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Natural Gas Industry B 2 (2015) 185–191

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Research article

Mechanical and mathematical models of multi-stage horizontal fracturing strings and their application☆

Lian Zhanghua[a,*], Zhang Ying[a], Zhao Xu[b], Ding Shidong[a], Lin Tiejun[a]

[a] *State Key Laboratory of Oil & Gas Reservoir Geology and Exploitation, Southwest Petroleum University, Chengdu, Sichuan 610500, China*

[b] *Sinopec Research Institute of Petroleum Engineering, Beijing 100101, China*

Received 21 October 2014; accepted 20 January 2015

Abstract

Multi-stage SRV fracturing in horizontal wells is a new technology developed at home and abroad in recent years to effectively develop shale gas or low-permeability reservoirs, but on the other hand makes the mechanical environment of fracturing strings more complicated at the same time. In view of this, based on the loading features of tubing strings during the multi-stage fracturing of a horizontal well, mechanical models were established for three working cases of multiple packer setting, open differential-pressure sliding sleeve, and open ball-injection sliding sleeve under a hold-down packer. Moreover, mathematical models were respectively built for the above three cases. According to the Lame formula and Von Mises stress calculation formula for the thick-walled cylinder in the theory of elastic mechanics, a mathematical model was also established to calculate the equivalent stress for tubing string safety evaluation when the fracturing string was under the combined action of inner pressure, external squeezing force and axial stress, and another mathematical model was built for the mechanical strength and safety evaluation of multi-stage fracturing strings. In addition, a practical software was developed for the mechanical safety evaluation of horizontal well multi-stage fracturing strings according to the mathematical model developed for the mechanical calculation of the multi-packer string in horizontal wells. The research results were applied and verified in a gas well of Tahe Oilfield in the Tarim Basin with excellent effects, providing a theoretical basis and a simple and reliable technical means for optimal design and safety evaluation of safe operational parameters of multi-stage fracturing strings in horizontal wells.

Keywords: Multi-stage fracturing; Fracturing string; Horizontal well; Mechanical model; Mathematical model; Hold-down packer; Sliding sleeve; Tarim Basin; Tahe oilfield

In order to enhance the recovery of low-permeability gas reservoirs and shale gas reservoirs, ball-injection sliding sleeve multi-stage fracturing for horizontal wells has been developed since 2007 [1,2]. Nowadays, multi-stage fracturing and multi-stage repeated volume fracturing in horizontal wells are the hot topics [3–10] in this field. However, in the application of these technologies, downhole tools and fracturing strings will be subject to complex stress environment, harsh mechanical condition. In view of the complicated mechanical issue, it is necessary to do safety evaluation of the multi-stage horizontal fracturing string. In 1962, Lubinski et al. [11] had initiated the study on downhole packer-string mechanics, and built mechanical-mathematical models for bulging effect, piston effect, temperature effect and helical buckling effect caused by pressure or temperature changes or various working conditions. Soon afterwards, Hammerlindl [12,13], Mitchell et al. [14] delved deeper into the study on the single packer and double-packer mechanical-mathematical models of vertical wells and buckle deformation of fracturing

☆ Fund project: Foundation for Doctors from the Chinese Ministry of Education (No. 20135121110005); National Major Science & Technology Project "Key technology for well bores in marine carbonate reservoirs" (No. 2011ZX05005-006).

* Corresponding author.

E-mail address: cwctlzh@swpu.edu.cn (Lian ZH).

Peer review under responsibility of Sichuan Petroleum Administration.

http://dx.doi.org/10.1016/j.ngib.2015.07.009

string, which laid the classical mechanical foundation for safety evaluation of fracturing strings. In China, Gao Deli [15], Li Zifeng [16], Lian Zhanghua and Ding Liangliang [17,18] conducted research on mechanical behavior and safety of oil well strings under different working conditions on the basis of previous studies and made some progress in conventional string mechanics study [11−16]. But the previous studies focus mainly on vertical wells with single packer, and there are few literature about the mechanics of horizontal well fracturing strings [17], especially under the working condition of multi-packers and multi-stage horizontal fracturing.

