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Unstructured Mixed Grid and SIMPLE Algorithm based Model for 2D-SWE

Xinhua Lu[a], Bingjiang Dong[b], Bing Mao[a], Xiaofeng Zhang[a], a*

[a]*State Key Laboratory of Water Resources and Hydropower Engineering Science, Wuhan Univ., Wuhan 430072, China*
[b]*Bureau of Hydrology Changjiang Water Resources Commission, 1863 Jiefang Avenue, Wuhan 430010, China*

Abstract

A 2D depth-averaged flow model was developed using implicit schemes on unstructured mixed grid. The implicit time-marching algorithm is adopted to make the model much stable. To suppress the numerical oscillation, the TVD (total-variation diminishing) based second-order convection scheme is employed in the framework of finite volume method. The new model is validated using measured data and compared with YGLai model (newly developed by Lai (2010)). Results show that the new model is consistent with the measured data fairly well. The comparison with YGLai model indicates that our new model is generally better with respect to accuracy.

Keywords: Unstructured mesh; Mixed grid; TVD; SIMPLE; Implicit solution; Depth-averaged flow

1. Introduction

In the solution of two-dimensional depth-averaged flow equations (2D-SWE), the explicit scheme is favoured by many researches. However, the well-known CFL criterion must be satisfied in using the explicit scheme. When a large time-scale problem is modelled such as a long time flow-sediment-river bed deformation simulation in a practical reservoir, the explicit scheme is of low efficiency.

Considering this, researchers resort to implicit time-marching scheme. Until now, the implicit discretization scheme under unstructured grid is less studied. This paper tries to develop a model using the implicit and bounded higher-order convection scheme on unstructured mixed grid which is relatively less involved in literatures. This model is rather stable due to the implicit discretization technique, and

* Corresponding author: Xinhua Lu. Tel.: +1-410-336-8552.
E-mail address: xhlu@yahoo.cn.

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presents a good resolution of sharp gradients for the using of bounded higher-order convection scheme, and can be flexibly implemented to arbitrary complex topography by using of unstructured mixed grids.

2. Governing equations and discretization strategies

The governing equations of 2D-SWE can be expressed as Eq.1 [1].

$$\frac{\partial h\Phi}{\partial t}+\nabla\cdot(h\vec{u}\Phi)=\nabla\cdot(h\Gamma_{\Phi}\nabla\Phi)+S_{\Phi} \tag{1}$$

where, Φ = the general variable and can be set as velocity u, v, turbulent k and its dissipation rate ε. h is water depth; $\vec{u}=(u,v)$ is the vector of depth-averaged velocity; Γ_{Φ} = the general diffusion coefficient of variable Φ; S_{Φ}=source term.

Eq.1 is discreted using finite volume method. All the conservative variables are stored at the center of the control volume. To an arbitrary control volume P_0, if implicit temporal discretization method is employed, Eq.2 is obtained.

$$\frac{h_{P_0}V_{P_0}(\Phi_{P0}-\Phi_{P_0}^{0})}{\Delta t}+\sum_{j=1}^{N}\left[(h\Phi)_j\left(\vec{u_j}\cdot\overrightarrow{SS}_j\right)\right]-\sum_{j=1}^{N}\left[(h\Gamma_{\Phi})_j\left((\nabla\Phi)_j\cdot\overrightarrow{SS}_j\right)\right]=\left(S_C+S_P\Phi_{P_0}\right)V_{P_0} \tag{2}$$

where, V_{P_0} =volume of cell P_0; $\Phi_{P_0}^{0}$ =value of Φ_{P0} at the last time step; $\overrightarrow{SS}_j=\left(SS_{jx},SS_{jy}\right)=\vec{S}_j\times\vec{k}$ and $\vec{S}_j=\left(S_{jx},S_{jy}\right)$ is the position vector of the j^{th} edge and $\vec{k}$ =outward unit normal vector of cell P_0; N is edge numbers of cell P_0; subscript j means values at the j^{th} edge. The source term S_{Φ} is linearized to $S_C+S_P\Phi_{P_0}$ [2].

In Eq.2, $(h\Phi)_j\left(\vec{u_j}\cdot\overrightarrow{SS}_j\right)$ and $(h\Gamma_{\Phi})_j\left((\nabla\Phi)_j\cdot\overrightarrow{SS}_j\right)$ are the convective term and diffusion term, respectively.

