ELSEVIER
SCIENCE DIRECT®
PHYSICS LETTERS B
Physics Letters B 578 (2004) 341–348
www.elsevier.com/locate/physletb

A CP asymmetry in $e^+e^- \to \tilde{\chi}_i^0 \tilde{\chi}_j^0 \to \tilde{\chi}_j^0 \tau \tilde{\tau}_k$ with tau polarization

A. Bartl [a], T. Kernreiter [a], O. Kittel [a,b]

[a] *Institut für Theoretische Physik, Universität Wien, A-1090 Vienna, Austria*
[b] *Institut für Theoretische Physik und Astrophysik, Universität Würzburg, D-97074 Würzburg, Germany*

Received 10 October 2003; accepted 20 October 2003

Editor: G.F. Giudice

Abstract

We propose a CP-odd asymmetry in the supersymmetric process $e^+e^- \to \tilde{\chi}_i^0 \tilde{\chi}_j^0 \to \tilde{\chi}_j^0 \tau^\mp \tilde{\tau}_k^\pm$ by means of the transverse $\tau^\mp$ polarization. We calculate the asymmetry and cross sections at a future linear collider in the 500 GeV c.m.s. energy range with longitudinal polarized beams and high luminosity. We work in the Minimal Supersymmetric Standard Model with complex parameters μ, M_1 and A_τ. The asymmetry can reach values up to 60%. We also estimate the sensitivity for measuring the τ polarization necessary to probe the CP asymmetry.

1. Introduction

In supersymmetric (SUSY) extensions of the Standard Model (SM), some parameters can be complex. In the neutralino sector of the Minimal Supersymmetric Standard Model (MSSM), these are the higgsino mass parameter μ and the gaugino mass parameter M_1, while M_2 can be chosen real by redefining the fields. In addition, in the scalar tau sector of the MSSM, also the trilinear scalar coupling parameter A_τ can be complex. The non-zero phases φ_μ, φ_{M_1} and φ_{A_τ} of these parameters give rise to CP-odd observables, which are not present if CP is maintained. Measurements of such CP-odd observables will allow us to determine these phases, in particular also their signs.

E-mail addresses: bartl@ap.univie.ac.at (A. Bartl), tkern@hephy.oeaw.ac.at, tkern@qhepu3.oeaw.ac.at (T. Kernreiter), kittel@physik.uni-wuerzburg.de (O. Kittel).

0370-2693 © 2003 Elsevier B.V. Open access under CC BY license.
doi:10.1016/j.physletb.2003.10.097

In this Letter we consider neutralino production

$$e^+e^- \to \tilde{\chi}_i^0 \tilde{\chi}_j^0, \quad i,j = 1,\ldots,4 \tag{1}$$

and the subsequent two-body decay of one neutralino

$$\tilde{\chi}_i^0 \to \tilde{\tau}_m^\pm \tau^\mp, \quad m = 1,2, \tag{2}$$

for a fixed τ-polarization. We would like to stress that without measuring the transverse $\tau^\mp$ polarization no sensitivity to the phase of A_τ, φ_{A_τ}, can be obtained, because (2) is a two-body decay. When summing over the τ^- polarization, we are sensitive only to CP violation in the production process [1,2]. The τ^- polarization is given by [3]

$$\mathbf{P} = \frac{\mathrm{Tr}(\varrho \boldsymbol{\sigma})}{\mathrm{Tr}(\varrho)}, \tag{3}$$

with ϱ being the Hermitean spin density matrix of the τ^- and σ_i the Pauli matrices. We use a convention for $\mathbf{P} = (P_1, P_2, P_3)$ where the component P_3 is the longitudinal polarization and P_1 is the transverse polarization in the plane formed by $\mathbf{p}_{e^-}$ and $\mathbf{p}_\tau$. The component P_2 is the polarization perpendicular to $\mathbf{p}_\tau$ and $\mathbf{p}_{e^-}$ and is proportional to the triple-product

$$\mathbf{s}_\tau \cdot (\mathbf{p}_\tau \times \mathbf{p}_{e^-}), \tag{4}$$

where $\mathbf{s}_\tau$ is the τ^- spin 3-vector. Since under time reversal the triple-product changes sign, the transverse τ^- polarization P_2 is a T-odd observable. Due to CPT invariance, P_2 is actually a CP-odd observable if absorbtive phases from final-state interactions are neglected.

In this Letter we study the asymmetry

$$\mathcal{A}_{\mathrm{CP}} = \frac{1}{2}(P_2 - \bar{P}_2), \tag{5}$$

which is CP-odd, even if absorbtive phases cannot be neglected. In Eq. (5), $\mathbf{P}$ denotes the τ^- polarization vector in the decay $\tilde{\chi}_i^0 \to \tilde{\tau}_m^+ \tau^-$ and $\bar{\mathbf{P}}$ denotes the τ^+ polarization vector in the decay $\tilde{\chi}_i^0 \to \tilde{\tau}_m^- \tau^+$. In Born approximation it follows from Eq. (5) that $\mathcal{A}_{\mathrm{CP}} = P_2$.

In Section 2 we briefly review stau mixing in the MSSM and define the part of the interaction Lagrangian which is relevant for our analysis. In Section 3 we define the τ spin density matrix ϱ and give the analytical formulae. In Section 4 we discuss the qualitative properties of the asymmetry $\mathcal{A}_{\mathrm{CP}}$. We present numerical results for $e^+e^- \to \tilde{\chi}_1^0 \tilde{\tau}_1 \tau$ in Section 5. We summarize and conclude in Section 6.

