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Theoretical and experimental study of strain localization and energy dissipation at fatigue crack tip

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Abstract

This work includes both experimental and theoretical studies of crack propagation under cyclic loading. The theoretical part is devoted to the description of energy balance under plastic deformation based on the results of statistical model of mesodefect ensemble proposed in the Institute of continuous media mechanics UB RAS and application of this model to the calculation of strain energy distribution at crack tip. The experimental part of the work is devoted to the study of the temperature evolution at crack tip with high spatial and time resolution. The thermodynamic measurement was combined with study of strain field at crack tip based on the digital image correlation technique. The analysis of experimental data allows us to determine the evolution of plastic deformation zone and dissipation energy at the crack tip.

Keywords: fatigue crack, numerical simulation, infrared thermography, digital image correlation;

1. Introduction

Nowadays investigation of crack propagation in metals and knowledge of its laws have great importance in the developments of various designs and constructions. To create new experimental and theoretical technologies and improve existing ones one has to reach a more deep understanding of physical nature of cracks. Most existing models assume that the volume defects evolution takes place at the final stage of plastic deformation. However, the
data obtained from systematic studies of defects evolution obtained by Betekhtin et al. (1997) shows that the defects play the important role in deformation process at every stage of plastic deformation. These defects emergence at the early stage of deformation and effect on the microplasticity and failure processes. The models described the interaction between damage accumulation and plasticity processes were developed by Naimark (2003). The description of damage accumulation includes a consideration of the mesodefect ensemble evolution, their coherent development and interaction, the effect on the relaxation properties of materials and merging into the main crack.

The numerical simulation of strain localization and crack propagation in plate vanadium specimen under cyclic loading were carried out in the finite-element package Simulia Abaqus using procedure UMAT. The big attention was paid on the calculation of plastic work and heat dissipation under investigated process. The results were qualitatively compared with original experimental data, and with the results obtained using standard (incorporated in Abaqus) elastic–plastic model.

The original experimental data include the infrared thermography and digital image correlation data. Infrared thermography is the simple way to measure the temperature of surface and to monitor of crack propagation during cycling test. In materials under cyclic deformation, fatigue cracks are initiated in the area of plastic deformation localization and lead to an intensive heat dissipation Shanyavskiy (2003). Investigation of the heat dissipative and absorption laws can take information about dissipative ability of material and current state of structural evolution. Investigation of the processes accompanying plastic deformation from thermodynamic point of view is actual problem that is discussed by the researchers. Basing principal of this approach and method for experimental infrared data processing are developed in works of Chrysochoos et al. (1989, 2000). The full-field measurements methods namely infrared thermography for temperature field measurements and digital image correlation (DIC) for strain fields measurements are presented by Dolinski (2010). In this work the digital image correlation technique was also used to study the strain localization at crack tip. The technique allows us to monitor the process of crack propagation and compare the strain and temperature fields in process zone. In the future development the combination of these techniques allows one to develop a real time monitoring technique forecast of fatigue crack propagation based on the dissipated energy estimation.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>( \vec{b} )</td>
<td>unit vector in the shear direction</td>
</tr>
<tr>
<td>( F )</td>
<td>specific free energy</td>
</tr>
<tr>
<td>( \vec{t} )</td>
<td>unit normal vector to the shear plane</td>
</tr>
<tr>
<td>( L_c )</td>
<td>the mean distance between the defects</td>
</tr>
<tr>
<td>( L_s )</td>
<td>the mean size of the defect</td>
</tr>
<tr>
<td>( n )</td>
<td>defect density</td>
</tr>
<tr>
<td>( \tilde{p} )</td>
<td>mesoscopic defect density tensor</td>
</tr>
<tr>
<td>(</td>
<td>p</td>
</tr>
<tr>
<td>( \check{s} )</td>
<td>microscopic defect tensor</td>
</tr>
<tr>
<td>( S )</td>
<td>shift intensity</td>
</tr>
<tr>
<td>( S_{r} )</td>
<td>yield stress</td>
</tr>
<tr>
<td>( \delta )</td>
<td>scale invariant structural parameter</td>
</tr>
<tr>
<td>( \varepsilon^e )</td>
<td>elastic strain tensor</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>deviator part of the elastic strain</td>
</tr>
<tr>
<td>( \varepsilon^p )</td>
<td>plastic strain tensor</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Cauchy stress tensor</td>
</tr>
<tr>
<td>(</td>
<td>\sigma</td>
</tr>
<tr>
<td>( \sigma_{y} )</td>
<td>deviator part of the stress tensor</td>
</tr>
<tr>
<td>( \tau_{i} )</td>
<td>characteristic relaxation times</td>
</tr>
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</table>

2. Mathematical model

Mesoscopic defects (mesoshears) can be described by the following tensor
\[
\tilde{s} = \frac{1}{2} S (\bar{b} \bar{b} + b \bar{b}) ,
\]  
(1)

where \( \bar{T} \) - a unit normal to the shear plane, \( \bar{b} \) - unit vector in the shear direction, \( S \) - shift intensity.

