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Physics Letters B 607 (2005) 87–95

PHYSICS LETTERS B

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Nuclear matrix elements of $\beta\beta$ decay from β-decay data

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Received 6 October 2004; received in revised form 27 November 2004; accepted 15 December 2004

Available online 23 December 2004

Editor: W. Haxton

Abstract

The evaluation of the nuclear matrix elements (NME) of the two-neutrino double beta ($2\nu\beta\beta$) decay and neutrinoless double beta ($0\nu\beta\beta$) decay using the proton–neutron quasiparticle random-phase approximation (pnQRPA) is addressed. In particular, the extraction of a proper value of the proton–neutron particle–particle interaction parameter, g_{pp}, of this theory is analyzed in detail. Evidence is shown, that it can be misleading to use the experimental half-life of the $2\nu\beta\beta$ decay to extract a value for g_{pp}. Rather, arguments are given in favour of using the available data on single beta decay for this purpose.
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PACS: 21.60.Jz; 23.40.Hc; 27.50.+e; 27.60.+j

Keywords: Double beta decay; Quasiparticle random-phase approximation; Nuclear matrix elements; Proton–neutron interaction

The recent large-scale neutrino-oscillation experiments, Super-Kamiokande [1], SNO [2], KamLAND [3], CHOOZ [4], have confirmed the existence of the neutrino mass. These experiments can only probe the differences of the squares of the masses, not the absolute mass scale of the neutrino. On the contrary, the neutrinoless double beta ($0\nu\beta\beta$) decay can probe the absolute mass scale using the effective neutrino mass, $\langle m_\nu \rangle$, extracted from the results of the underground double-beta-decay experiments. To extract the absolute neutrino masses one needs information about the involved nuclear matrix elements [5,6], neutrino mixing [7], and the associated CP phases [8]. As a matter of fact, knowing the underlying nuclear matrix elements accurately enough, one can extract from the double-beta experiments information about the CP phases of the neutrino-mixing matrix [8].

One more fundamental piece of information would emerge if the $0\nu\beta\beta$ decay were detected, namely that the neutrino would be a Majorana particle, i.e., an object for whom the particle and antiparticle states coincide. The $0\nu\beta\beta$ decay then immediately implies also nonconcervation of the lepton number, changing the lepton number by two units. Majorana neutrinos are naturally contained in various particle-physics theories going beyond the Standard Model, such as grand-

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doi:10.1016/j.physletb.2004.12.048

unification theories and supersymmetric extensions of the Standad Model.

Given the above impressive list of important qualitative and quantitative neutrino properties, potentially probed by the $0\nu\beta\beta$ decay, one cannot stress enough the importance of a reliable calculation of the involved nuclear matrix elements (NME). Lack of accuracy in the values of these matrix elements is the source of inaccuracy in the information on the neutrino masses and CP phases, extracted from the $0\nu\beta\beta$-decay experiments. In particular, in view of the planned near-future large-scale underground experiments, with detectors in the ton scale, knowledge of the most promising nuclear candidates for detection is of paramount importance.

Contrary to the $0\nu\beta\beta$ decay, the two-neutrino double beta ($2\nu\beta\beta$) decay, with two neutrinos and two electrons in the final state, can proceed as a perturbative process within the Standard Model. It can also be used as a test bench for the nuclear models, since the decay proceeds via only the 1^+ states of the intermediate double-odd nucleus. Success in describing this decay mode is a prerequisite for a reliable calculation of the NME's related to the $0\nu\beta\beta$ decay. During the last two decades a host of different nuclear models have been used to compute values of the matrix elements involved in both types of double-beta-decay transition [5,9,10]. The mostly used nuclear models in the evaluation of the NME's of double beta decay are the nuclear shell model and the proton–neutron quasiparticle random-phase approximation (pnQRPA), designed for spherical or nearly spherical nuclei.

After the first shell-model attempts, the problem of the NME's of the $2\nu\beta\beta$ and $0\nu\beta\beta$ decays was viewed in a fresh new way by the introduction of the pnQRPA with an adjustable particle–particle part of the proton–neutron two-body interaction. Determination of the value of the corresponding strength parameter, $g_{\rm pp}$, has been a key issue since the mid 80's. As noticed in the early works [11,12], the NME of the $2\nu\beta\beta$ decay is very sensitive to the value of this parameter, leading to the so-called $g_{\rm pp}$ problem of the pnQRPA. On the other hand, the NME of the $0\nu\beta\beta$ decay is much less dependent on the value of $g_{\rm pp}$, as discussed, e.g., in [5,13]. Many extensions of the pnQRPA have come to light during the last nine years. The first of these was the so-called renormalized pnQRPA (pnRQRPA of Ref. [14]). Other extensions of the pnQRPA, used in the $\beta\beta$-decay calculations, are cited, e.g., in [5,6,15,16]. A common feature of all these extensions is the attempt to introduce the Pauli exclusion principle into the pnQRPA by improving on the quasiboson commutation relations, adopted at the pnQRPA level. In these theories different types of correction to the boson commutators of the bifermionic operators are introduced, leading to renormalization factors at the level of the pnQRPA equations of motion.