In view of the complicated mechanical issue, we developed the mechanical models and mathematical models for multi-stage horizontal fracturing strings in four kinds of working condition, packer setting, opening differential pressure sliding sleeve, opening ball sliding sleeve, and fracturing, mathematical models for these mechanical models, and developed practical software for mechanical safety evaluation of the strings.

1. Mechanical and mathematical models of the fracturing strings and packers

The fracturing string will contract or extend due to the changes of wellhead operation pressure in the fracturing process, creating tensile or compressive force on the packers. Therefore, the packer will generate an axial tensile or compressive force on the fracturing string. The mechanical model of multi-packer fracturing string in openhole horizontal wells is shown in Fig. 1, which shows the fracturing string with multi-packer location and its mechanical model. In Fig. 1a, point A is the wellhead location, and point D is the deflection point, AD is the vertical well section, DPC is the building up section, and BC is the horizontal section. Point P is the hold-down packer, P_1-P_{n0} are the openhole packers of horizontal interval, S_1 is differential pressure sliding sleeve and S_2-S_{m0} are the ball sliding sleeves.

1.1. Mathematical model of pretension force on the tubing string after the setting of multi-packers

1.1.1. Case 1: under the working condition of setting multi-packers, with differential pressure ΔP_k

As is shown in Fig. 1b, the mechanical-mathematical model of string section L_k is established by statics equilibrium relationship [17]:

$$T_k = W_k \cos\theta_k \pm N_k \mu \tag{1}$$

$$N_k = \mu W_k \sin\theta_k \tag{2}$$

where, T_k is the axial force of k section fracturing string, N; W_k is buoyant weight of k section fracturing string, N; N_k is the normal force of k section fracturing string, N; θ_k is the angle of inclination at k section fracturing string, °; μ is the friction coefficient between fracturing string and borehole wall.

Substituting Eq. (1) with Eq. (2), the tension acting on any given infinitesimal section k is deduced and the equation is expressed in Eq. (3).

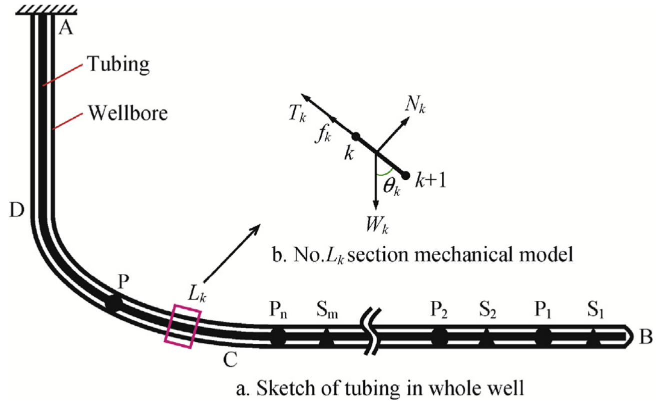

Fig. 1. Forces on the multi-stage fracturing tubing in a horizontal well.

$$T_k = W_k(\cos\theta_k \pm \mu \sin\theta_k) \tag{3}$$

The axial tension at the hold-down packer location P is the sum of T_k acting on each section of string, which is written as Eq. (4).

$$T = \sum_{k=1}^{n} T_k = \sum_{k=1}^{n} W_k(\cos\theta_k \pm \mu \sin\theta_k) \tag{4}$$

The sketch map of the fracturing string below hold-down packer P with openhole packers and sliding sleeves is shown in section BC of Fig. 1a. After setting packers, the pressure in the fracturing string increases by Δp_k, and the pretension force F_0 on the fracturing string between horizontal openhole packers is calculated by Eq. (5).

$$F_0 = \Delta p_k A_i \tag{5}$$

where, F_0 is the axial force between the openhole packers, N; Δp_k is setting pressure, MPa; A_i is internal surface area of the fracturing string, mm^2.

