well:

\[
\lim_{r \to -\infty} \epsilon_v = 0 \quad \text{and} \quad \lim_{r \to -\infty} \nabla \epsilon_v = 0.
\]

(18)

First, some considerations on the expected behavior of the model are discussed in order to derive requirements to the FE-model. A plastic zone forms at the crack tip even for arbitrary small loads. In the initial stage the plastic zone is small compared to the intrinsic length scale \(l_{\text{nl}}\). Correspondingly, the local plastification does not lead to an increase of the non-local strain \(\epsilon_{\text{fail}}\). Thus, in this stage the crack tip blunts stably as large deformations are taken into account. A relevant increase of \(\epsilon_{\text{fail}}\) does not happen until the size of the region of large plastic deformations becomes comparable to \(l_{\text{nl}}\) as shown schematically in Fig. 1. During crack tip blunting large plastic deformations occur within a zone with width of about twice the crack tip opening displacement (CTOD) \(d_t\) (McMeeking, 1977; Rice and Johnson, 1970). For dimensional reasons \(d_t\) is proportional to \(J : d_t = n J / \sigma_0\). The dimensionless proportionality factor lies in the range \(n_s \approx 0.25 \ldots 0.5\). In the following we use the displacement of the initial crack tip along the crack plane as measure of the crack tip opening displacement, see Fig. 1. This measure has the advantage that it is defined even after crack initiation (in contrast to the common definition with 45°-intersection lines) and that is easily accessible in the numerical simulations.

In the blunting stage the maximum of the hydrostatic stress lies in a distance of about \(x_{\sigma_{\text{max}}} \approx 2 l_{\text{nl}}\) in front of the crack tip. The GTN-yield condition (1) contains the hydrostatic stress so that dilatational plastic flow occurs at this place. This local flow is the source of the evolution of the non-local volumetric plastic strain and thus of void growth. Hence, it is expected that softening by void growth does initiate not at but in front of the crack tip as found by Simone et al. (2003) and Linse et al. (2012).

Subsequently, the softening zone coalesces with the initial crack tip and moves along the crack plane and so does the stress field in front of the crack tip. We employ the position of the current maximum stress in the ligament \(x_{\sigma_{\text{max}}}\) (with respect to the initial configuration) as measure of crack growth

\[
\Delta \alpha := x_{\sigma_{\text{max}}}
\]

(19)

where \(x = 0\) refers to the initial crack tip. This definition has the advantage that it incorporates the blunting stage natively where \(x_{\sigma_{\text{max}}}\) increases proportionally with \(J\). For dimensional reasons the computed crack growth resistance curves can be specified in the form

\[
J_r \sigma_{\text{fail}} \left( \frac{l_{\text{nl}}}{N} \right) = \text{function} \left( \frac{\Delta \alpha_{\text{cr}}}{l_{\text{nl}}}, f_e, \frac{E}{\sigma_0}, N, v \right).
\]

(20)

The parameters \(E/\sigma_0, N\) and \(v\) describe the elastic–plastic behavior of the matrix material and are fixed in the present study. Due to the small-scale yielding conditions a steady-state is reached finally where the softening zone shifts congruently along the crack plane at constant loading.

3.2. Numerical implementation

The described model is implemented in the finite element method using a so-called boundary layer approach where a circular domain of radius \(A_0\) is spatially discretized. Only a half-model needs to be taken into account due to symmetry. The outer radius is chosen as \(A_0 = (65.000 \ldots 175.000) l_{\text{nl}}\) ensuring small-scale yielding conditions. The \(K_I\)-solution is prescribed as displacement boundary condition at the circumference of the boundary layer as well as the trivial NEUMANN boundary condition (11) for the non-local field. A region of width \(B_0 = (120 \ldots 330) l_{\text{nl}}\) in front of the crack tip is fine-meshed with elements of edge length \(b_e = (1/16 \ldots 1/6) l_{\text{nl}}\) fulfilling the condition \(b_e < 0.25 l_{\text{nl}}\) established by Linse et al. (2012). A typical mesh is depicted in Fig. 2.