To discretize the convcective term, Darwish and Moukalled [3] (abbreviated as DM) proposed a TVD-based method, which method allows the implementation of TVD schemes under unstructured grid. For TVD schemes, the face value is calculated by

$$\Phi_j=\Phi_{P_0}+\frac{1}{2}\sigma_j\left(\Phi_{P_j}-\Phi_{P_0}\right) \tag{3}$$

where, σ_j is the limiter corresponding to the jth edge. A number of limiters have been proposed, of which three representative ones, namely, Minmod limiter, Vanleer limiter and van Albada limiter are chosen for study.

The diffusion term can be calculated using values at the auxiliary points [4].

The general discretized equation at control volume P_0 is

$$a_{P_0}\Phi_{P_0}+\sum_{j=1}^{N}a_{P_j}\Phi_{P_j}=b_0 \tag{4}$$

The different discretization strategies contribute to different models. Here after, if DM method is used for convective term, the model is called DM-type model, such as DM-Minmod model, DM-Vanleer model. In Lai's model [1], a simple central difference scheme with a damping term is employed for the discretization of the convective term and the diffusion term is decomposed into "normal" and "cross" diffusion parts.

Eq.4 is employed for the solution of any conservative variable. To tackle the velocity-water stage coupling problem, the SIMPLE algorithm is adopted. The water stage correction equation can be derived by using of the continuity equation and the momentum interpolation method.

3. Simulation results

To test the performance of the DM-type models and the Lai model, various test cases are available [1][3]. Among the massive cases, lots of models are verified to be consistent quite well with the simple theoretical solutions or simple open channel flows test cases, but may perform rather bad when the simulation case is complicated. Here a complex test case, that is to say, a two-dimensional diversion flow test case studied in Lai [1], is chosen for verification of the new model.

The studied domain consists of a 6.0m×0.3m (*x*-direction by *y*-direction) main channel. At the center of the main channel there is a side channel with a size of 0.3m×3.0m (*x*-direction by *y*-direction). The main channel and the side channel are perpendicular to each other. The entire channel is horizontal and smooth with Manning's coefficient around 0.012. The flow rate in the main channel is 0.00567m^3/s and the water stage at the exit of the main channel and side channel are 0.0555m and 0.0465m, respectively. The sketch of the channel and the mesh is shown in Fig. 1. Note that two partial enlarged drawings are presented to illustrate the local grids which are arbitrarily disorganized to contain both the triangles and quadrilaterals. Totally 9303 cells are employed, in which 640 cells are quadrilaterals. The minimum and maximum cell area is 1.56×10^{-4} m^2 and 7.00×10^{-4} m^2, respectively.

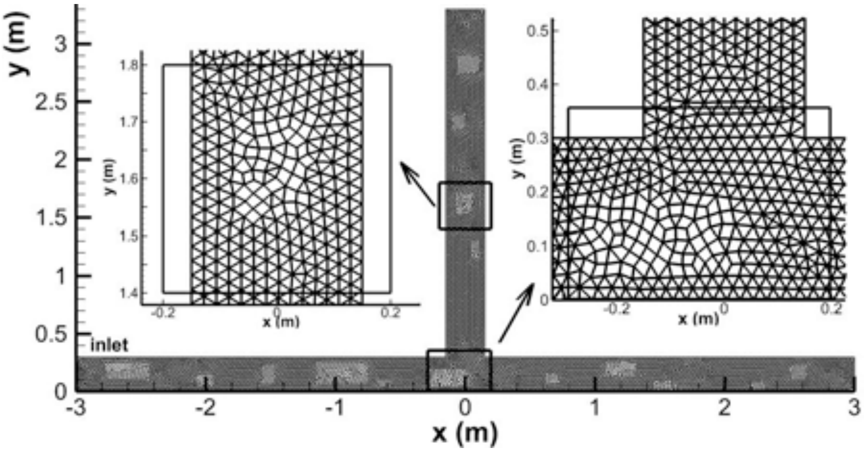

Figure 1. Sketch of the solution domain and grid layout

Fig. 2 illustrates the simulated distribution of water stage along the right- and left- bank of main channel and side channel. Also shown are the measured values.