For the pretension force F_1 at the hold-down packer location P, tension T caused by buoyant weight of the string should be added, so the calculating model of F_1 is Eq. (6). The fracturing string will elongate when setting, in this case the friction and axial force are of opposite directions, which means minus will be applied in Eq. (6), otherwise plus sign should be applied.

$$F_1 = T + F_0 = \sum_{k=1}^{n} W_k(\cos\theta_k - \mu \sin\theta_k) + \Delta p_k A_i \tag{6}$$

1.2. Mechanical-mathematical model of open differential pressure sliding sleeve strings

1.2.1. Case 2: under the working condition of open differential pressure sliding sleeve at differential pressure $\Delta p'$

After the setting of downhole string, the differential pressure sliding sleeve S_1 will be opened to do the first stage of fracturing of P_1-B section. The mechanical model of the tubing string before opening differential pressure sliding sleeve S_1 is shown in Fig. 2a−d. Fixed at P_1, and free at B, the

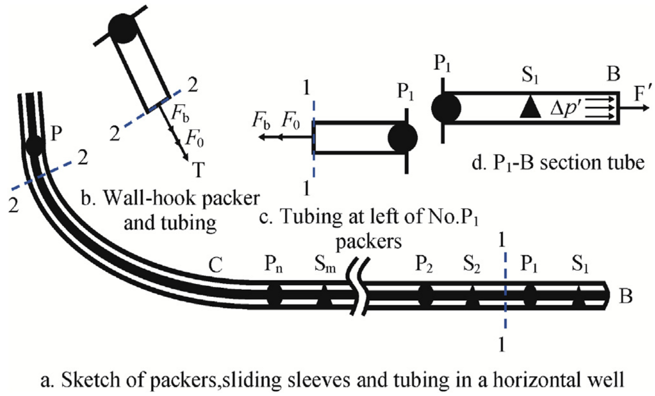

Fig. 2. Mechanical model of the tubing string when opening the No.S_1 differential pressure sliding sleeve.

P_1-B section of the tubing string is at the state of free stretching under the opening pressure $\Delta p'$.

The stress condition and stress size of the fracturing string sections between openhole packers can be obtained by using the recursive relationship and stress analysis diagram of DP sliding sleeve string shown in Fig. 2d. Fig. 2c is the mechanical model of the first openhole packer P_1 in the horizontal section. Other openhole packers in the horizontal section bear the same force with cross section 1-1. The forces at hold-down packer location P will add by an axial force T caused by buoyant weight, as is shown in Fig. 2b.

Fig. 2d shows the mechanical model of the differential pressure sliding sleeve S_1 section, the string is subject to tensile force F' at cross section 1-1. The string section of differential pressure sliding sleeve S_1 is subject to not only tensile force F' but internal pressure p_i and external pressure p_o. Under differential pressure $\Delta p'$, the mathematical model of axial tension F' on the string is

$$F' = \Delta p' A_i \tag{7}$$

and the model of axial stress S_a is

$$S_a = \frac{F'}{A_r} \tag{8}$$

where A_i is inner cross-section area of the tubing, mm^2, S_a is the axial stress on the string, MPa.

When differential pressure changes, the string between openhole packers is subject to bulging force F_b [11–13], and the mathematical model for bulging force is

$$F_b = 0.6A_i\Delta p_i - 0.6A_o\Delta p_o \tag{9}$$

Where: F_b is the bulging force, N; Δp_i is the pressure change in the string, MPa; Δp_o is the pressure change outside the string, MPa; A_o is the outer surface area of the string, mm^2.