2. Stau mixing and Lagrangian

We give a short account of $\tilde{\tau}_L$–$\tilde{\tau}_R$ mixing for complex $\mu = |\mu| e^{i\varphi_\mu}$, $A_\tau = |A_\tau| e^{i\varphi_{A_\tau}}$ and $M_1 = |M_1| e^{i\varphi_{M_1}}$. The masses and couplings of the τ-sleptons follow from the Hermitian 2×2 mass matrix which in the basis $(\tilde{\tau}_L, \tilde{\tau}_R)$ reads [4,5]

$$\mathcal{L}_M^{\tilde{\tau}} = -\big(\tilde{\tau}_L^\dagger, \tilde{\tau}_R^\dagger\big) \times \begin{pmatrix} M^2_{\tilde{\tau}_{LL}} & e^{-i\varphi_{\tilde{\tau}}} \big|M^2_{\tilde{\tau}_{LR}}\big| \\ e^{i\varphi_{\tilde{\tau}}} \big|M^2_{\tilde{\tau}_{LR}}\big| & M^2_{\tilde{\tau}_{RR}} \end{pmatrix} \begin{pmatrix} \tilde{\tau}_L \\ \tilde{\tau}_R \end{pmatrix}, \tag{6}$$

with

$$M^2_{\tilde{\tau}_{LL}} = M^2_{\tilde{L}} + \left(-\frac{1}{2} + \sin^2\Theta_W\right)\cos 2\beta\, m_Z^2 + m_\tau^2, \tag{7}$$

$$M^2_{\tilde{\tau}_{RR}} = M^2_{\tilde{E}} - \sin^2\Theta_W \cos 2\beta\, m_Z^2 + m_\tau^2, \tag{8}$$

$$M^2_{\tilde{\tau}_{RL}} = \big(M^2_{\tilde{\tau}_{LR}}\big)^* = m_\tau\big(A_\tau - \mu^* \tan\beta\big), \tag{9}$$

$$\varphi_{\tilde{\tau}} = \arg\big[A_\tau - \mu^* \tan\beta\big], \tag{10}$$

where m_τ is the mass of the τ-lepton, Θ_W is the weak mixing angle, $\tan\beta = v_2/v_1$ with v_1 (v_2) being the vacuum expectation value of the Higgs field H_1^0 (H_2^0), and $M_{\tilde{L}}$, $M_{\tilde{E}}$, A_τ are the soft SUSY-breaking parameters of the $\tilde{\tau}_i$ system. The $\tilde{\tau}$ mass eigenstates are $(\tilde{\tau}_1, \tilde{\tau}_2) = (\tilde{\tau}_L, \tilde{\tau}_R)\mathcal{R}^{\tilde{\tau}\,T}$ with

$$\mathcal{R}^{\tilde{\tau}} = \begin{pmatrix} e^{i\varphi_{\tilde{\tau}}}\cos\theta_{\tilde{\tau}} & \sin\theta_{\tilde{\tau}} \\ -\sin\theta_{\tilde{\tau}} & e^{-i\varphi_{\tilde{\tau}}}\cos\theta_{\tilde{\tau}} \end{pmatrix}, \tag{11}$$

and

$$\cos\theta_{\tilde{\tau}} = \frac{-|M^2_{\tilde{\tau}_{LR}}|}{\sqrt{|M^2_{\tilde{\tau}_{LR}}|^2 + (m^2_{\tilde{\tau}_1} - M^2_{\tilde{\tau}_{LL}})^2}},$$
$$\sin\theta_{\tilde{\tau}} = \frac{M^2_{\tilde{\tau}_{LL}} - m^2_{\tilde{\tau}_1}}{\sqrt{|M^2_{\tilde{\tau}_{LR}}|^2 + (m^2_{\tilde{\tau}_1} - M^2_{\tilde{\tau}_{LL}})^2}}. \tag{12}$$

The mass eigenvalues are

$$m^2_{\tilde{\tau}_{1,2}} = \frac{1}{2}\Big(\big(M^2_{\tilde{\tau}_{LL}} + M^2_{\tilde{\tau}_{RR}}\big) \mp \sqrt{(M^2_{\tilde{\tau}_{LL}} - M^2_{\tilde{\tau}_{RR}})^2 + 4|M^2_{\tilde{\tau}_{LR}}|^2}\Big). \tag{13}$$

The part of the interaction Lagrangian of the MSSM relevant to study the decay (2) reads (in our notation and conventions we follow closely [7,8]):

$$\mathcal{L}_{\tilde{\tau}\tau\tilde{\chi}^0} = \tilde{\tau}_k \bar{\tau}\big(b^{\tilde{\tau}}_{ki} P_L + a^{\tilde{\tau}}_{ki} P_R\big)\tilde{\chi}_i^0 + \text{h.c.}, \quad i = 1, \ldots, 4,\ k = 1, 2, \tag{14}$$

with

$$a^{\tilde{\tau}}_{ki} = g\big(\mathcal{R}^{\tilde{\tau}}_{kn}\big)^* \mathcal{A}^\tau_{in}, \qquad b^{\tilde{f}}_{ki} = g\big(\mathcal{R}^{\tilde{\tau}}_{kn}\big)^* \mathcal{B}^\tau_{in} \quad (n = L, R), \tag{15}$$

$$\mathcal{A}_i^\tau = \begin{pmatrix} f^\tau_{Li} \\ h^\tau_{Ri} \end{pmatrix}, \qquad \mathcal{B}_i^\tau = \begin{pmatrix} h^\tau_{Li} \\ f^\tau_{Ri} \end{pmatrix}, \tag{16}$$

and

$$h^\tau_{Li} = \big(h^\tau_{Ri}\big)^* = Y_\tau N^*_{i3},$$
$$f^\tau_{Li} = -\frac{1}{\sqrt{2}}(\tan\Theta_W N_{i1} + N_{i2}),$$
$$f^\tau_{Ri} = \sqrt{2}\tan\Theta_W N^*_{i1}, \tag{17}$$

where $Y_\tau = m_\tau/\sqrt{2}m_W \cos\beta$, $P_{L,R} = 1/2(1 \mp \gamma_5)$ and g is the weak coupling constant. N is the 4×4 unitary neutralino mixing matrix, which diagonalizes the neutral gaugino–higgsino mass matrix $Y_{\alpha\beta}$, $N^*_{i\alpha} Y_{\alpha\beta} N^*_{k\beta} = m_{\tilde\chi^0_i}\delta_{ik}$, in the basis $(\tilde B, \tilde W^3, \tilde H^0_1, \tilde H^0_2)$ [7]. The part of the Lagrangian for the neutralino production (1) can be found, e.g., in [10,11].