Averaging \( \tilde{s} \) over an elementary volume with corresponding distribution function allows us to introduce the tensor parameter \( \tilde{p} \), which can be considered as a deformation caused by the defects:

\[
\tilde{p} = n \langle \tilde{s} \rangle ,
\]  
(2)

where \( n \) - defect density.

The solution of this statistical problem proposed by Naimark (2003) shows that material susceptibility to the defect growth in the deformation process can be described by one dimensionless parameter

\[
\delta = \left( \frac{L_n}{L_c} \right)^3 ,
\]  
(3)

where \( L_n \) - the mean size of the defect nuclei, \( L_c \) - the mean distance between the defects.

It is also shown that these characteristic lengths \( (L_c, L_n) \) are determined by non-linear kinetics of \( \tilde{p} \) and thus, the defect density distribution determines the structural sensitivity to its further growth.

Elastic strains defined by linear Hooke’s law. Full strain rate can be represented as the sum of three components: elastic strain rate \( \hat{\varepsilon}^e \), plastic strain rate \( \hat{\varepsilon}^p \) and strain rate caused by defects:

\[
\hat{\varepsilon} = \hat{\varepsilon}^e + \hat{\varepsilon}^p + \hat{\varepsilon}^d .
\]  
(4)

Assuming a quasilinear relationship between the thermodynamic forces and flows, there were obtained constitutive equations for calculating kinetics of plastic and structural strains:

\[
\hat{\varepsilon}^p = \Gamma_\sigma \tilde{\sigma} + \Gamma_{p\sigma} (\tilde{\sigma} - \frac{\partial F}{\partial \tilde{p}}) ,
\]  
(5)

\[
\hat{p} = \Gamma_p (\tilde{\sigma} - \frac{\partial F}{\partial \tilde{p}}) + \Gamma_{p\sigma} \tilde{\sigma} ,
\]  
(6)

where \( F \) is a part of the free energy, which is responsible for the energy defect subsystem, parameters \( \Gamma_\sigma , \Gamma_p , \) are kinetic coefficients having following form

\[
\Gamma_\sigma = \frac{1}{\tau_\sigma} \frac{1}{1 + \exp \left( \frac{\left| \sigma \right| - S_c}{a_1} \right)} ,
\]  
(7)

\[
\Gamma_p = \frac{1}{\tau_p} \frac{1}{1 + \exp \left( \frac{H(\left| \sigma \right|, p, \delta, \rho_c, \sigma_c) - S_y}{a_2} \right)} ,
\]  
(8)

\[
\Gamma_{p\sigma} = \frac{1}{\tau_{p\sigma}} .
\]  
(9)
where \( \tau_\sigma, \tau_\mu, \tau_{\rho_{\sigma}} \) - characteristic relaxation times, \( |\sigma| \) - stress intensity tensor, \( S, a_1, a_2 \) - material constants, \( S_\gamma \) - yield stress, \( |\rho| \) - intensity of \( \rho_\gamma \), \( \sigma, \gamma, \rho, \sigma \) - scaling factors, \( H(|\sigma|,|\rho|,\delta,\rho) = |\sigma| - 2\mu\sigma \left[ \delta(f + 1)|\rho|\rho - |\rho|\rho \right] \) - material function (it can be considered as “degree of system nonequilibrium”).

It is supposed that thermodynamic force \( \tilde{\sigma} - \frac{\partial F}{\partial \rho} \) can be written as:

\[
\tilde{\sigma} - \frac{\partial F}{\partial \rho} = \left[ \frac{1}{\delta} \left( \frac{\tilde{\sigma}}{2G\sigma} + \frac{\tilde{\rho}}{\rho_{\gamma}} \right) - \left( f \left( \frac{|\rho|}{\rho_{\gamma}} \right) \rho_{\gamma} + 1 \right) \frac{\tilde{\rho}}{\rho_{\gamma}} \right],
\]

where \( f(|\rho|) \) denotes a power function for modeling of nonlinear hardening process:

\[
f \left( \frac{|\rho|}{\rho_{\gamma}} \right) = k \left( \frac{|\rho|}{\rho_{\gamma}} \right)^a,
\]

\( k \) is a scaling factor, \( a \) is the exponent.

The system of equations (4)-(11) can be considered as a closed system of equation for modeling of damage accumulation and plasticity process in metals.

3. Numerical simulation

Numerical simulation was carried out using the finite element package Abaqus. Material behaviour is described by the aforementioned model introducing above using the procedure UMAT. There was considered a numerical experiment on the cyclic loading of the steel specimen, containing a central crack. The gage part of the samples was 3x55x250 mm. The specimen contains a central crack; its length is 17 mm.