Under the effect of differential pressure sliding sleeve, the pressure in the string $\Delta p_i = \Delta p'$, the external pressure of string does not change relatively before the opening of DP sliding sleeve, $\Delta p_0 = 0$. The mathematical model calculating bulging force can be deduced by Eq. (9),

$$F_b = 0.6A_i\Delta p_i \tag{10}$$

The total axial force on each section between openhole packers in the horizontal section is ($F_0 + F_b$) in Fig. 2c. The axial stress can be calculated by the following equation,

$$S_a = \frac{F_0 + F_b}{A_r} \tag{11}$$

The axial stress at the hold-down packer P is

$$S_a = \frac{F_0 + F_b + T}{A_r} \tag{12}$$

1.3. Mechanical-mathematical model for open ball sliding sleeve strings

1.3.1. Case 3: open ball sliding sleeve at differential pressure Δp_{s2}

After ball sliding sleeve S_1 is open, the fracturing is proceeded at the right side of openhole packer P_1 to B. After the fracturing of this section, the second stage fracturing between P_1 and P_2 section should be carried out. At this point, the ball is dropped to S_2 to block off the sliding sleeve S_2, and the ground pressure is increased to build the pressure at S_2. The mechanical model of the sliding sleeve S_2 string is shown in Fig. 3a–d. At this point, the fractured section P_1 to B can be neglected, because there is no force acting on string section P_1 to B after the setting of packer P_1 and the sealing function of the sliding sleeve S_2. Based on the mechanical separation method, the mechanical model of the string from cross section 1.1 to openhole packer P_1 is shown in Fig. 4d. This string section is subject to pretension force F_0 before the setting, and drag force F_f of openhole packer P_1 and compressive force caused by the differential pressure on the ball sliding sleeve S_2.

On the left side of S_2, cross section 2-2 is subject to not only pretension force F_0 but bulging force F_b from differential pressure Δp_{s2} and axial tension F_{s2} from building pressure of S_2. The mechanical model of cross section 2-2 is shown in Fig. 3c. When the differential pressure at S_2 increases to Δp_{s2}, ground pressure drops suddenly, meaning that the sliding sleeve S_2 is open and fracturing process can be started.

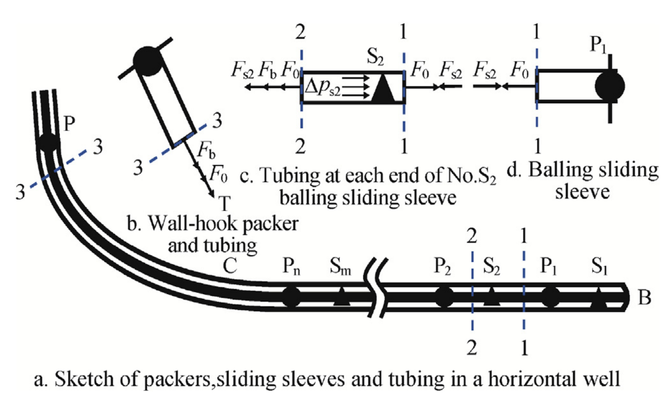

Fig. 3. Mechanical model of the string when opening the No.S_2 ball sliding sleeve.

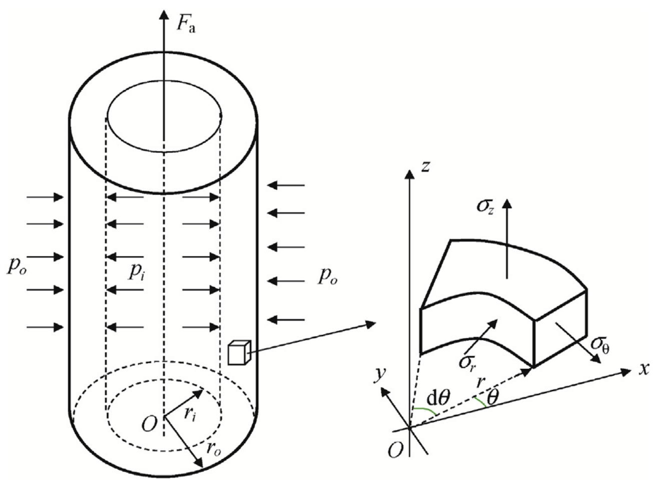

Fig. 4. Sketch of internal pressure, external pressure and axial force on the string.

