A CA model for bidirectional pedestrian streams

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Abstract

The modeling of pedestrian flows is of particular importance for the planning of pedestrian facilities and the preparation for emergency situations. Even though there is a wide variety of simulation models on the market, a simulation model that deals with bidirectional pedestrian flow adequately is still missing. This contribution proposes an event-based cellular automaton model that is capable of simulating bi- and unidirectional pedestrian streams. The model is built on a theoretically sound foundation. Its performance is demonstrated by a comparison to empirical data.

Keywords: bidirectional pedestrian flows, fundamental diagram, cellular automata, event-based simulation

1. Introduction

The modeling of pedestrian flows is of particular importance for the planning of pedestrian facilities and the preparation for emergency situations. Over the last 20–25 years, computer simulation models have been developed with that aim. Existing models, however, are either still too demanding in terms of computational costs to simulate large areas with hundreds of thousands of pedestrians or they reduce the level of detail and behavioral complexity for the sake of faster computation. The aim of this contribution is to provide a model (i) that is able to reproduce flow dynamics as observed in real world situations; (ii) that is build on a realistic fundamental diagram for stationary flow; (iii) that can deal with large-scale scenarios.

Experimental studies on pedestrian dynamics dating back more than 50 years. Some studies are based on field observations, while others are conducted under laboratory conditions. Cheung and Lam1 study bidirectional pedestrians flows in the Hong Kong Mass Transit Railway. One focus of their study is the influence of the directional distribution of both flows on the capacity and speed. Similar studies are conducted by Lam et al.2,3. One finding is that not only density but also the flow composition (flow vs counter-flow) influences the dynamics. This finding is supported by experiments performed under laboratory conditions4,5. An important observation is that in the bidirectional case even at high densities a substantial flow is possible.

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Nomenclature

• pedestrian that moves from left to right
○ pedestrian that moves from right to left
∅ an empty cell
∅< \text{z} an empty cell; last pedestrian left cell less than \text{z} time units ago
∅≥ \text{z} an empty cell; last pedestrian left cell at least \text{z} time units ago
\[c_1, c_2, \ldots, c_n\] a configuration of cells 1, 2, \ldots, n with \(c_i \in \{∅, •, ◦\}\)
\[\ldots, i/ j, \ldots\] a cell that is occupied by either i or j with i, j ∈ \{∅, •, ◦\}
TTA(\text{a}_i, t) TTA event concerning pedestrian \text{a}_i (with a \in \{•, ◦\}) that will be executed at time t
SWAP(\text{a}_i, t) SWAP event concerning pedestrian \text{a}_i (with a \in \{•, ◦\}) that will be executed at time t

This is in contrast to unidirectional situations where the flow rapidly decreases when the density exceeds a certain threshold. Other bidirectional experiments are performed by Wong et al.\textsuperscript{6}. The study investigates crossing pedestrian streams for various intersection angles. Similar experiments are performed by Plaue et al.\textsuperscript{7}. Zhang et al.\textsuperscript{8} present a comprehensive study on bidirectional pedestrian flows in a channel, which also supports the earlier findings of a substantial bidirectional flow even for higher densities.

A common approach to simulate pedestrian dynamics is based on cellular automata (CA). Early works include those of Blue and Adler\textsuperscript{9} and Fukui and Ishibashi\textsuperscript{10}. Nowadays, it is a widely accepted approach with a large variety of different models (see, Zheng et al.\textsuperscript{11} for an overview). CA are capable of reproducing realistic fundamental diagrams of stationary flow\textsuperscript{12} and of capturing flow dynamics that are consistent with the Kinematic Wave Model (KWM)\textsuperscript{13,14,15}.

This work proposes a novel event-based CA model that is capable of simulating uni- and bidirectional pedestrian streams and reproduces similar dynamics as observed in laboratory experiments. The remainder of this text is organized as follows. Section 2 introduces a basic instance of the model that only allows for longitudinal movement. A generalization that allows for lateral movement is given in Section 3. The model’s performance is demonstrated by a comparison with empirical data in Section 4. Section 5 concludes.