The results of the $\beta\beta$-decay calculations are quite scattered [17] (see also [5,9] for a detailed discussion of the matrix elements up to the year 1998), and recently it has been suggested that this shortcoming could be overcome in the framework of the pnQRPA and its renormalized extensions. In this scheme it has been suggested [16] that one could use data on the $2\nu\beta\beta$ decay to extract a more accurate value for the NME corresponding to the neutrino-mass mode (i.e., decay mode mediated by the mass of the neutrino) of the $0\nu\beta\beta$ decay. The essentials of this method are summarized as follows: the value of the interaction strength parameter $g_{\rm pp}$ of the pnQRPA (or any of its renormalized extensions) can be determined by fitting the value of the computed NME to the one extracted from the experimental half-life of the corresponding $2\nu\beta\beta$ transition. This fitted value of $g_{\rm pp}$ is then used in the computation of the $0\nu\beta\beta$ NME. This suggestion has recently been made also in [15]. In the following, the implications and pitfalls of this scheme are analyzed in detail by using the simple and transparent framework of the plain pnQRPA. The same qualitative features persist largely also in its renormalized extensions. At the same time, arguments are given in favour of an other approach, namely fitting $g_{\rm pp}$ by the data on single beta decay(s).

To have an idea of the suggested procedure [16], and its alternative, advocated in this Letter, it is instructive to write down an expression for the $2\nu\beta\beta$-decay half-life, $t_{1/2}^{(2\nu)}$, for a transition from the initial ground state, $0_{\rm I}^+$, to the final ground state, $0_{\rm F}^+$. This expression reads

$$\left[t_{1/2}^{(2\nu)}\left(0_{\rm I}^+ \to 0_{\rm F}^+\right)\right]^{-1} = G^{(2\nu)}\left|M_{\rm DGT}^{(2\nu)}\right|^2, \tag{1}$$

where $G^{(2\nu)}$ is an integral over the phase space of the leptonic variables [5]. The nuclear double Gamow–Teller matrix element, $M_{\rm DGT}^{(2\nu)}$, corresponding to the

Table 1
Experimental EC- and β^--decay $\log ft$ values for heavy double-odd nuclei involved as intermediate nuclei in double β^- and double β^+ decays. For completeness, also the Q values of the $\beta\beta$ decays are given in the second column

$\beta\beta$ mode	Q [MeV]	Init. nucl.	Final nucl.	Mode	$\log ft$	Ref.
$\beta^-\beta^-$	3.03	^{100}Tc	^{100}Mo	EC	4.45	[18]
		^{100}Tc	^{100}Ru	β^-	4.6	[19]
$\beta^-\beta^-$	1.30	^{104}Rh	^{104}Ru	EC	4.3	[19]
		^{104}Rh	^{104}Pd	β^-	4.5	[19]
$\beta^+\beta^+$	0.73	^{106}Ag	^{106}Pd	EC	4.9	[19]
		^{106}Ag	^{106}Cd	β^-	$\geqslant 4.2$	[19]
$\beta^-\beta^-$	2.01	^{110}Ag	^{110}Pd	EC	4.1	[19]
		^{110}Ag	^{110}Cd	β^-	4.7	[19]
$\beta^-\beta^-$	0.53	^{114}In	^{114}Cd	EC	4.9	[19]
		^{114}In	^{114}Sn	β^-	4.5	[19]
$\beta^-\beta^-$	2.80	^{116}In	^{116}Cd	EC	4.39	[20]
		^{116}In	^{116}Sn	β^-	4.7	[19]
$\beta^-\beta^-$	0.87	^{128}I	^{128}Te	EC	5.0	[19]
		^{128}I	^{128}Xe	β^-	6.1	[19]
$\beta^+\beta^+$	0.54	^{130}Cs	^{130}Xe	EC	5.1	[19]
		^{130}Cs	^{130}Ba	β^-	5.1	[19]
$\beta^+\beta^+$	0.37	^{136}La	^{136}Ba	EC	4.6	[19]
		^{136}La	^{136}Ce	β^-	?	[19]