Pure crack tip blunting is expected in the initial stage. Thus, a strain singularity appears at the crack tip which cannot be handled by standard elements. For this reason we follow the approach of

\[1\] The asymptotic field is known for the displacement as well as the non-local field so that either DIRICHLET or NEUMANN boundary conditions can be prescribed equivalently.
McMeeking (1977) and introduce an initial radius \( r_i \) at the crack tip. McMeeking (1977) discussed that for increasing loading the theoretical solution for an ideal sharp crack is reached asymptotically. However, if the crack tip opening displacement \( \delta_t \) becomes considerably larger than \( r_i \), the elements at the initial radius distort.

Hence, for the present model the initial radius \( r_i \) at the crack tip has to be chosen small enough so that the final stages of blunting are captured adequately. Though, \( r_i \) must not be so small that the elements distort before the initiation of softening. In principle, the distortion could be avoided if enough elements were placed at the circumference of the initial rounding. However, then it is difficult to realize a passably regular transition to a coarser mesh on the crack plane. The latter is necessary to keep the number of elements on a manageable level as a large amount of crack growth is to be simulated. In addition, at full failure the solution is not steadily differentiable so that the results converge slowly and many increments are necessary.

The initial radius is set to \( r_i = (0.05 \ldots 0.6) l_0 \) such that the crack tip opening displacement at initiation of softening \( \delta_t \) lies in the range \( 5 \leq \delta_t/r_i \leq 7 \). In some cases this could not be obtained but only \( \delta_t \approx 3 r_i \).

For the following results the values of the crack tip opening displacement \( \delta_t \) are not picked from the node in the crack plane but from the upper node of the first element since the displacement of the node directly in the crack plane has no meaning anymore when the adjacent element fails due to numerical implementation described in Section 2.4.

As discussed in Section 3.1, a steady-state is expected to occur finally where the crack propagates at practically constant loading. In this stage the amount of crack growth is not uniquely determined by the loading anymore. This behavior cannot be handled by a standard load application scheme. Either a generalized arc length method or a dynamic computation needs to be applied. In this study, a dynamic simulation under quasi-static loading with implicit time integration is performed. This means that the time elastic waves need to propagate through the boundary layer has to be small compared to the time scale of loading \( \tau_i \). That is why the mass density \( \rho \) is specified such that \( A_0/c_s = 0.0025 \tau_i \) holds. In this context \( c_s \) denotes the velocity of the slower shear waves. The procedure can be seen as a dynamic stabilization scheme.

In each increment iterations are performed until the non-local residuals fall below a bound

\[
\max(R^{\text{nl}}) < \left(3 \cdot 10^{-6} \ldots 3 \cdot 10^{-5}\right) E^* \quad (21)
\]

depending on the employed mesh resolution. All in all, 10,000–20,000 elements are used in a simulation where 6000 to 200,000 increments are necessary in each case.

### 4. Results

#### 4.1. Initial crack tip blunting

In a first step the initial crack tip blunting is investigated. Technically, this can easily be achieved by setting the nodal values of the non-local strain to zero everywhere. Then, the void volume fraction maintains its initial value \( f_0 \). Consequently, the blunting is influenced by \( f_0 \) and the hardening parameters of the material only.

The evolution of the maximum normal stress in the ligament \( \sigma_{\text{yy}}^{\max} \) with increasing crack tip opening displacement \( \delta_t \) is depicted in Fig. 3. As discussed \( \delta_t \) is proportional to the far-field energy-release rate \( J \). The results show that the stress level of the asymptotic sharp-crack solutions is reached with reasonable accuracy already when the crack tip opening displacement reaches more than twice to three times the value of the initial radius at the crack tip \( r_i \). This value is slightly lower than the requirement \( \delta_t/r_i \geq 5 \) proposed by McMeeking (1977). Of course this value depends on the desired level of accuracy.

The distribution of the normal stress \( \sigma_{\text{yy}} \) in the ligament is plotted in Fig. 4 for the converged state \( \delta_t \gg r_i \). The solution for the low void volume fraction \( f_0 = 0.001 \) differs hardly from the one for compact material \( f_0 = 0 \). Between \( f_0 = 0.01 \) and \( f_0 = 0.1 \) a considerable drop of the stress level is observed.