3. Tau spin density matrix

The unnormalized, Hermitean, 2×2 spin density matrix of the τ^- is defined by:

$$\varrho^{\lambda_k\lambda'_k} \equiv \int \left(|\mathcal{M}|^2\right)^{\lambda_k\lambda'_k} \mathrm{dLips}, \tag{18}$$

where $\mathcal{M}$ and dLips are the amplitude squared and the Lorentz invariant phase space element, respectively, for the whole process of neutralino production (1) and decay (2). The τ^- helicities are denoted by λ_k and λ'_k. In the spin density matrix formalism (as used, e.g., in [9,10]) the amplitude squared can be written as

$$\left(|\mathcal{M}|^2\right)^{\lambda_k\lambda'_k} = 2\left|\Delta\left(\tilde\chi^0_i\right)\right|^2 \sum_{\lambda_i,\lambda'_i} (\rho_P)^{\lambda_i\lambda'_i} (\rho_D)^{\lambda_k\lambda'_k}_{\lambda'_i\lambda_i}. \tag{19}$$

It is composed of the un-normalized spin density matrices ρ_P for the production (1) and ρ_D for the decay (2), the propagator $\Delta(\tilde\chi^0_i) = 1/[p^2_{\chi_i} - m^2_{\chi_i} + i m_{\chi_i}\Gamma_{\chi_i}]$, with p_{χ_i}, m_{χ_i}, Γ_{χ_i} being the four-momenta, masses and widths of the decaying neutralino, respectively. ρ_P and ρ_D carry the helicity indices λ_i, λ'_i of the neutralinos and/or the helicity indices λ_k, λ'_k of the τ^-. The factor 2 in Eq. (19) is due to the summation of the $\tilde\chi^0_j$ helicities, whose decay is not considered. We introduce a set of spin basis vectors $s^a_{\chi_i}$ $(a = 1, 2, 3)$ for the neutralino $\tilde\chi^0_i$, which fulfill the orthonormality relations $s^a_{\chi_i}\cdot s^b_{\chi_i} = -\delta^{ab}$ and $s^a_{\chi_i}\cdot p_{\chi_i} = 0$. Then the spin density matrices can be expanded in terms of the Pauli matrices:

$$(\rho_P)^{\lambda_i\lambda'_i} = P\delta_{\lambda_i\lambda'_i} + \Sigma^a_P \sigma^a_{\lambda_i\lambda'_i},$$

$$(\rho_D)^{\lambda_k\lambda'_k}_{\lambda'_i\lambda_i} = \left[D^{\lambda_k\lambda'_k}\delta_{\lambda'_i\lambda_i} + \left(\Sigma^a_D\right)^{\lambda_k\lambda'_k}\sigma^a_{\lambda'_i\lambda_i}\right]. \tag{20}$$

The analytical formulae of P and Σ^a_P can be found in [10]. Introducing also a set of spin basis vectors s^b_τ for the τ^-, $D^{\lambda_k\lambda'_k}$ and $(\Sigma^a_D)^{\lambda_k\lambda'_k}$ can be expanded:

$$D^{\lambda_k\lambda'_k} = D\delta_{\lambda_k\lambda'_k} + D^b\sigma^b_{\lambda_k\lambda'_k}, \tag{21}$$

$$\left(\Sigma^a_D\right)^{\lambda_k\lambda'_k} = \Sigma^a_D\delta_{\lambda_k\lambda'_k} + \Sigma^{ab}_D\sigma^b_{\lambda_k\lambda'_k}. \tag{22}$$

The expansion coefficient are given by

$$D = \mathrm{Re}\left(b^{\tilde\tau}_{mi}{}^* a^{\tilde\tau}_{mi}\right) m_\tau m_{\tilde\chi_i} + \frac{1}{2}\left(\left|b^{\tilde\tau}_{mi}\right|^2 + \left|a^{\tilde\tau}_{mi}\right|^2\right)(p_\tau\cdot p_{\tilde\chi_i}), \tag{23}$$

$$D^b = \frac{1}{2} m_\tau\left(\left|b^{\tilde\tau}_{mi}\right|^2 - \left|a^{\tilde\tau}_{mi}\right|^2\right)\left(p_{\tilde\chi_i}\cdot s^b_\tau\right), \tag{24}$$

$$\Sigma^a_D = \frac{1}{2} m_{\tilde\chi_i}\left(\left|a^{\tilde\tau}_{mi}\right|^2 - \left|b^{\tilde\tau}_{mi}\right|^2\right)\left(p_\tau\cdot s^a_{\tilde\chi_i}\right), \tag{25}$$

$$\begin{aligned}\Sigma^{ab}_D = {}& \mathrm{Re}\left(b^{\tilde\tau}_{mi}{}^* a^{\tilde\tau}_{mi}\right)\left(p_\tau\cdot s^a_{\tilde\chi_i}\right)\left(p_{\tilde\chi_i}\cdot s^b_\tau\right) \\ & - \left(s^a_{\tilde\chi_i}\cdot s^b_\tau\right)\left[\frac{1}{2}\left(\left|b^{\tilde\tau}_{mi}\right|^2 + \left|a^{\tilde\tau}_{mi}\right|^2\right) m_\tau m_{\tilde\chi_i} + \mathrm{Re}\left(b^{\tilde\tau}_{mi}{}^* a^{\tilde\tau}_{mi}\right)(p_\tau\cdot p_{\tilde\chi_i})\right] \\ & + \mathrm{Im}\left(b^{\tilde\tau}_{mi}{}^* a^{\tilde\tau}_{mi}\right)\epsilon^{\mu\nu\rho\sigma} p_{\tau\,\mu} p_{\tilde\chi_i\,\nu} s^a_{\tilde\chi_i\,\rho} s^b_{\tau\,\sigma},\end{aligned} \tag{26}$$

with $\epsilon^{0123} = 1$. The last term in Eq. (26) contains the triple product (4). This term is proportional to $\mathrm{Im}(b^{\tilde\tau}_{mi}{}^* a^{\tilde\tau}_{mi})$ and is therefore sensitive to the phases φ_{A_τ}, φ_μ and φ_{M_1}. Inserting the density matrices of Eq. (20) into Eq. (19) yields

$$\begin{aligned}\left(|\mathcal{M}|^2\right)^{\lambda_k\lambda'_k} = {}& 4\left|\Delta\left(\tilde\chi^0_i\right)\right|^2\left[\left(PD + \Sigma^a_P\Sigma^a_D\right)\delta_{\lambda_k\lambda'_k} \right.\\ & \left. + \left(PD^b + \Sigma^a_P\Sigma^{ab}_D\right)\sigma^b_{\lambda_k\lambda'_k}\right].\end{aligned} \tag{27}$$