As is seen, generally all the three DM-type models and YGLai model are agree quite well with the measured values, both in the main channel and the side channel. One may notice that in Fig. 2(d) at the junction near the left bank of the side channel, the simulated water stage is lower than it should be. It's still an unsolved problem using 2D-SWE in the published literatures. This discrepany may be caused by the turbulent closure model chosen and the three-dimensional characteristics of the flow at the junction which can't be accurately resolved by 2D-SWE.

The simulation results by the DM-type models nearly overlap with each other, and are slightly better than the results from YGLai model especially at the right bank of the main channel as marked by a dash-dot-dot rectangle in Fig. 2(a).

To quantitatively assess the performance of different models, Ghostine et al. [5] introduced an estimator of quality, namely, EM_ϕ expressed as Eq.5 to demonstrate the approximation level between predicted and measured values. However, this estimator can not reflect the global approximation level, namely the average relative errors between predicted and measured values at all of the measured points. Considering this, here we introduce another estimator of EA_ϕ which is expressed in Eq.6. Obviously a small value of EM_ϕ and EA_ϕ means a good performance of the numerical model and vice versa.

$$EM_\phi = 100 \cdot \max\left(\left|\phi_p - \phi_m\right| / \left|\phi_m\right|\right)_i . \tag{5}$$

$$EA_\phi = 100 / n \cdot \sum_{i=1}^{n}\left(\left|\phi_p - \phi_m\right| / \left|\phi_m\right|\right)_i . \tag{6}$$

where ϕ=u, v, h, z or any other variables; ϕ_p and ϕ_m =predicted and observed values at measured point i; n=the number of measured points.

Table 1 gives the results of EM_h and EA_h in assesement of the simulated sidewall water depth using different models. At the main channel, EM_h and EA_h of YGLai model is about 0.26~0.29 and 0.013~0.018 larger than DM-type model. The significant disparity demonstrates that DM-type model is better than YGLai model with respect to the accuracy. At the side channel, all the models give similar approximation levels between simulated and measured data. However, generally the DM-Type model especially the DM-Minmod model tends to be more accurate.

Table 1. EM_h and EA_h for different models at the main channel

Model	Right Bank, main channel (n=16)		Left Bank, main channel (n=22)		Right Bank, side channel (n=18)		Left Bank, side channel (n=17)	
	EM_h	EA_h	EM_h	EA_h	EM_h	EA_h	EM_h	EA_h
DM_Minmod	2.677	0.167	2.587	0.118	5.919	0.329	8.130	0.478
DM_Vanleer	2.649	0.166	2.580	0.117	5.905	0.328	8.356	0.492
DM_Albada	2.661	0.166	2.577	0.117	5.927	0.329	8.250	0.485
YGLai	2.938	0.184	2.848	0.130	6.026	0.335	8.239	0.485

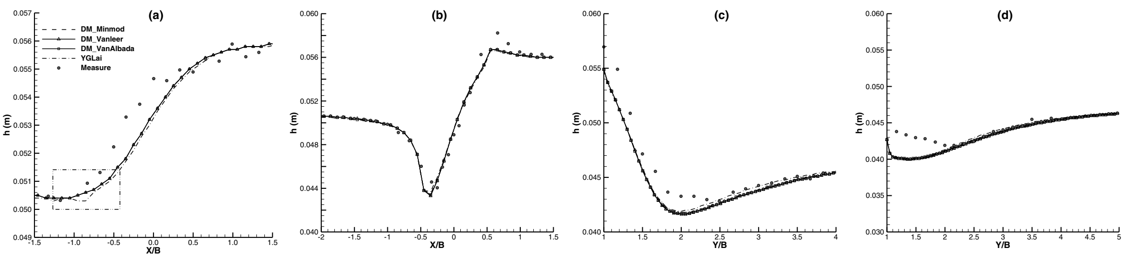

Fig. 2. Water stage distribution. (a) (b) at right- and left- bank of main channel; (c) (d) at right- and left- bank of side channel

4. Conclusion

The implicit time-marching method and TVD based second-order convection scheme are used in the development of a 2D depth-averaged model. This model is stable, robust and presents no numerical oscillation. This model is validated by using of a complicated two-dimensional diversion flow test case. It shows that in the main channel of the test case, the new models presents lower values of EM_h and EA_h than the YGLai model. This indicates that with respect to accuracy, our new model performs much better.

Acknowledgments

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