#### 4.2. Crack initiation

If void growth is activated in the simulations, the crack tip blunting is followed by initiation of void growth. The evolution of the void volume fraction \( f \) in the ligament with increasing loading is shown in Fig. 5a for an intermediate value of the initial void

![Fig. 2. Mesh in the outer region and at the crack tip.](image)

![Fig. 3. Maximum normal stress at the ligament.](image)
volume fraction $f_0 = 0.01$. The void growth begins directly at the crack tip whereas the first complete failure occurs at a distance of about $x_{init} \approx 1 l_{nl}$ to the initial crack tip. From this location the failure zone grows in both directions and coalesces with the main crack. Finally, the crack propagates along the ligament. Fig. 5a proves that the failure boundary condition equation (13) is met in reasonable accuracy with the employed penalty technique.

The distribution of the normal stress $\sigma_{yy}$ in the ligament at the same load levels $J$ is plotted in Fig. 5b. After crack initiation the stress profile shifts along the ligament. Exemplary, the current amount of crack growth $\Delta \alpha$ is incorporated for one curve to illustrate definition (19). Comparing the stresses in Fig. 5b with the corresponding sharp-crack solution in Fig. 4, one can see that the maximum stress before crack initiation still lies below the theoretical blunting level by 8%. This is attributed to the compromise which had to be made with respect to the initial radius at the crack tip as described in Section 3.1.

The evolution of the maximum void volume fraction $f_{max}$ in the ligament and the crack tip opening displacement $\delta_t$ with loading is plotted in Fig. 6 for two parameter sets. The graph of $\delta_t$ for pure crack tip blunting is incorporated, too. Fig. 6a shows that for $f_0 = 0.01$ the void volume fraction grows steadily at the beginning when evolution of the crack tip opening $\delta_t$ corresponds still to the blunting line. Obviously, the influence of the initial radius $r_l$ at the crack tip decays faster in the evolution of $\delta_t$ than it does for the maximum stress level in the ligament. At a certain point when the maximum void volume fraction $f_{max}$ reaches the critical value $f_c$, the void volume fraction accumulates abruptly. The corresponding load level $J = J_c$ can be defined as crack initiation toughness. At this point there is a distinct kink in the curve for the crack tip opening displacement $\delta_t$ in Fig. 6b which was defined as initiation event by Wu (2000). Thus, for the low value of the initial void volume fraction $f_0 = 0.01$ it does not make any practical difference whether the initiation toughness $f_c$ is defined as the point when $f_{max}$ reaches the critical or final values $f_c$ respectively $f_t$ as the kink point in the curve of $\delta_t$.

For the larger initial void volume fraction $f_0 = 0.10$ in Fig. 6b there is no such sharp initiation point. Rather, the void volume fraction $f_{max}$ increases steadily with the loading $J$. Different values of the crack initiation toughness $J_c$ are obtained depending on which of the definitions discussed above is employed. The possible range of $J_c$ is incorporated in Fig. 6b. This smooth transition to crack growth is attributed to the continuous softening caused by the pre-critical void growth.

### 4.3. Crack propagation

The distribution of the increase $f - f_0$ of the void volume fraction around the initial crack tip is depicted in Fig. 7 at first complete failure of a material point (Fig. 7a) and and after some crack growth (Fig. 7b). The blunt crack tip is clearly visible in both stages. During crack propagation a zone of active void growth forms in front of the current crack tip which moves along the crack plane. Fig. 7b indicates that a nearly linear crack opening profile remains in the wake behind the current tip as predicted by Rice et al. (1980).

The computed crack growth resistance curves for different values of the void volume fraction at initiation of coalescence $f_c$ are depicted in Fig. 8a for an initial void volume fraction of $f_0 = 0.01$ (parameter set #2 to #4 in Table 1) and and $f_0 = 0.001$ (parameter set #1 in Table 1). A distinct crack initiation point is visible for all of these curves. The $R$-curves for $f_0 = 0.01$ share the blunting line since the latter depends on $f_0$ only as discussed in the preceding section but the curves exhibit a slightly concave behavior due to the void growth preceding the critical point, see Fig. 6.