![Fig. 1. (a) stress distribution in the vicinity of the crack tip, (b) plastic work distribution at crack tips (numerical simulation).](image-url)

The extended finite element method (XFEM) capability in Abaqus was used to model crack propagation. XFEM models a crack as an enriched feature by adding degrees of freedom in elements with special displacement functions. XFEM does not require the mesh to match the geometry of the discontinuities. It can be used to simulate initiation and propagation of a discrete crack along an arbitrary, solution – dependent path without the requirement of remeshing.

A maximum principal stress criterion was used to model the damage initiation. Figure 1 displays the zoomed stress field near crack tip and the plastic work distribution near the crack tip. In figure 1b we can see several areas...
with the centre at the crack tip and increasing radius, that is confirms the experimental data presented in Fig. 2.

Figures 2 present the experimental temperature field on the specimen surface near crack tip and experimental obtained neat power near crack tip, respectively (The description of infrared monitoring technique is presented in next section).

![Fig. 2. (a) experimental temperature field at the crack tip; (b) corresponding heat sources distribution at the crack tip.](image)

4. Experimental conditions and results

Experimental study of temperature evolution at the fatigue crack tip was carried out on the plane specimens of titanium alloy (Ti-4.2Al-1.6Mn). The specimens were manufactured from a commercial titanium sheet with a thick of 3 mm. Modulus of elasticity 64 GPa, yield stress 800 MPa, ultimate stress 900 MPa, fatigue limit 460 MPa, fracture toughness 75.6 MPa/√m. The gage part of the samples was 3x55x250 mm. The fatigue crack (about 10 mm) was initiated at the initial stage of the experiment by high amplitude cyclic loading. Then the load was decreased (Pmin -18 MPa, Pmax 367 MPa, loading frequency 10 Hz) to slow down the rate of crack propagation.

The surface of the specimens was polished in several stages by the abrasive paper (at the final stage of polishing the grit size does not exceed 3 μm). Before starting the experiment, the polished surface was covered by a thin layer of amorphous carbon.

The temperature evolution was recorded by infrared camera FLIR SC 5000. The spectral range of the camera is 3-5 μm. The maximum frame size is 320×256 pixels; the spatial resolution is 10^-4 meters. The temperature sensitivity is 25 mK at 300 K. Calibration of the camera was made based on the standard calibration table. It was used FLIR SC5000 MW G1 F/3.0 close-up lens (distortion is less than 0.5%).

Digital image correlation LaVision Strain Master (SM) was used to track surface displacements. This method allows us to recover the displacement and deformation evolution of the sample surface of different materials under different types of loading with a high precision. The mathematical apparatus of DIC method is described by Sutton et al. (2009). The digital camera has a resolution of 1600x1200 pixels and it is set in approximately 30 cm from the sample. Pixel on recorded images represents an approximately 3.4 μm square on the specimen surface. Displacement and strain fields were calculated for the surface area of the sample size of 12 mm by 15 mm.

To calculate the heat source distribution an original numerical algorithm was developed. The algorithm includes filtration and relative motion compensation procedures. The spatially fixed temperature signal of the smooth specimen was processed using the two-dimensional discrete Fourier transform with a standard Gaussian kernel to increase data accuracy and eliminate the influence of random temperature fluctuations. To compensate this relative motion, the algorithm described in detail by Fedorova (2012) was used. The power of heat source at crack tip is shown in Fig. 2.

The figure 5 presents the comparison of heat sources distribution and strain localization at crack tip. The maxima of heat dissipation and strain localization observed far from the crack tip (point 20 in figure 5(a)). The crack propagation process exhibits jumping behavior. The crack jumps distribution is presented in figure 5(b).
5. Conclusions

The results of experimental and theoretical study of fatigue crack propagation are presented. Based on the original statistical model of defect evolution the constitutive equations for elastic-plastic medium with mesodefects were proposed. These equations were generalized for three-dimensional modeling of cyclic loading. Numerical simulation was performed using finite-element package Abaqus by subroutine UMAT. Fracture modeling was carried out using XFEM capability in Abaqus. Simulation results are compared with experimental study of crack propagation using infrared thermography and digital image correlation.

The infrared technique has been applied to investigate the effect of heat dissipation under quasi-static loading and its localization at the crack tip under cyclic loading. The data processing algorithm allows us to calculate the values of heat dissipation at the crack tip and to determine the area of plastic deformation localization. To study the plastic deformation localization the digital image correlation technique was used. It allows us to show the coincidence of the maxima of strain localization and heat dissipation and determine the position of this point respectively crack tip. The DIC allows us to determine the shape of strain localization zone and show the jump behavior of crack.

The mutual application of developed experimental technique and theoretical description of defect evolution open interesting perspectives for study of fatigue crack propagation in metals. At present we have show, that the experimental and numerical results have good qualitative agreement.

Acknowledgements

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References