Based on the mechanical model of opening the sliding sleeve S_2 in Fig. 3, the mathematical models of F_{s2} and F_b are as follows:

$$F_{S2} = \Delta p_{S2} A_i \tag{13}$$

$$F_b = 0.6 \Delta p_{S2} A_i \tag{14}$$

where Δp_{S2} is the differential pressure opening the ball sliding sleeve, MPa; F_b is the bulging force caused by the opening of the S_2 sliding sleeve, N.

The mathematical model of axial stress S_a on cross section 2-2 is

$$S_a = \frac{F_0 + F_b + F_{S2}}{A_{st1}} \tag{15}$$

The axial stress S_a at the hold-down packer location P is

$$S_a = \frac{F_0 + F_b + T}{A_{st1}} \tag{16}$$

2. Evaluation criterion for string mechanical strength

Based on the multi-stage horizontal fracturing mechanical-mathematical models Eqs. (1–16), the axial force F_a and axial stress S_a at each packer can be calculated. In the course of fracturing, the string is subject to internal pressure p_i, external pressure p_o and axial stress S_a jointly (Fig. 4).

According to the Lame formula of elastic dynamic theory Eqs. (17–19) and Von-Mises Eq. (20), the mathematical models of tri-axial stresses for the string safety evaluation are as follows:

$$\sigma_r = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} - \frac{(p_i - p_o) r_o^2 r_i^2}{\left(r_o^2 - r_i^2\right) r^2} \tag{17}$$

$$\sigma_\theta = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} + \frac{(p_i - p_o) r_o^2 r_i^2}{\left(r_o^2 - r_i^2\right) r^2} \tag{18}$$

$$\sigma_z = S_a \tag{19}$$

$$\sigma_{VME} = \sqrt{\frac{(\sigma_r - \sigma_\theta)^2 + (\sigma_\theta - \sigma_z)^2 + (\sigma_z - \sigma_r)^2}{2}} \tag{20}$$

where, r_i is inner radius of string, mm; r_o is outer radius of string, mm; p_i is inner pressure of the string, MPa; p_o is outer pressure of the string, MPa.

The result of triaxial stress intensity in Eq. (20) is substituted into $n_{Real} = Y_P/\sigma_{VME}$, and the actual factor of safety can be obtained. Based on the Von Mises yield strength criterion, if $n_{Real} > n_s$, the string is in safe state, otherwise it is in a dangerous state. Where Y_P is yield stress of the string, MPa; n_s is the design safety factor.

Based on the mathematical-mechanical models we build, the mechanical safety evaluation software is developed by Visual Basic 2010 for multi-stage horizontal fracturing string. Based on the hole trajectory data, structure and size of multi-packer string, borehole structure and size, fluid parameters, internal and external pressure and fracturing parameters, this software can directly evaluate the safety of every section of multi-stage fracturing string with multi-packers, providing a simple and reliable means for the safety evaluation of horizontal fracturing string in different working conditions.

3. Case study

Take an openhole completed horizontal gas well in Tahe oilfield as an example. The hold-down packer, openhole packer, differential pressure sliding sleeve, ball sliding sleeve and fracturing string location are all saved in the database of well track. The well has N80 steel grade casing of Φ177.8 × 8.05 mm, N80 steel grade tubing of Φ88.9 × 6.45 mm with a yield stress of 552 MPa, formation pressure coefficient of 1.30 MPa/100 m, average vertical depth of horizontal section of 3066 m, formation pore pressure of horizontal section of 39.86 MPa, and fracturing fluid density of 1.25, design safety factor of tubing string of 1.2. The basic parameters of three kinds of working conditions are as follows: packer setting differential pressure Δp_k is 15 MPa, starting pressure of DP

Table 1
Axial force on the tubing in three working conditions.