4. Transverse tau polarization and CP asymmetry

From Eqs. (3) and (27) we obtain for the transverse τ^- polarization

$$P_2 = \frac{\int |\Delta(\tilde\chi^0_i)|^2 \Sigma^a_P \Sigma^{a2}_D \,\mathrm{dLips}}{\int |\Delta(\tilde\chi^0_i)|^2 PD \,\mathrm{dLips}}, \tag{28}$$

which follows because in the numerator we have used $\int |\Delta(\tilde\chi^0_i)|^2 PD^2 \,\mathrm{dLips} = 0$ and in the denominator we have used $\int |\Delta(\tilde\chi^0_i)|^2 \Sigma^a_P\Sigma^a_D \,\mathrm{dLips} = 0$. As can be seen from Eq. (28), P_2 is proportional to the spin correlation term Σ^{a2}_D, Eq. (26), which contains the CP-sensitive part $\mathrm{Im}(b^{\tilde\tau}_{mi}{}^* a^{\tilde\tau}_{mi})\epsilon^{\mu\nu\rho\sigma} p_{\tau\,\mu} p_{\tilde\chi_i\,\nu} s^a_{\tilde\chi_i\,\rho} s^2_{\tau\,\sigma}$. In order to study the dependence of P_2 on the parameters, we expand

$$\begin{aligned}
&\mathrm{Im}\big(b_{1i}^{\tilde{\tau}\,*}a_{1i}^{\tilde{\tau}}\big) \\
&\quad = g^2\Big[\cos^2\theta_{\tilde{\tau}}\,Y_\tau\,\mathrm{Im}\big(f_{Li}^{\tau}N_{i3}\big) \\
&\qquad\qquad + \sin^2\theta_{\tilde{\tau}}\,Y_\tau\sqrt{2}\tan\Theta_W\,\mathrm{Im}(N_{i1}N_{i3}) \\
&\qquad\qquad + \sin\theta_{\tilde{\tau}}\cos\theta_{\tilde{\tau}}\Big(Y_\tau^2\,\mathrm{Im}\big(N_{i3}N_{i3}e^{i\varphi_{\tilde{\tau}}}\big) \\
&\qquad\qquad + \sqrt{2}\tan\Theta_W\,\mathrm{Im}\big(f_{Li}^{\tau}N_{i1}e^{-i\varphi_{\tilde{\tau}}}\big)\Big)\Big], \qquad (29)
\end{aligned}$$

using Eqs. (15)–(17) for $m = 1$. If CP violation is solely due to $\varphi_{A_\tau} \neq 0 \ (\mathrm{mod}\,\pi)$, we find from (29) that $P_2 \propto \sin 2\theta_{\tilde{\tau}} \sin\varphi_{\tilde{\tau}}$. We note that the dependence of $\varphi_{\tilde{\tau}}$ on φ_{A_τ} is weak if $|A_\tau| \ll |\mu|\tan\beta$, see Eq. (10). Thus, we expect that P_2 increases with increasing $|A_\tau|$.

Details concerning phase space and kinematics necessary for the calculation of P_2 from Eq. (28) can be found in [2]. The τ^- spin vectors s_τ^b are chosen by:

$$s_\tau^1 = \left(0, \frac{\mathbf{s}_2 \times \mathbf{s}_3}{|\mathbf{s}_2 \times \mathbf{s}_3|}\right), \qquad s_\tau^2 = \left(0, \frac{\mathbf{p}_\tau \times \mathbf{p}_{e^-}}{|\mathbf{p}_\tau \times \mathbf{p}_{e^-}|}\right),$$
$$s_\tau^3 = \frac{1}{m_\tau}\left(|\mathbf{p}_\tau|, \frac{E_\tau}{|\mathbf{p}_\tau|}\mathbf{p}_\tau\right). \qquad (30)$$

In order to measure P_2 and the CP asymmetry $\mathcal{A}_{\mathrm{CP}}$, Eq. (5), the τ^- from the decay (2) and the τ^+ from the subsequent $\tilde{\tau}_m^+$ decay, $\tilde{\tau}_m^+ \to \tilde{\chi}_1^0\tau^+$, have to be distinguished. This can be accomplished by measuring the energies of the τ's and making use of their different energy distributions [2].

A potentially large background may be due to stau production $e^+e^- \to \tilde{\tau}_l^+\tilde{\tau}_m^- \to \tau^+\tau^-\tilde{\chi}_1^0\tilde{\chi}_1^0$. However, these reactions would generally lead to "two-sided events", whereas the events from $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_i^0 \to \tau^+\tau^-\tilde{\chi}_1^0\tilde{\chi}_1^0$ are "one-sided events". Moreover, the background reaction is CP-even and will not give rise to a CP asymmetry, because the staus are scalars with a two-body decay.

5. Numerical results

We present numerical results for $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0$ and the subsequent decay of the neutralino into the lightest stau $\tilde{\chi}_2^0 \to \tilde{\tau}_1\tau$ for a linear collider (LC) with $\sqrt{s} = 500$ GeV and longitudinal polarized beams with $(P_{e^-}, P_{e^+}) = (0.8, -0.6)$ or $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$. This choice favors right or left selectron exchange in the neutralino production process, respectively [10]. We study the dependence of the asymmetry $\mathcal{A}_{\mathrm{CP}}$ and the production cross sections $\sigma \equiv \sigma_p(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0) \times \mathrm{BR}(\tilde{\chi}_2^0 \to \tilde{\tau}_1^+\tau^-)$ on the parameters φ_μ, $|\mu|$, φ_{M_1}, $|M_1|$, φ_{A_τ}, $|A_\tau|$ and $\tan\beta$.