The results in Fig. 8a indicate that $f_c$ affects the crack initiation toughness $J_c$ and influences the tearing behavior strongly. For $f_c = 0.15$ a steep and almost linear tearing up to a multiple of the initiation toughness occurs. The slope of $R$-curve in the initial tearing region even reaches that of the blunting line. In contrast, there is only moderate and regressive tearing for $f_c = 0.03$ and the steady-state toughness $J_{ss}$ amounts to about twice the initiation value only. The curve for $f_c = 0.05$ lies in between both graphs. The considerable difference between the results for $f_c = 0.03$ and $f_c = 0.05$ is remarkable. The curve for the lower initial porosity $f_0 = 0.001$ exhibits strong tearing as with the set $f_0 = 0.01$, $f_c = 0.15$. The abrupt softening succeeding a regime of very slow
void growth lead to numerical problems why no more crack growth than $\Delta a \approx 35 l_{nl}$ could be simulated with $f_0 = 0.001$.

For the higher initial void volume fraction $f_0 = 0.10$ in Fig. 8b there is no such distinct initiation point as already found in Section 4.2. In the diagram the points are marked when $f$ reaches the critical and the final value for the first time spanning a region of steady crack initiation. Again, this initiation region as well as the tearing behavior are affected by the critical void volume fraction $f_c$.

The influence of the hardening exponent $N$ on the predicted R-curve is investigated in Fig. 9. The results show that a stronger hardening with growth than void growth leads to numerical problems why no more crack initiation is observed in cell model computations. However, micro-mechanical models (Gao et al., 2005; Tvergaard and Hutchinson, 2002; Hütter et al., 2012) where a void-by-void growth mechanism is considered to be determinable uniquely from cell model computations. However, micro-mechanical simulations (Hütter et al., 2012) showed that the deformation history in front of the crack tip deviates considerably from that assumed in cell models. Hence, the $f_c$ can hardly be considered to be determinable uniquely from cell model computations. Recalling in addition the purely heuristic nature common to all non-local approaches we propose to fade out the origin of $f_c$ at this point and consider $f_c$ as a free parameter. Our results for

**5. Discussion**

In the present study the ductile crack initiation and propagation is simulated by means of a non-local GTN-model. The problem of the handling of fully failed material is pointed out. Introducing a DIRICHLET boundary condition for the non-local degree of freedom at the respective points allows to simulate large amounts of ductile crack growth $\Delta a$ amounting up to 250 times the intrinsic length $l_{nl}$ (Fig. 8). This is considerable more than in studies where ductile crack growth is simulated with a non-local damage model without addressing the problem of fully failed material and which are thus limited to $\Delta a \leq 15 l_{nl}$ (Bargellini et al., 2009; Jakewicz and Kuna, 2003; Linse et al., 2012; Reusch et al., 2008; Samal et al., 2008a,b; Seidenfuss et al., 2011). Large amounts of ductile crack growth could also be simulated with the approaches by Aslan et al. (2011) and Mediavilla et al. (2006).

The non-local GURSON model employed in the present study predicts an initial stage of crack tip blunting which is followed by crack initiation and stable crack propagation. This way, a realistic simulation of all three stages of ductile fracture could be achieved. Since the underlying boundary value problem is well-posed the solution can be numerically determined at least theoretically with arbitrary accuracy. This is of significant advantage compared to computational cell approaches as employed in (e.g., in Xia and Shih, 1995; Steglich and Brocks, 1998; Pardoen and Hutchinson, 2003; Ruggieri and Dodds, 1996). There, the classical FE-implementation of a damage model like GTN is employed and the element size in front of the crack tip is incorporated as a further material parameter. Consequently, crack tip blunting cannot be resolved with such models. Howard et al. (2000) introduced cells of fixed size and coupled the behavior of the arbitrary small elements therein in the softening regime. This approach is similar to an integral type non-local formulation such that this model predicts initial crack tip blunting.