$2\nu\beta\beta$ decay, can be written as

$$M_{\mathrm{DGT}}^{(2\nu)} = \sum_n \frac{(0_{\mathrm{F}}^+ \| \sum_j \sigma(j) t_j^- \| 1_n^+)}{(\frac{1}{2} Q_{\beta\beta} + E_n - M_{\mathrm{I}})/m_{\mathrm{e}} + 1} \times \left(1_n^+ \Big\| \sum_j \sigma(j) t_j^- \Big\| 0_{\mathrm{I}}^+\right), \tag{2}$$

where the transition operators are the usual Gamow–Teller operators for β^- transitions, $Q_{\beta\beta}$ is the $2\nu\beta\beta$ Q value, E_n is the energy of the nth intermediate state, M_{I} is the mass energy of the initial nucleus, and m_{e} is the rest-mass of the electron.

As an alternative to the proposed [15,16] use of the measured $2\nu\beta\beta$ decay half-life to determine the value of g_{pp}, the use of the measured single-beta-decay half-lives is advocated in this work. The available data on Gamow–Teller transitions of heavy double-odd nuclei, involved in double β^- and double β^+/EC transitions, have been summarized in Table 1. At the moment, it is believed that the double β^- decays are better accessible to experiments than the double β^+/EC decays. Nevertheless, it is instructive to show the available beta-decay data for nuclei involved in the double β^+/EC decays, as well.

As the first, clean-cut test case one can take the decay of ^{116}Cd which is a nearly spherical, almost semi-magic nucleus. The corresponding final nucleus of the $2\nu\beta\beta$ decay is ^{116}Sn, a genuine spherical semi-magic nucleus. Both these nuclei are well describable by the spherical pnQRPA.

The calculation of the matrix element of Eq. (2) proceeds on the following lines. The single-particle energies of the spherical mean field are obtained from a Woods–Saxon single-particle potential, including the Coulomb and spin–orbit parts in the Bohr–Mottelson parametrization [21]. The single-particle valence space is taken typically to span two to three oscillator major shells around the proton and neutron Fermi surfaces. The adopted two-body interaction is a realistic one, based on the one-boson-exchange potential of the Bonn type, transformed to nuclear matter by the G-matrix technique. The finite-size effects have been taken into account in an approximate way by using simple scaling parameters for the short-range monopole part, and separate scalings for the $J^\pi = 1^+$ multipole in the particle–hole and particle–particle channels. Details of the calculation can be read in [22].

The strong short-range correlations between nucleons have been treated by using the BCS approximation. The associated pairing strengths are adjusted to reproduce the empirical pairing gaps, extracted from the experimental separation energies of protons and