For the calculations we assume $|M_1| = 5/3\tan^2\Theta_W M_2$, $m_\tau = 0$ and use in Eqs. (7) and (8) the renormalization group equations (RGEs) for the selectron masses [6], $M_{\tilde{L}}^2 = m_0^2 + 0.79M_2^2$ and $M_{\tilde{E}}^2 = m_0^2 + 0.23M_2^2$, taking $m_0 = 100$ GeV. The size of $|A_\tau|$ is restricted due to the tree-level vacuum stability conditions [12]. The restrictions on the masses of the SUSY particles are $m_{\tilde{\chi}_1^\pm} > 104$ GeV, $m_{\tilde{\tau}_1} > 100$ GeV and $m_{\tilde{\tau}_1} > m_{\tilde{\chi}_1^0}$. For the calculation of $\mathrm{BR}(\tilde{\chi}_2^0 \to \tilde{\tau}_1^+\tau^-)$ we concentrate on the parameter domain where two-body decays are allowed and neglect three-body decays. We consider the two-body decays

$$\begin{aligned}
&\tilde{\chi}_2^0 \to \tilde{\tau}_m\tau,\ \tilde{\ell}_{R,L}\ell,\ \tilde{\chi}_2^0 Z,\ \tilde{\chi}_n^\mp W^\pm,\ \tilde{\chi}_1^0 H_1^0, \\
&\quad \ell = e, \mu,\ m, n = 1, 2, \qquad (31)
\end{aligned}$$

with H_1^0 being the lightest neutral Higgs boson. The Higgs mass parameter m_A is chosen as $m_A = 1000$ GeV, which means that explicit CP violation is not important for the lightest Higgs state [13]. Furthermore, the neutralino decays into charginos and the charged Higgs bosons $\tilde{\chi}_2^0 \nrightarrow \tilde{\chi}_n^\pm H^\mp$, as well as decays into the heavy neutral Higgs bosons $\tilde{\chi}_2^0 \nrightarrow \tilde{\chi}_1^0 H_{2,3}^0$, are ruled out in this scenario.

In Fig. 1 we show the contour lines for $\mathcal{A}_{\mathrm{CP}}$ in the φ_{A_τ}–$|A_\tau|$ plane. As can be seen $\mathcal{A}_{\mathrm{CP}}$ is proportional to $\sin 2\theta_{\tilde{\tau}}\sin\varphi_{\tilde{\tau}}$, which is expected from Eq. (29). $\mathcal{A}_{\mathrm{CP}}$ increases with increasing $|A_\tau| \gg |\mu|\tan\beta$, which also follows from Eq. (29). Furthermore, in the parameter region shown the cross section σ varies between 20 fb and 30 fb.

In Fig. 2 we show the dependence of $\mathcal{A}_{\mathrm{CP}}$ on $\tan\beta$ and φ_{M_1}. Large values up to $\pm 20\%$ are obtained for $\tan\beta \approx 5$. Note that these values are obtained for $\varphi_{M_1} \approx \pm 0.8\pi$ rather than for maximal $\varphi_{M_1} \approx \pm 0.5\pi$. This is due to the complex interplay of the spin correlation terms in Eq. (28). In the region shown in Fig. 2, the cross section σ varies between 10 fb and 30 fb.

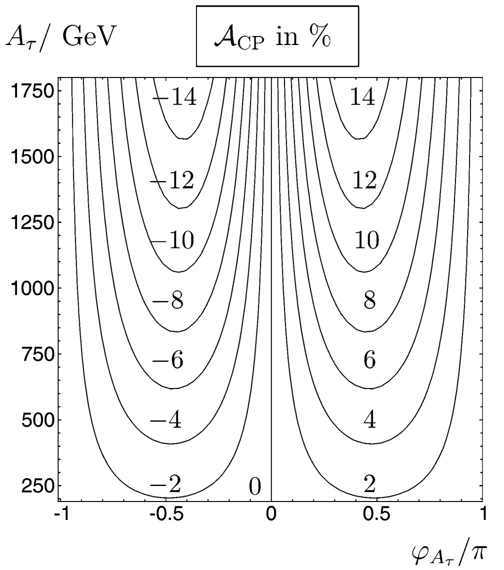


Fig. 1. Contour lines of $\mathcal{A}_{\mathrm{CP}}$ in Eq. (5) for $M_2 = 200$ GeV, $|\mu| = 250$ GeV, $\tan\beta = 5$, $\varphi_{M_1} = \varphi_\mu = 0$ and $(P_{e^-}, P_{e^+}) = (0.8, -0.6)$.

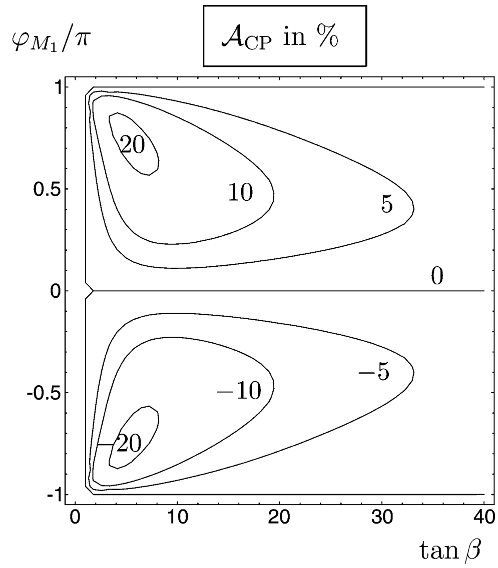


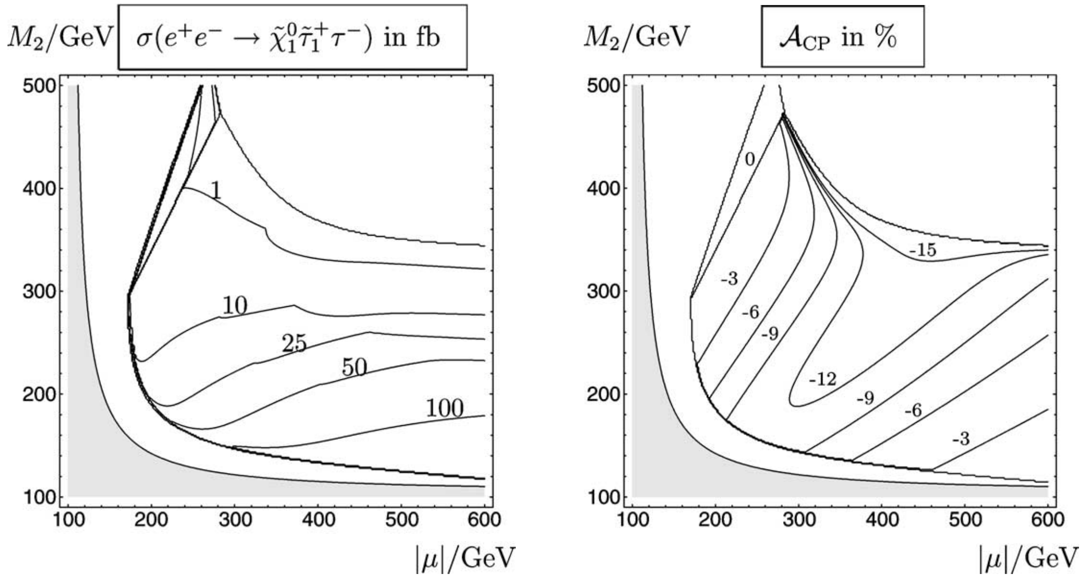
Fig. 2. Contour lines of $\mathcal{A}_{\mathrm{CP}}$ in Eq. (5) for $A_\tau = 1$ TeV, $M_2 = 300$ GeV, $|\mu| = 250$ GeV, $\varphi_{A_\tau} = \varphi_\mu = 0$ and $(P_{e^-}, P_{e^+}) = (0.8, -0.6)$.