In Section 4.2 it is found that the present model predicts crack initiation at a distinct point for low values of the initial void volume fraction $f_0$. However, for higher values of $f_0$ there is no such distinct point but a region of continuous transition to crack propagation is observed. This behavior is in accordance with micromechanical models (Gao et al., 2005; Tvergaard and Hutchinson, 2002; Hütter et al., 2012) where a void-by-void growth mechanism is observed at crack initiation for low initial porosities whereas multiple void interaction occurs for high initial porosities without a distinct point of crack initiation.

In the tearing regime a linear crack opening profile is obtained with the present model (see Fig. 7b) as predicted by Rice et al. (1980). This way a crack tip opening angle (CTOA) can be defined and measured which is in accordance with experimental experience (e.g., Heerens and Schödel, 2003; McCabe, 1986).

The non-local material model employed in the present study is based on the GTN-model incorporating accelerated void growth when the void volume fraction reaches the critical value $f_c$. This approach was introduced by Tvergaard and Needleman (1984) to capture the transition to a uniaxial mode of deformation which was obtained in cell model computations. However, micromechanical simulations (Hütter et al., 2012) showed that the deformation history in front of the crack tip deviates considerable from that assumed in cell models. Hence, the $f_c$ can hardly be considered to be determinable uniquely from cell model computations. Recalling in addition the purely heuristic nature common to all non-local approaches we propose to fade out the origin of $f_c$ at this point and consider $f_c$ as a free parameter. Our results for

**Fig. 7.** Void growth in front of the crack tip (a) at first full failure and (b) during crack growth at $J = 4.5 \sigma_0 l_{nl}$ ($f_0 = 0.01, f_c = 0.05$).
crack propagation show that \( f_c \) has a strong influence on the predicted tearing behavior so that it should be determined from corresponding experiments.

The tearing is often quantified by the dimensionless tearing modulus \( T_R \) according to Paris et al. (1979). In the normalization employed here (Eq. (20)) the tearing modulus is obtained as

\[
T_R = \frac{E}{\sigma_0} \frac{\partial \Delta R}{\partial \Delta \alpha} = \frac{E}{\sigma_0} \frac{\partial \left( \frac{\Delta \alpha}{\Delta \alpha_{nl}} \right)}{\partial \left( \frac{\Delta \alpha}{\Delta \alpha_{nl}} \right)}.
\]

Note, that under small-scale yielding the value \( T_R \) does not depend on the intrinsic length scale \( \Delta \alpha_{nl} \) but only on the initial and critical void volume fraction and on the elastic-plastic properties of the matrix material. Hence, varying \( f_c \) is a suitable procedure to adjust the tearing modulus from simulations to experiments. The results in Fig. 8a indicate that the whole range of \( T_R \) can be recovered reaching from nearly vanishing tearing up to the slope of the blunting line.

If the parameters of the employed non-local GTN-model are to be determined from particular experiments we propose the following procedure:

1. Determine \( f_0 \) from micrographs, chemical composition or the like.
2. Determine Young’s modulus and the yield curve of the matrix material from tensile test (taking into account \( f_0 \) for \( f_0 \approx 0.01 \)).
3. Choose \( f_c \) to fit the tearing modulus from simulations to fracture experiments.
4. Calculate \( l_a \) from ratio of measured fracture initiation toughness to computed dimensionless value.
5. Large-scale yielding: Iterate over points 3-4.

In this context it has to be remarked that \( f_c \) is determined by the initial tearing behavior in a single experiment. As there are no further parameters in the model the upper region of the R-curve and the constraint sensitivity are fixed then. It needs to be verified whether especially the constraint sensitivity “extracted” this way from a single experiment by means of the presented model allows reliable predictions on specimens with different crack tip constraint. The tearing and the CTOA are directly related (Rice et al., 1980) so that \( f_c \) could also be determined by fitting CTOA. However, the latter is mostly more difficult to access practically. Of course, the initial void volume fraction \( f_0 \) can hardly be measured accurately so that a slight adjustment of \( f_0 \) is still possible. If incorporated in the model, point 1 applies also to the nucleable porosity.