Case 1: $\Delta p_k = 15$ MPa				Case 2: $\Delta p' = 42$ MPa		Case 3: $\Delta p_{S2} = 20$ MPa		
F (kN)	T (kN)	F_0 (kN)	F_1 (kN)	F_b (kN)	S_a (MPa)	F_0 (kN)	F_{s2} (kN)	F_b (kN)
2.826	11.946	93.1	105.04	114.32	156.04	93.107	124.14	54.437

Table 2
Multi-packer setting condition (Case 1).

Location	p_o (MPa)	p_i (MPa)	S_a (MPa)	$\sigma\sigma_{VME}$ (MPa)	n_{Real}
Hold-down packer	36.36	53.56	62.87	122.94	4.49
Openhole packer	39.87	54.87	55.73	111.03	4.97

Table 3
Differential pressure sliding sleeve opening condition (Case 2).

Location	p_o (MPa)	p_i (MPa)	S_a (MPa)	$\sigma\sigma_{VME}$ (MPa)	n_{Real}
Hold-down packer	36.36	53.6	131.3	276.25	2.00
Openhole packer	39.87	81.87	156.04	282.4	1.95

Table 4
Ball sliding sleeve opening condition (Case 3).

Location	p_o (MPa)	p_i (MPa)	S_a (MPa)	$\sigma\sigma_{VME}$ (MPa)	n_{Real}
Hold-down packer	36.26	53.56	95.46	152.08	3.63
Openhole packer	39.86	59.86	162.61	196.26	2.81

sliding sleeve $\Delta p'$ is 42 MPa, starting pressure of ball sliding sleeve Δp_{s2} is 20 MPa, respectively.

Based on the packers, differential pressure sliding sleeve and well track database, the internal stress, axial pressure, triaxial stress σ_{VME} and string safety factor of three working conditions were calculated with the developed safety evaluation software for horizontal fracturing string with multi-packers. The calculation results are listed in Tables 1–4. Fig. 5 shows the exact position of multi-packers and sliding sleeves drawn automatically by this software according to the well track database. It can be seen from Fig. 5 that there are eight ball sliding sleeves (numbering S_2, S_3 … …S_9), one pressure differential sliding sleeve (S_1) and eight openhole packers (numbering P_2, P_3 … …P_8) in the horizontal section, and one hold-down packer (P) in deflection section. Table 1 shows the numerical value of prestressing force, drag force, bulging force and axial force of three different working conditions. Von Mises stress, safety factor, compression, extrusion force and axial force at the hold-down packer and openhole packers are listed in Tables 2–4.

It can be seen from Tables 2 and 3 that the actual safety factor at the hold-down packer is 2.0–4.49 and the actual safety factor at openhole packer is 1.95–4.97, both greater than the designed safety factor of 1.2, indicating that the N80 tubing string (Φ88.9 × 6.45 mm) meets the safety requirements in these three working conditions.

Case 4 is the fracturing working condition. In the first three cases, fracturing string is connected to formation, while in

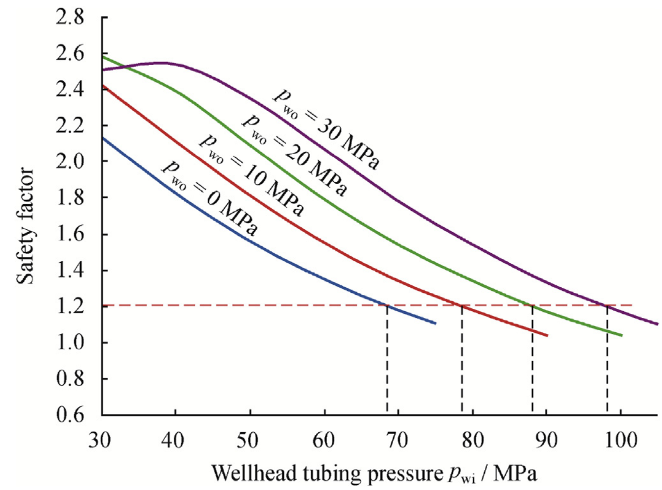

Fig. 6. Relationship of wellhead tubing safety factor with tubing pressure and casing pressure.