Fig. 3(a) and (b) show, for $\varphi_{A_\tau} = 0.5\pi$ and $\varphi_{M_1} = \varphi_\mu = 0$, the $|\mu|$–M_2 dependence of the cross section σ and the asymmetry $\mathcal{A}_{\mathrm{CP}}$, respectively. The asymmetry reaches values up to -15% due to the large value of $|A_\tau| = 1$ TeV and the choice of the beam polarization $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$. This choice also enhances the cross section, which reaches values of more than 100 fb. The gray shaded area excludes chargino masses $m_{\tilde{\chi}_1^\pm} < 104$ GeV. In the blank area either the sum of the masses of the produced neutralinos exceeds $\sqrt{s} = 500$ GeV or the two-body decay $\tilde{\chi}_2^0 \to \tilde{\tau}_1^+ \tau^-$ is not open.

For $\varphi_{M_1} = 0.5\pi$ and $\varphi_\mu = \varphi_{A_\tau} = 0$ we show in Fig. 4(a), (b) the contour lines of σ and $\mathcal{A}_{\mathrm{CP}}$ in the $|\mu|$–M_2 plane, respectively. It is remarkable that in a large region the asymmetry is larger than -10% and reaches values up to -40% while also the cross section is large. Unpolarized beams would reduce the largest values of σ by a factor 3, whereas $\mathcal{A}_{\mathrm{CP}}$ would only be marginally reduced.

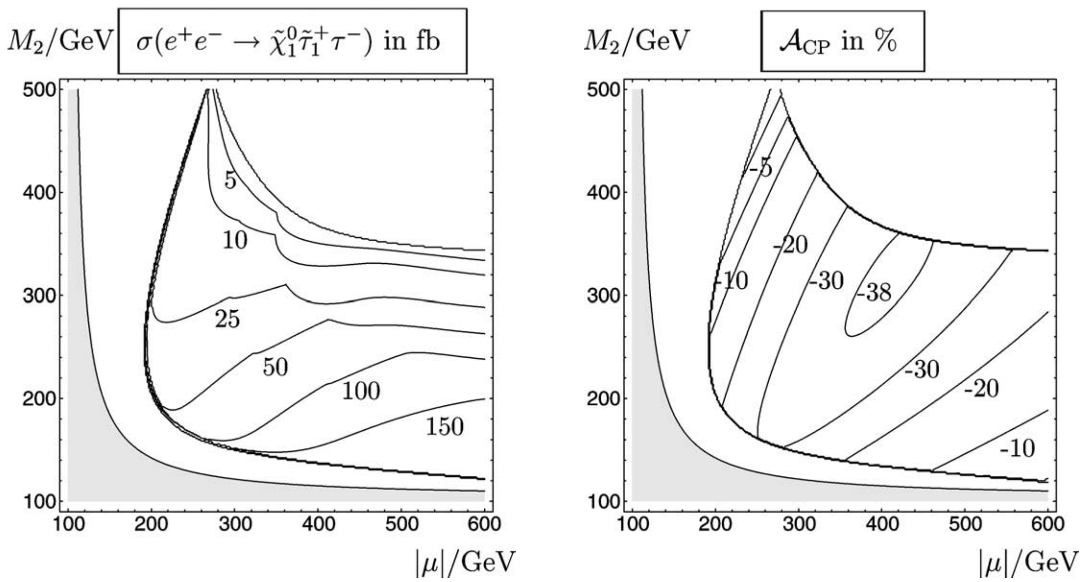
For $|\mu| = 300$ GeV and $M_2 = 400$ GeV, we show in Fig. 5(a), (b) contour lines of σ and $\mathcal{A}_{\mathrm{CP}}$, respectively, in the φ_μ–φ_{M_1} plane. As can be seen the asymmetry $\mathcal{A}_{\mathrm{CP}}$ is very sensitive to variations of the phases φ_{M_1} and φ_μ. Even for small phases, $\mathcal{A}_{\mathrm{CP}}$ can be sizable. Small values of the phases, especially of φ_μ, are suggested by constraints on electron and neutron electric dipole moments (EDMs) [15] for a typical SUSY scale of the order of a few 100 GeV (for a review see, e.g., [16]).

The polarization of the τ is analyzed through its decay distributions. The sensitivities for measuring the polarization of the τ lepton for the various decay modes are quoted in [17]. The numbers quoted are for an ideal detector and for longitudinal τ polarization and it is expected that the sensitivities for transversely polarized τ leptons are somewhat smaller [14]. Combining informations of all τ decay modes a sensitivity of $S = 0.35$ [18] has been obtained. Following [17], the relative statistical error of P_2 (and of $\bar{P}_2$ analogously) can be calculated as $\delta P_2 = \Delta P_2/|P_2| = \sigma^s/(S|P_2|\sqrt{N})$, for σ^s standard deviations, where $N = \sigma\mathcal{L}$ is the number of events with integrated luminosity $\mathcal{L}$ and cross section $\sigma = \sigma_p(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0) \times \mathrm{BR}(\tilde{\chi}_2^0 \to \tilde{\tau}_1^+\tau^-)$. Then for $\mathcal{A}_{\mathrm{CP}}$, Eq. (5), it follows that $\Delta\mathcal{A}_{\mathrm{CP}} = \Delta P_2/\sqrt{2}$. We show in Fig. 6 the contour lines of the sensitivity $S = \sqrt{2}/(|\mathcal{A}_{\mathrm{CP}}|\sqrt{N})$ which is needed to measure $\mathcal{A}_{\mathrm{CP}}$ at 95% CL ($\sigma^s = 2$) for $\mathcal{L} = 500$ fb^{-1}, for the parameters $\varphi_{A_\tau} = 0.2\pi$, $\varphi_{M_1} = \varphi_\mu = 0$. In Fig. 7 we show the contour lines of the sensitivity S for the parameters $\varphi_{M_1} = 0.2\pi$ and $\varphi_\mu = \varphi_{A_\tau} = 0$. It is