In the literature there are also other approaches than that of Tvergaard and Needleman (1984) to describe the final stages of ductile softening. So Benzerga and Leblond (2010) incorporate a more sophisticated void growth and coalescence model (however, within local material theory) and propose a “top-down approach” to identify the material parameters. The steps to identify the micro-structural parameters and the hardening behavior of the matrix material are similar to steps 1–2 in the listing above. In addition, they have the element size as a free length parameter for the computational cell simulation analogous to the non-local length scale \( l_a \) under step 5 in the present study. However, lacking a further adjustable parameter the predicted tearing behavior and the constraint sensitivity are fixed then. Gao and Kim (2006) employ a similar model but need to calibrate the parameter describing the initiation of void coalescence in order to capture the experimental fracture data. With this calibration the behavior of specimens with lower crack tip constraint can reliably be predicted.

In order to demonstrate the procedure proposed here the experimental results of McCabe (1986) for a pressure-vessel steel are processed exemplarily. The corresponding R-curve is plotted in Fig. 10. These data were obtained with a relatively large, high-constraint specimen C (T)-10T so that they are comparable to the small-scale yielding results of the present study. Furthermore, a relatively large amount of crack growth is encompassed. For step 1 of the fitting procedure an initial void volume fraction of \( f_0 = 0.01 \) is assumed arbitrarily since no other data are available and the simulations above were performed with this \( f_0 \). Values of this order of magnitude are typically used as nucleable porosity in the literature for engineering metals. The yield stress is given by \( \sigma_0 = 379 \text{ MPa} \). If the measured R-curve is plotted in the form \( J_R/\sigma_0 = f(\Delta \alpha) \) and compared with the computed curves in Fig. 8a, it is found that the tearing can be well fitted with the parameter set \( f_c = 0.05, f_0 = 0.13 \). (This means that the tearing modulus is used in contrast to Eq. (22) without the pre-factor \( E/\sigma_0 \), since the latter amounts to about \( E/\sigma_0 \approx 500 \) in the experiment which is larger than the value assumed in the present study.) According to step...
the intrinsic length $l_{0}$ is determined as $l_{0} \approx 0.2$ mm by equili-
brating the fracture initiation points if the suspected kink in the
measured R-curve in Fig. 10 is defined as initiation point. Compar-
ing the fitted curve to the measured one shows that a passable fit
was achieved despite the relatively low effort. The blunting regime
is experimentally difficult to access so that a slight offset in the
curves occurs. In the upper region $J_{R} \approx 600 \ldots 700$ N/mm the inter-
action of the plastic zone with the specimen geometry can presum-
ably not be neglected.

6. Summary

In the present study a non-local GTN-model is employed in an
implicit gradient-enriched formulation to simulate ductile crack
growth under small-scale yielding conditions numerically. It is
pointed out that in contrast to the equilibrium condition the
non-local partial differential equation is not fulfilled identically
at material failure. We propose to introduce a DIRICHLET-bound-
ary condition for the non-local variable where material fails.
Numerically, this is implemented by a penalty method which is
advantageous since it can be managed at element level.

It is shown that the model predicts correctly an initial stage of
stable crack tip blunting followed by crack initiation and propaga-
tion. In order to handle the strain singularity at the crack tip
accompanying the blunting we introduce an initial rounding at
the crack tip in the numerical model. This approach and the pen-
alty technique allow to adequately resolve the crack initiation pro-
cess and to simulate numerically considerable larger amounts of
crack growth than in comparable preceding studies.

It is found that the model predicts the transition from a sharp
fracture initiation mechanism to a continuous damage initiation for
increasing values of the initial void volume fraction which is in
accordance with micromechanical models from literature. The cri-
tical void volume fraction $f_{c}$ was originally introduced by Tvergaard
and Needleman (1984) to capture the transition to a uniaxial state
of deformation in cell model computations. This quantity has a
strong influence on the computed tearing behavior and we propose
to take $f_{c}$ as a adjustable parameter which can be determined from
the measured tearing of a single fracture experiment. This proced-
er is carried out exemplarily for a data set from literature. It re-
 mains open whether a parameter set extracted this way already
allows reliable predictions for other specimens with different crack
tip constraint.

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