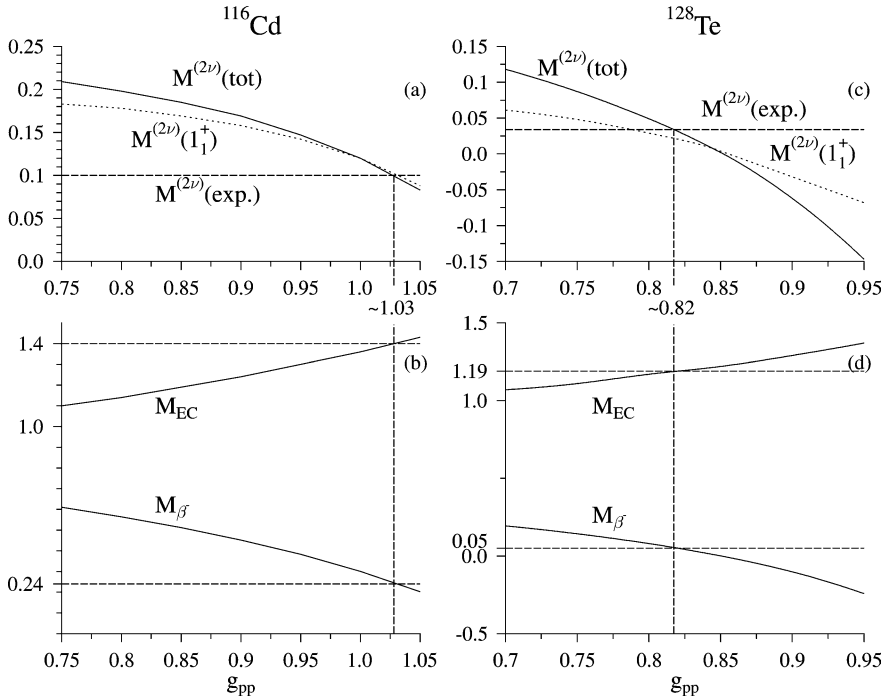

Fig. 1. (a) The NME's corresponding to the $2\nu\beta\beta$ decay of ^{116}Cd shown as functions of $g_{\rm pp}$. The complete NME, $M^{(2\nu)}(\text{tot})$, the NME with only the lowest intermediate contribution included, $M^{(2\nu)}(1_1^+)$, and the experimental NME, $M^{(2\nu)}(\text{exp})$, have been shown. (b) The left-branch, EC NME, $M_{\rm EC}$, and the right-branch NME, β^- NME, M_{β^-}, shown as functions of $g_{\rm pp}$. (c), (d) The same as (a) and (b) for the $2\nu\beta\beta$ decay of ^{128}Te.

neutrons, in a way described in [23]. The proton–neutron correlations are treated at the pnQRPA level by fixing the scale of the particle–hole $J^\pi = 1^+$ two-body matrix elements to reproduce the empirical location of the Gamow–Teller giant resonance, whereas the particle–particle part of the same interaction is scaled by the interaction strength constant $g_{\rm pp}$, left as a free parameter in the calculations. This method was used for realistic interactions in the context of the $2\nu\beta\beta$ decay in [12], and in description of single beta decays in [23].

In Fig. 1(a), the NME $M^{(2\nu)}(\text{tot})$, corresponding to the $2\nu\beta\beta$ decay of ^{116}Cd, is drawn as a function of $g_{\rm pp}$. In the same figure a rough value of the extracted experimental NME, $M^{(2\nu)}(\text{exp})$, has been shown as a horizontal line, since its value is independent of $g_{\rm pp}$. Here the uncertainties in the value of this extracted NME, arising from the experimental error in the measured half-life, and the uncertainty in the proper value of the axial-vector coupling constant, $g_{\rm A}$, for medium-heavy and heavy nuclei, have been omitted. The intersection point of these two curves gives now the fitted value, $g_{\rm pp}(\beta\beta) \simeq 1.03$, of $g_{\rm pp}$. As can be seen from the curve denoted by $M^{(2\nu)}(1_1^+)$ in Fig. 1, the NME including only the contribution arising from the virtual transition through the first 1^+ state, 1_1^+, of the intermediate nucleus ^{116}In, almost coincides with the complete NME, $M^{(2\nu)}(\text{tot})$, especially for $g_{\rm pp}$ values around unity. This is a characteristic of the so-called single-state dominance (SSD), studied extensively, e.g., in [22].

In the case of such a SSD, the NME (2) of the $2\nu\beta\beta$ decay can be approximately written as

$$M^{(2\nu)} \simeq \frac{M_{\rm EC} M_{\beta^-}}{(\frac{1}{2}Q_{\beta\beta} + E_1 - M_{\rm I})/m_{\rm e} + 1}. \tag{3}$$

The two branches of the $2\nu\beta\beta$ transition, $M_{\rm EC}$ and M_{β^-} are drawn as functions of $g_{\rm pp}$ in panel (b) of Fig. 1. It is remarkable that the magnitudes of the left-branch NME, corresponding to the electron-capture (EC) decay of the 1_1^+ state in ^{116}In to the ground state of ^{116}Cd, and the right-branch NME, corresponding to

the β^- decay of the same state to the ground state of ^{116}Sn, can in some cases be determined from experimental data on the corresponding decay half-lives. It is also clear that in this kind of a simple case the study of the relation between the single and double beta decays is most transparent, in particular, related to the determination of the $g_{\rm pp}$ parameter.

Using the extracted value of $g_{\rm pp}(\beta\beta)$, one immediately obtains, due to the SSD, the values of the left- and right-branch NME's, as shown in panel (b) of Fig. 1. From Fig. 1 one obtains $M_{\rm EC} \simeq 1.4$ and $M_{\beta^-} \simeq 0.24$. These values of the NME's can, in turn, be used to compute the half-lives of the EC and β^- decays from the 1_1^+ state in ^{116}In. Comparison of these computed values with the corresponding experimental ones, extracted from Table 1, yields

$$\frac{t_{1/2}^{(\rm EC)}(\rm exp)}{t_{1/2}^{(\rm EC)}(\rm th)} \simeq 2.6, \qquad \frac{t_{1/2}^{(\beta^-)}(\rm exp)}{t_{1/2}^{(\beta^-)}(\rm th)} \simeq 0.16, \tag{4}$$