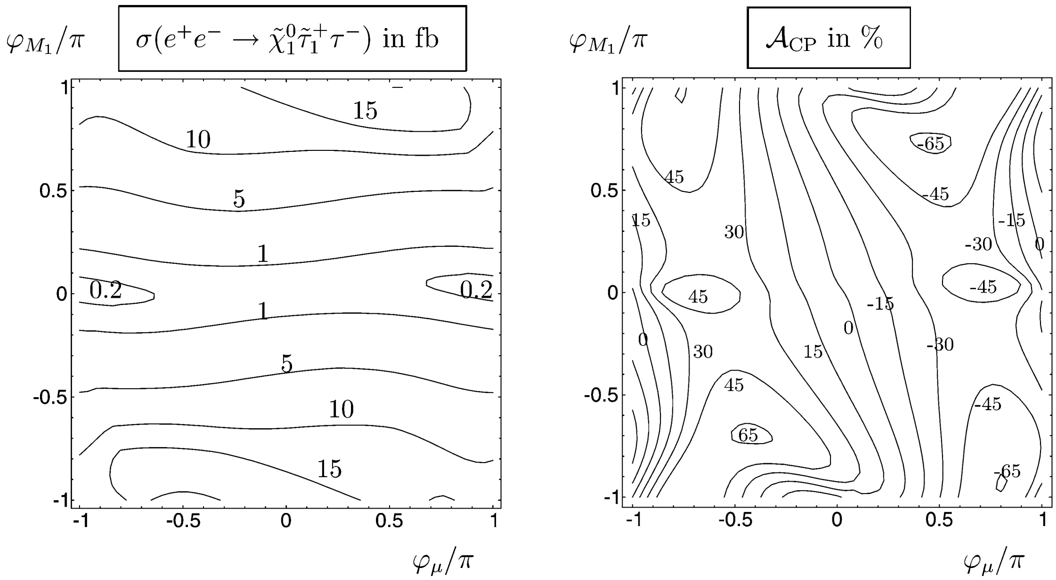
(a) (b)

Fig. 3. Contour lines of σ and $\mathcal{A}_{\rm CP}$ in the $|\mu|$–M_2 plane for $\varphi_{A_\tau} = 0.5\pi$, $\varphi_{M_1} = \varphi_\mu = 0$, $A_\tau = 1$ TeV, $\tan\beta = 5$ and $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$. The blank area outside the area of the contour lines is kinematically forbidden since here either $\sqrt{s} < m_{\tilde{\chi}_1} + m_{\tilde{\chi}_2}$ or $m_{\tilde{\tau}_1} + m_\tau > m_{\tilde{\chi}_2}$. The gray area is excluded since $m_{\tilde{\chi}_1^\pm} < 104$ GeV.



(a) (b)

Fig. 4. Contour lines of σ and $\mathcal{A}_{\rm CP}$ in the $|\mu|$–M_2 plane for $\varphi_{M_1} = 0.5\pi$, $\varphi_{A_\tau} = \varphi_\mu = 0$, $A_\tau = 250$ GeV, $\tan\beta = 5$ and $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$. The blank area outside the area of the contour lines is kinematically forbidden since here either $\sqrt{s} < m_{\tilde{\chi}_1} + m_{\tilde{\chi}_2}$ or $m_{\tilde{\tau}_1} + m_\tau > m_{\tilde{\chi}_2}$. The gray area is excluded since $m_{\tilde{\chi}_1^\pm} < 104$ GeV.



(a) (b)

Fig. 5. Contour lines of σ and $\mathcal{A}_{\rm CP}$ in the φ_μ–φ_{M_1} plane for $M_2 = 400$ GeV, $|\mu| = 300$ GeV, $\tan\beta = 5$, $\varphi_{A_\tau} = 0$, $A_\tau = 250$ GeV and $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$.

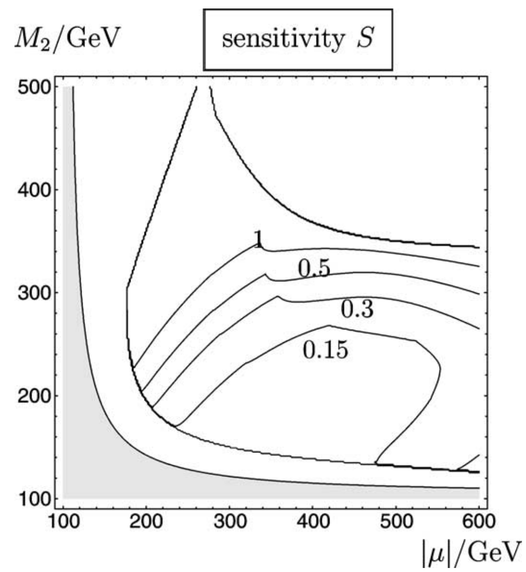


Fig. 6. Contour lines of S for $\varphi_{A_\tau} = 0.2\pi$, $\varphi_{M_1} = \varphi_\mu = 0$, $A_\tau = 1$ TeV, $\tan\beta = 5$ and $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$. The blank area outside the area of the contour lines is kinematically forbidden since here either $\sqrt{s} < m_{\tilde{\chi}_1} + m_{\tilde{\chi}_2}$ or $m_{\tilde{\tau}_1} + m_\tau > m_{\tilde{\chi}_2}$. The gray area is excluded since $m_{\tilde{\chi}_1^\pm} < 104$ GeV.