case 4 based on the previous three cases, wellhead tubing pressure (internal pressure of wellhead string) or wellhead casing pressure (internal pressure of casing) is increased to make the pressure of horizontal fracturing section higher than the formation pressure of 39.86 MPa and force the creation of fractures. The pressure must meet the safety requirement of the string in all these working conditions. Based on the mechanical modeling and force analysis, the most dangerous locations of the string are the wellhead and hold-down packer. Based on the mathematical-mechanical model constructed in this paper, when wellhead tubing pressure and casing pressure increase continuously, the relationship between safety factor at wellhead and hold-down packer location and tubing pressure and casing pressure is shown in Figs. 6 and 7.

Wellhead casing pressure is the balance pressure outside the string from wellhead to the hold-down packer location. Casing pressure can only transmits to the hold-down packer, not to the formation, so wellhead casing pressure cannot increase infinitely and only serves as the balance pressure to adjust the deformation of the string above hold-down packer. In consideration of this, Figs. 6 and 7 only show the working conditions at p_{wo} of 0 MPa, 10 MPa, 20 MP and 30 MPa, and wellhead oil pressure changing continuously from 30 MPa to

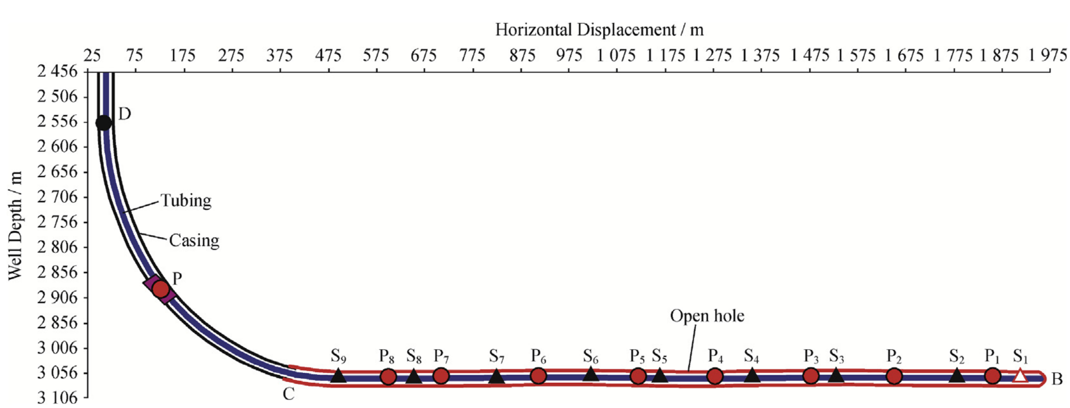

Fig. 5. Packer positions and track of a well in the Tahe oilfield

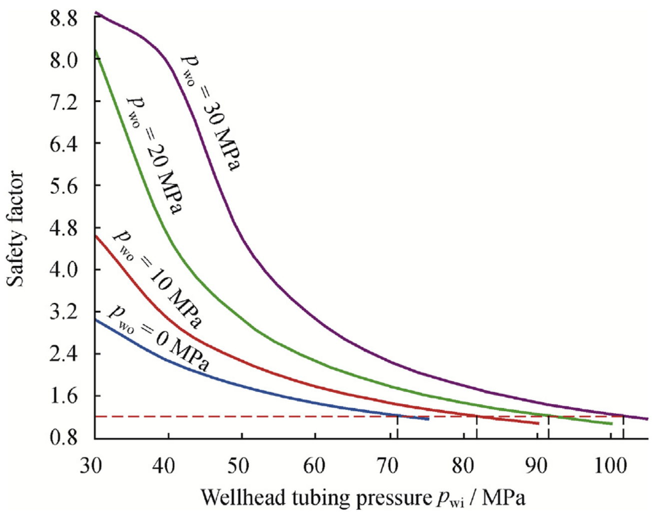

Fig. 7. Relationship of hold-down packer safety factor with tubing pressure and casing pressure.