indicating that for $g_{\rm pp}(\beta\beta) \simeq 1.03$ one obtains too fast an EC transition and much too slow a β^- transition. Fitting the β^- decay half-life, instead of the $2\nu\beta\beta$ decay half-life, would yield a value $g_{\rm pp}(\beta^-) \simeq 0.85$, which also would result in a more reasonable matrix element for the EC branch, namely $M_{\rm EC} \simeq 1.2$. The corresponding experimental magnitude is $M_{\rm EC}(\rm exp) \simeq 0.8$, the exact value depending on the adopted value for $g_{\rm A}$. As can be seen, the proper determination of the value of the $g_{\rm pp}$ parameter, by using the β^- decay half-life, can lead to a notably different value from the one extracted by using the $2\nu\beta\beta$ decay half-life, even in the simple case of the SSD. Summarizing the above: use of the value $g_{\rm pp}(\beta\beta) \simeq 1.03$ reproduces the $2\nu\beta\beta$ half-life via two compensating errors: too large an EC NME is compensated by too small a β^- NME.

As the second test case one can take the $2\nu\beta\beta$ decay of ^{128}Te to the ground state of ^{128}Xe. This case can be analyzed using the very methods deviced for ^{116}Cd in Fig. 1(a) and (b). A corresponding scheme is shown for the ^{128}Te decay in Fig. 1(c) and (d). As can be seen from panel (c), the curves for the total matrix element and the $M^{(2\nu)}(1_1^+)$ matrix element are very much separated everywhere but at the values of $g_{\rm pp}$ close to the point which reproduces the value of the experimental matrix element. Hence, in this case one cannot speak about SSD, and the situation is more complicated than in the ^{116}Cd case. In this case the intersection point of the curves, corresponding to the total and experimental matrix elements, gives the fitted value, $g_{\rm pp}(\beta\beta) \simeq 0.82$, of $g_{\rm pp}$.

The two branches of the matrix element $M^{(2\nu)}(1_1^+)$, $M_{\rm EC}$ and M_{β^-}, are drawn as functions of $g_{\rm pp}$ in panel (d) of Fig. 1. Using the extracted value of $g_{\rm pp}(\beta\beta)$, one obtains for the left- and right-branch matrix elements $M_{\rm EC} \simeq 1.19$ and $M_{\beta^-} \simeq 0.05$. These values of the NME's can, in turn, be used to compute the half-lives of the EC and β^- decays from the 1_1^+ state of ^{128}I. Comparison of these computed values with the corresponding experimental ones, extracted from Table 1, yields

$$\frac{t_{1/2}^{(\rm EC)}(\rm exp)}{t_{1/2}^{(\rm EC)}(\rm th)} \simeq 9.7, \qquad \frac{t_{1/2}^{(\beta^-)}(\rm exp)}{t_{1/2}^{(\beta^-)}(\rm th)} \simeq 0.17, \tag{5}$$

indicating that for $g_{\rm pp}(\beta\beta) \simeq 0.82$ one obtains much too fast an EC transition and much too slow a β^- transition. Fitting the β^- decay half-life, instead of the $2\nu\beta\beta$ decay half-life, would yield a value $g_{\rm pp}(\beta^-) \simeq 0.755$, which would only slightly change the value of the matrix element for the EC branch, namely to $M_{\rm EC} \simeq 1.15$. The corresponding experimental magnitude is $M_{\rm EC}(\rm exp) \simeq 0.38$, for $g_{\rm A} = 1.0$. As can be seen, in this case the proper determination of the value of the $g_{\rm pp}$ parameter, by using the β^- decay half-life, does not lead to a notably different value of $M_{\rm EC}$ from the one extracted by using the $2\nu\beta\beta$ decay half-life. The reason for this discrepancy is not clear, but deformation effects could play some role. Even so, the above tells us that the use of the value $g_{\rm pp}(\beta\beta) \simeq 0.82$ reproduces the $2\nu\beta\beta$ half-life via two compensating errors: too large an EC NME is compensated by too small a β^- NME.

As the third case, the $2\nu\beta\beta$ decay of ^{76}Ge to the ground state of ^{76}Se will be discussed. This case can be analyzed along the lines of the previous two cases. A corresponding scheme is shown for the ^{76}Ge decay in Fig. 2(a) and (b). As can be seen from Fig. 2, there exists no SSD, and the situation is in this respect similar to the ^{128}Te case. In this case the intersection point of the curves, corresponding to the total and experimental matrix elements, gives the fitted value, $g_{\rm pp}(\beta\beta) \simeq 0.94$, of $g_{\rm pp}$.