2. Basic model

This section considers the simulation environment as a homogenous channel that is as wide as one pedestrian. Thus, the unit of density is [pedestrians per meter] and the unit of flow is [pedestrians per second]. The pedestrian movement is modeled through a one-dimensional CA. The stationary behavior of this model has been thoroughly theoretically analyzed by Flötteröd and Lämmel\textsuperscript{12}. The CA model is controlled by three rules.

1. The free speed travel time \(t_{\text{free}} = (\hat{\nu} \hat{\rho})^{-1}\) is the time it takes for a pedestrian to advance one cell unhindered. \(\hat{\nu}\) is the maximum or jam density and \(\hat{\nu}\) is the free speed.
2. The time gap \(\text{z}\) is the minimal time headway a pedestrian keeps to the pedestrian in front of it, given that the pedestrian in front is moving in the same direction. In other words, after a pedestrian has left a cell the next pedestrian moving in the same direction cannot enter that cell before \text{z} time units have been passed.
3. The conflict delay \(D\) is the additional time it takes for two pedestrians who are moving in opposite direction to move past each other (switch position). Thus, the total time for two opposing pedestrians to switch position is \(t_{\text{swap}} = D + t_{\text{free}}\).

The first two rules correspond to the original work of Daganzo\textsuperscript{15}, a CA model that produces flow dynamics that are consistent with the KWM. The third rule is the addend by Flötteröd and Lämmel\textsuperscript{12}, which makes the model applicable for bidirectional flows.

The CA model is implemented as a discrete event simulation. The state of the system is defined by the individual cell states, which are either occupied by one pedestrian that wants to move in one of two possible directions or empty. In its current implementation, the simulation holds one global list of pending events. Events are chronologically sorted.
using a fast priority queue implementation\textsuperscript{16}. Events are parametrized by type, execution time, and by the pedestrian to which they refer. There are two types of events. One is called TTA (try-to-advance). A TTA event triggers a procedure that tries to move the concerning pedestrian one cell forward. Two is called SWAP. A SWAP event triggers a procedure that let the corresponding pedestrian and that directly in front of it switch positions. The execution follows the same scheme for both event types:

1. Evaluation of the current system state (pre-condition) in the surrounding of the pedestrian in question.
2. Performing an action based on this evaluation.
3. Evaluation of the system state in the surrounding of the pedestrian in question after the action has been performed (post-condition).
4. Generation of new events depending on the post-condition evaluation.

Depending on the pre-condition, the execution of an event can lead to different post-conditions and subsequently to different new events. The event logic is given in Table 1. The first column denotes the type of event, the concerned pedestrian, and the event time. To give an example, the first entry in the first row (TTA(i,t)) means that pedestrian \(\bullet_i\) will try to proceed by a try-to-advance action at time \(t\). Assume \(\bullet_i\) faces the following pre-condition

\[
[\ldots, \bullet_h, \bullet_i, \emptyset_{c_{z\ast}}, \ldots].
\]

which means that the next cell in the direction of travel is empty for at least \(z\) time units, denoted by \(\emptyset_{c_{z\ast}}\). Thus, \(\bullet_i\) can proceed to the next cell without any delay. This leads to the following post-condition:

\[
[\ldots, \bullet_h, \emptyset, \bullet_i, \ldots].
\]

Following the second rule of the CA model pedestrian \(\bullet_h\) implements a time gap of \(z\) time units to its predecessor \(\bullet_i\), and thus the corresponding event TTA(\(\bullet_h, t+z\)) is scheduled. Further, as stated in the post-condition, after \(\bullet_i\) has moved it faces an oncoming \(\emptyset\)-pedestrian. This conflict has to be resolved by a swap action. This leads to the scheduling of the event SWAP(\(\bullet_i, t + t^{\text{swap}}\)).

In case a pedestrian, triggered by a TTA event, attempts to enter an empty cell \(i\) at time \(t\) with a configuration \(c_i = \emptyset_{c_{z\ast}}\), the attempt will fail. Instead, a new TTA event will be generated and scheduled for the earliest time where the cell’s configuration changes to \(c_i = \emptyset_{c_{z\ast}}\). The amount of time the pedestrian has to wait until the configuration changes is denoted by \(z^*\). Thus, the new TTA event will scheduled for time \(t + z^*\).