105 MPa. In Figs. 6 and 7, the horizontal dotted line of safety factor 1.2 intersects with corresponding curves, and the abscissas of intersections are the ultimate value of wellhead tubing pressure. Comparing Figs. 6 and 7, we can see that the ultimate tubing pressure in Fig. 6 is smaller than the ultimate tubing pressure at the hold-down packer, which means that the most dangerous location of the string is the wellhead A rather than hold-down packer P. Besides, when wellhead casing pressure is constant, string safety factor will decrease non-linearly with the increase of tubing pressure until reaching the designed safety factor 1.2.

In order to further analyze and study the safety parameters under fracturing conditions, we extracted the ultimate oil pressure and casing pressure in Fig. 6 and obtained the safe operating range under fracturing condition (Fig. 8). In Fig. 8, curve $n_s = 1.2$ is the safety extreme boundary of tubing pressure and casing pressure. If the tubing pressure and casing pressure values are in the green area of Fig. 8, the safety factor $n_{Real} \geq 1.2$, the whole string is safe, otherwise it is dangerous. Fig. 8 provides quantitative data for the optimization design of fracturing string safety factor in Tahe oilfield. Similarly, the

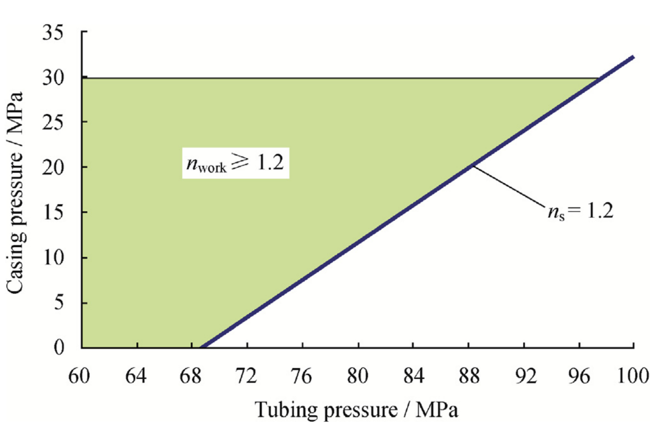

Fig. 8. Relationship of tubing pressure and casing pressure with safe working range.

established mathematical-mechanical models and developed software can be applied in the safety evaluation of different strings of other multi-stage fracturing process. Now, this research results have been applied and verified in Sichuan oil and gas field.

4. Conclusions

(1) The mechanical models are established for three working cases, i.e. multi-packer setting, opening differential pressure sliding sleeve, and opening balling sliding sleeve. Besides, the string mechanical-mathematical models are built based on the mechanical characteristics of three working conditions.

(2) According to the Lame formula of elastic dynamic theory and Von-Mises formula, the mathematical model of equivalent stress for safety evaluation of the string under the combined effect of compression, extrusion and axial stress has been derived. Also, the multi-stage fracturing string strength evaluation model has been constructed.

(3) Based on the mathematical-mechanical models in this paper, a piece of mechanical safety evaluation software has been developed on Visual Basic 2010 platform for multi-stage horizontal fracturing string, which can directly evaluate the safety of multi-stage fracturing string with multi-packers, providing a simple and reliable tool for the safety evaluation of horizontal tubing string in different working conditions.

(4) According to the models proposed in this paper, the safe operating range of casing pressure and tubing pressure can be obtained, which provides a theoretical basis for the optimization design of multi-stage horizontal fracturing string safety parameters. This research results have been applied and verified in Tahe oilfield.

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