The two branches of the matrix element $M^{(2\nu)}(1_1^+)$, $M_{\rm EC}$ and M_{β^-}, are drawn as functions of $g_{\rm pp}$ in panel

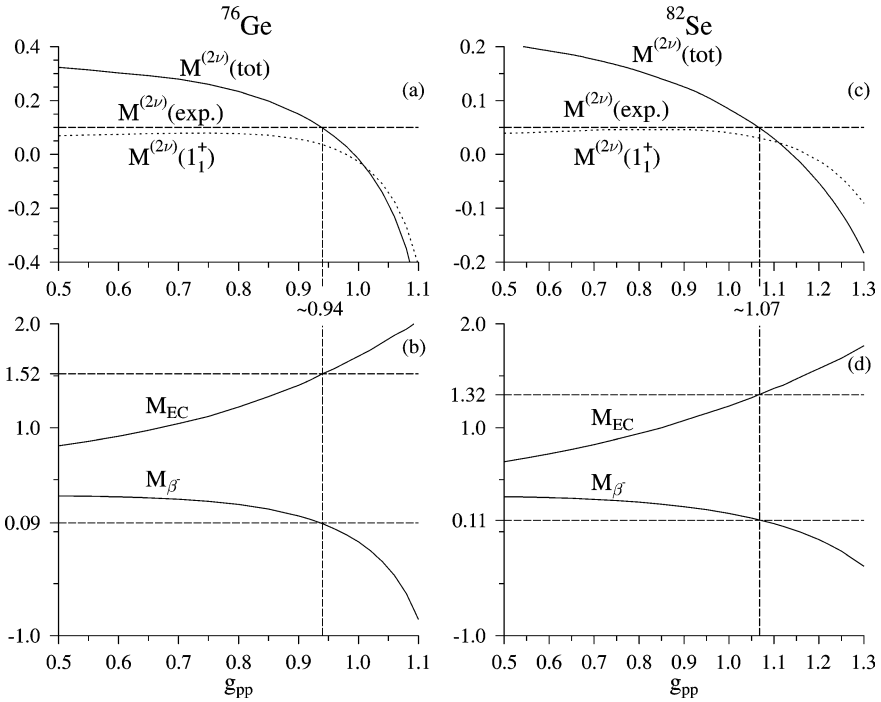

Fig. 2. The same as Fig. 1 for NME's corresponding to the $2\nu\beta\beta$ decays of ^{76}Ge and ^{82}Se.

(b) of Fig. 2. Using the extracted value of $g_{\rm pp}(\beta\beta)$, one obtains the values $M_{\rm EC} \simeq 1.52$ and $M_{\beta^-} \simeq 0.09$ of the left- and right-branch NME's, which give for the corresponding $\log ft$ values

$$\log ft(\mathrm{EC}) \simeq 3.9, \qquad \log ft\left(\beta^-\right) \simeq 6.4, \tag{6}$$

for $g_{\rm A} = 1.0$. The lowest state in the intermediate nucleus, ^{76}As, is a 2^- state, and hence the Gamow–Teller decays of the lowest 1^+ state are hard to observe due to the fast gamma decays to this 2^- state.

As the next example of the $g_{\rm pp}(\beta\beta)$ problem, the $2\nu\beta\beta$ decay of ^{82}Se to the ground state of ^{82}Kr is discussed in Fig. 2(c) and (d). As for the ^{76}Ge case, also here the SSD is not applicable. From Fig. 2 one can read for the intersection point of the total and experimental matrix elements the value $g_{\rm pp}(\beta\beta) \simeq 1.07$, giving for the EC and β^- NME's the values $M_{\rm EC} \simeq 1.32$ and $M_{\beta^-} \simeq 0.11$. These, in turn, give for the corresponding $\log ft$ values

$$\log ft(\mathrm{EC}) \simeq 4.0, \qquad \log ft\left(\beta^-\right) \simeq 6.2, \tag{7}$$

for $g_{\rm A} = 1.0$. The lowest two states in the intermediate nucleus, ^{82}Br, are a 5^- state and a 2^- state, and hence the Gamow–Teller decays of the lowest 1^+ state have not been observed.