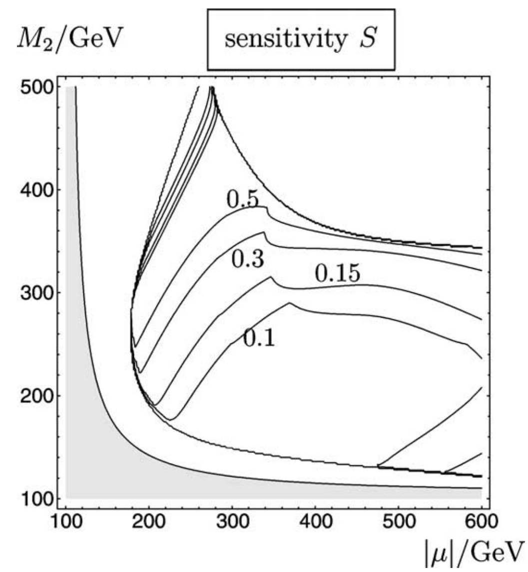


Fig. 7. Contour lines of S for $\varphi_{M_1} = 0.2\pi$, $\varphi_{A_\tau} = \varphi_\mu = 0$, $A_\tau = 250$ GeV, $\tan\beta = 5$ and $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$. The blank area outside the area of the contour lines is kinematically forbidden since here either $\sqrt{s} < m_{\tilde{\chi}_1} + m_{\tilde{\chi}_2}$ or $m_{\tilde{\tau}_1} + m_\tau > m_{\tilde{\chi}_2}$. The gray area is excluded since $m_{\tilde{\chi}_1^\pm} < 104$ GeV.

interesting to note that in a large region in the $|\mu|$–M_2 plane in Figs. 6 and 7 we obtain a sensitivity $S < 0.35$, which means that the asymmetries can be measured at 95% CL.

6. Summary and conclusion

We have proposed and analyzed the CP-odd asymmetry $\mathcal{A}_{\mathrm{CP}}$ in Eq. (5) in neutralino production $e^+e^- \to \tilde{\chi}_i^0\tilde{\chi}_j^0$ and the subsequent two-body decay of one neutralino $\tilde{\chi}_i^0 \to \tilde{\tau}_k^\pm\tau^\mp$. The asymmetry is due to the transverse $\tau^\mp$ polarization, which is non-vanishing if CP-violating phases of the trilinear scalar coupling parameter A_τ and/or the gaugino and higgsino mass parameters M_1, μ are present. The asymmetry occurs already at tree level and is due to spin effects in the neutralino production and decay process. In a numerical study for $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0$ and neutralino decay $\tilde{\chi}_2^0 \to \tilde{\tau}_1^\pm\tau^\mp$ we have shown that the asymmetry can be as large as 60%. It can be sizeable even for small phases of μ and M_1, which is suggested by the experimental limits on EDMs. Depending on the MSSM scenario, the asymmetry should be accessible in future electron–positron linear collider experiments in the 500 GeV range. Longitudinally polarized electron and positron beams can considerably enhance both the asymmetry and the production cross section.

Acknowledgements

This work is supported by the 'Fonds zur Förderung der wissenschaftlichen Forschung' (FWF) of Austria, projects No. P13139-PHY and No. P16592-N02, by the European Community's Human Potential Programme under contract HPRN-CT-2000-00149 and by the Deutsche Forschungsgemeinschaft (DFG) under contract Fr 1064/5-1.

References

[1] A. Bartl, H. Fraas, T. Kernreiter, O. Kittel, hep-ph/0306304.
[2] A. Bartl, H. Fraas, O. Kittel, W. Majerotto, hep-ph/0308141.
[3] F.M. Renard, Basics of Electron Positron Collisions, Editions Frontieres, Dreux, 1981.
[4] J. Ellis, S. Rudaz, Phys. Lett. B 128 (1983) 248.
[5] J.F. Gunion, H.E. Haber, Nucl. Phys. B 272 (1986) 1;
J.F. Gunion, H.E. Haber, Nucl. Phys. B 278 (1986) 449;
J.F. Gunion, H.E. Haber, Nucl. Phys. B 402 (1993) 567, Erratum.
[6] L.J. Hall, J. Polchinski, Phys. Lett. B 152 (1989) 335.
[7] H.E. Haber, G.L. Kane, Phys. Rep. 117 (1985) 75.
[8] A. Bartl, K. Hidaka, T. Kernreiter, W. Porod, Phys. Lett. B 538 (2002) 137;
A. Bartl, K. Hidaka, T. Kernreiter, W. Porod, Phys. Rev. D 66 (2002) 115009.
[9] H. E. Haber, in: L. DeProcel, Ch. Dunwoodie (Eds.), Proceedings of the 21st SLAC Summer Institute on Particle Physics, Stanford, CA, 1993, p. 231.
[10] G. Moortgat-Pick, H. Fraas, A. Bartl, W. Majerotto, Eur. Phys. J. C 9 (1999) 521;
G. Moortgat-Pick, H. Fraas, A. Bartl, W. Majerotto, Eur. Phys. J. C 9 (1999) 549, Erratum.
[11] A. Bartl, H. Fraas, W. Majerotto, Nucl. Phys. B 278 (1986) 1.
[12] J.P. Derendinger, C.A. Savoy, Nucl. Phys. B 237 (1984) 307;
J.A. Casas, S. Dimopoulos, Phys. Lett. B 387 (1996) 107.
[13] M. Carena, J. Ellis, A. Pilaftsis, C.E.M. Wagner, Nucl. Phys. B 586 (2000) 92.
[14] A. Bartl, T. Kernreiter, W. Porod, Phys. Lett. B 538 (2002) 59, hep-ph/0202198.
[15] I.S. Altarev, et al., Phys. Lett. B 276 (1992) 242;
I.S. Altarev, et al., Phys. At. Nucl. 59 (1996) 1152, Yad. Fiz. 59 (1996) 1204 (in Russian);
E.D. Commins, S.B. Ross, D. DeMille, B.C. Regan, Phys. Rev. A 50 (1994) 2960.
[16] P. Nath, Talk at the 9th International Conference on Supersymmetry and Unification of Fundamental Interactions, Dubna, 11–17 June 2001, hep-ph/0107325 and references therein.
[17] M. Davier, L. Duflot, F. Le Diberder, A. Rougé, Phys. Lett. B 306 (1993) 411.
[18] D. Atwood, G. Eilam, A. Soni, Phys. Rev. Lett. 71 (1993) 492.