Table 2
EC- and β^--decay $\log ft$ values for selected decays of double-odd nuclei in the pf shell. The data is taken from [19]

Init. nucl.	Final nucl.	Mode	$\log ft$
^{70}Ga	^{70}Zn	EC	4.7
^{70}Ga	^{70}Ge	β^-	5.1
^{78}Br	^{78}Se	EC	4.8
^{78}Br	^{78}Kr	β^-	?
^{80}Br	^{80}Se	EC	4.7
^{80}Br	^{80}Kr	β^-	5.5

Although no measured EC or β^- NME can be extracted for the ^{76}Ge and ^{82}Se cases, one can compare the computed $\log ft$ values of Eqs. (6) and (7) to the $\log ft$ values of similar cases in the same mass region. In the relevant mass region there are three double-odd nuclei with a 1^+ ground state and decay patterns analogous to the ones of ^{76}As and ^{82}Br, namely the ones listed in Table 2. From this table one immediately notices that the $\log ft$ values of the relevant EC decays range between $\log ft(\mathrm{EC}) = 4.7$–$4.8$ and values of the

relevant β^- decays range between $\log ft(\beta^-) = 5.1$–5.5. This would suggest that the extracted $\log ft$ values of Eqs. (6) and (7) for the EC decays are too small and the corresponding extracted $\log ft$ values for the β^- decays far too large. Much better agreement between the theoretical and experimental EC and β^- $\log ft$ values, around $\log ft(\mathrm{EC}) \simeq 4.6$ and $\log ft(\beta^-) \simeq 5.3$ could be obtained for smaller g_{pp} values than the one, suggested by the $2\nu\beta\beta$-decay half-life. For the two discussed decays a value $g_{\mathrm{pp}}(\beta^-) \simeq 0.8$ would do quite well.

Based on the previous analysis one can say that the conclusions arising from the analysis of the $2\nu\beta\beta$ decays of ^{76}Ge and ^{82}Se coincide with the ones arising from the $2\nu\beta\beta$ decays of ^{116}Cd and ^{128}Te: cancellation of errors in the two matrix elements, M_{EC} and M_{β^-}, conspire to produce a $2\nu\beta\beta$ NME which exactly reproduces the corresponding experimental matrix element. This demonstrates that it can be dangerous to determine the value of g_{pp} by fitting the $2\nu\beta\beta$ decay half-life.

In fact, determination of the value of g_{pp} by the data on single beta decay leaves the $2\nu\beta\beta$-decay half-life as a prediction of the theory. Comparison of this prediction to the experimental half-life would tell about the predictive power of the adopted theoretical framework, in terms of the size of the adopted single-particle space, the adopted single-particle energies, etc. A roughly correct prediction for the $2\nu\beta\beta$-decay half-life would shed more confidence on the theoretical predictions concerning the other multipoles, involved in the $0\nu\beta\beta$ decay.

As the final example of the $g_{\mathrm{pp}}(\beta\beta)$ problem, the $2\nu\beta\beta$ decay of ^{100}Mo to the ground state of ^{100}Ru is discussed in Fig. 3. In this case the SSD is roughly applicable. From Fig. 3 one can see that in this particular nuclear-structure calculation, the one of Ref. [22] where one can read more details of the used single-particle basis, etc., the computed total NME never reaches the experimental NME, extracted by using $g_{\mathrm{A}} = 1.0$. Hence, in this case one is forced to use the experimental β^--decay $\log ft$ value, quoted in Table 1, to determine the value of g_{pp}, resulting in $g_{\mathrm{pp}}(\beta^-) \simeq 1.02$. This gives for the EC the value $M_{\mathrm{EC}} \simeq 1.97$, and for the corresponding $\log ft$ value

$$\log ft(\mathrm{EC}) \simeq 3.7, \tag{8}$$

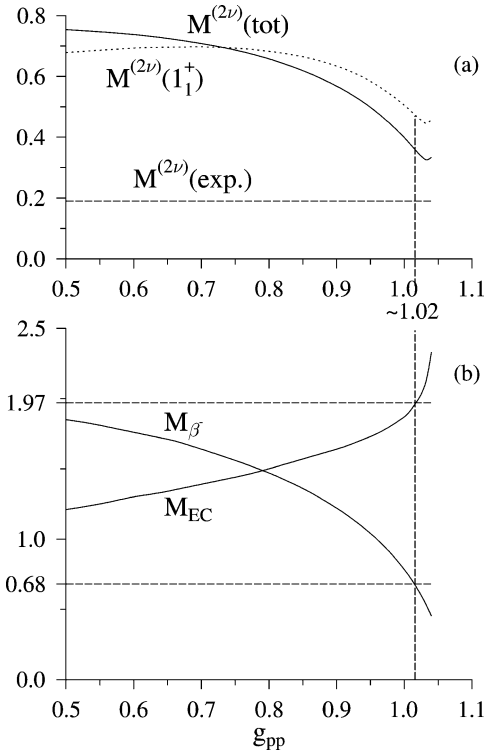

Fig. 3. The same as Fig. 1 for NME's corresponding to the $2\nu\beta\beta$ decay of ^{100}Mo.

using $g_{\mathrm{A}} = 1.0$. This is too low a value for this decay, as seen from the data of Table 1, indicating that some nuclear-structure effects, e.g., deformation, beyond the reach of the spherical pnQRPA, might be present.

Summarizing the above presented results, the problem of determination of the proton–neutron interaction strength, g_{pp}, in a pnQRPA type of calculation, be it the plain pnQRPA or one of its renormalized extensions, has been addressed. The apparent solution of the "g_{pp} problem" by fitting g_{pp} to available data on $2\nu\beta\beta$-decay half-lives has been critically analyzed. Fitting this parameter to the existing data on single β^- transitions is found to be a more meaningful solution to the problem. Arguments favouring this method have been summarized in Table 3 where the positive points (+) and negative points (−) of the two fitting methods have been listed. Below few comments concerning the listed points of the table are made.

Concerning point one, one can even perform a systematical study of the beta-decay properties of a given nuclear region in the fit to beta decays. This approach

Table 3
Pros (+) and cons (−) of the two discussed recipes to fit the parameter g_{pp}. For more explanation on the various points see the text

Point	Fit to β^- and/or EC decay(s)	Fit to $2\nu\beta^-\beta^-$ decay
1	One, two or more observables can be used for the fit (+)	Only one observable can be used for the fit (−)
2	Direct access to grass-root-level deficiencies of a nuclear model (+)	Two or more compensating errors may conspire to produce a good $2\nu\beta^-\beta^-$ decay rate (−)
3	The beta-decay properties better reproduced (+)	The $2\nu\beta^-\beta^-$ decay properties better reproduced (+)
4	Error limits from comparison of the experimental and computed $2\nu\beta^-\beta^-$ decay rate (+)	Advisable to check against data on β^- decays
5	Largely eliminates the model-space dependence of the computed $0\nu\beta^-\beta^-$ decay rates (+)	Largely eliminates the model-space dependence of the computed $0\nu\beta^-\beta^-$ decay rates (+)
6	Can be extended to study of forbidden contributions, e.g., 2^-, in $0\nu\beta^-\beta^-$ decay (+)	No access to a possible variation of g_{pp} from multipole to multipole (−)
7	Can access $\beta\beta$ decays where no $2\nu\beta\beta$ data exists, see Table 2 (+)	Can access $\beta\beta$ decays where no direct β-decay data exists (^{76}Ge and ^{82}Se) (+)
Balance	7 × (+)	3 × (+) and 3 × (−)

would correspond to a shell-model [24] type of application of the beta-decay data. Referring to point two, a fit to β^- data can reveal deficiencies in the predictive power of the used nuclear model in the case of the EC rates of the other branch. This seems to be the case, e.g., in the present calculation.

In regard to point four, the first method can be used to draw some conclusions about the error limits in the $\beta\beta$ calculations, whereas in the second method one necessarily should check the consistency of the calculations against the available beta-decay observables. This is a necessary procedure, not warranting either a plus or a minus mark. In point five the similar behaviour of the two discussed fitting methods comes, on one hand, from the fact that in both methods one fixes first the pairing parameters by semiempirical pairing gaps. This is an essential step and produces, for each single-particle space, the consistent quasiparticle mean field. On the other hand, as the next step, both methods use experimental data to fit the g_{pp} parameter. This two-step fitting procedure is enough to eliminate almost completely the dependence of the computed $0\nu\beta\beta$-decay rates on the size of the model space.

The point number six is a very important one considering the actual computation of the $0\nu\beta\beta$-decay rates. In the first procedure a separate g_{pp} analysis of higher multipoles can be performed, e.g., in the pf shell where data on beta decays of 2^- states are available. In the second method the same value of the g_{pp} parameter has to be assumed for all multipoles. Finally, it is to be noted that in the previous analysis the axial-vector coupling constant, g_A, has been assumed to be roughly the same for both the β and $\beta\beta$ decays. Since no exhaustive studies of this matter have been performed, we take this assumption at face value in this work.

Concluding, the last line of Table 3 sums up the positive and negative points of each method. This final balance clearly supports the argument that the beta-decay fitting should be favoured, rather than the $2\nu\beta\beta$-decay fitting.

Acknowledgements

I thank M. Kortelainen for assistance in drawing the figures of this Letter. This work has been supported by the Academy of Finland under the Finnish Centre of Excellence Programme 2000–2005 (Project No. 44875, Nuclear and Condensed Matter Programme at JYFL).

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