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<thead>
<tr>
<th>Event</th>
<th>Pre-condition</th>
<th>Post-condition</th>
<th>New event(s)</th>
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<tr>
<td>TTA((\bullet_i, t))</td>
<td>([\ldots, \emptyset, \bullet_i, \emptyset_{c_{z\ast}}, \ldots] )</td>
<td>([\ldots, \emptyset, \emptyset, \bullet_i, \emptyset, \ldots] )</td>
<td>TTA((\bullet_i, t + t^{\text{free}}))</td>
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3. Multi-lane model

The model above is defined for a single long channel of one pedestrian width, which is of limited practical relevance. This requirement is in the following subsequently relaxed and the model is extended to multi-lane model with density-dependent parameter settings. Flötteröd and Lämmel\(^{12}\) show that if the time gap and conflict delay are made density dependent according to

\[ D(\rho) = \alpha + \beta \left( \frac{\rho}{1m^2} \right)^\gamma, \]

(1)

then a good fit against experimental data can be achieved, where \((\alpha, \beta, \text{and} \gamma)\) are additional parameters. Furthermore, Flötteröd and Lämmel\(^{12}\) introduce a continuity constraint that requires

\[ z = D + \frac{1}{\hat{\nu}\hat{\rho}}. \]

(2)

If (2) would not hold, then the dynamic of the model would behave discontinuously when going from arbitrarily small but strictly positive counter-flow to a unidirectional flow. By assuming (2), the model is controlled by three parameters \((D, \hat{\nu}, \text{and} \hat{\rho})\). Every time an event is triggered, \(D\) is recalculated according to (1) and using a kernel-based estimate of the density in the proximity of the considered agents. A smooth transition from one density regime to another requires also a smooth transition in the estimated density \(\rho\). The following formula is used for estimating the density around cell \(c_i\).

\[ \rho(c_i) = \sum_{c_k} w(c_k) W(|k - i|). \]

(3)

The weight \(w(c_k)\) for a cell \(c_k\) is either 0 (empty) or \(\hat{\rho}\) (occupied). \(W(|k - i|)\) is the kernel, and \(|k - i|\) is the distance \(d\) between \(c_k\) and \(c_i\). To limit the computational burden of evaluating (3), a spline-based kernel with a compact support is used.

Flötteröd and Lämmel\(^{12}\) point out that the assumption of independent and unconnected channels imply the assumption of implausible queues where the available space is not used uniformly. The problem disappears only if the density is smoothed over the entire system, which is not practical. Flötteröd and Lämmel\(^{12}\) observe that the occurrence of these implausible queues is a consequence of lacking lane changing. In this contribution, lane changing behavior is implemented as follows. Every time a pedestrian moves forward (either by TTA or by SWAP), it checks the available headway ahead of the new cell and add it to a choice set of cells. Same evaluation is performed for corresponding cells in the direct neighboring channels. The pedestrian moves then to the cell out of the choice set that offers the biggest available headway. Note that lane changing leads to a homogenization of density. Flötteröd and Lämmel\(^{12}\) show that introducing density-homogenizing lane changing does not change the form of the fundamental diagram, given that the continuity constraint is satisfied.

4. Experiments

The proposed model is tested against uni- and bidirectional laboratory experiments\(^1\) conducted by Zhang et al.\(^{18}\) and Zhang et al.\(^8\). A sketch of the layouts is depicted in Figure 1. The datasets from the laboratory experiments are

\(^1\) The authors would like to thank Jun Zhang for providing the disaggregated data of the experimental studies.
Based on the setups from the laboratory experiments, a number of simulation runs using the proposed CA model are performed. The CA model has neither been calibrated nor are the boundary conditions of the laboratory experiments known in detail. For that reason it is focused on a qualitative and visual analysis of the proposed CA model. The resulting fundamental diagrams are given in Figure 2(a) for the unidirectional case and in Figure 2(b) for the bidirectional case. It is shown that the simulation based fundamental diagrams compare well with those from empirical data\citep{8,18} and the macroscopic model\citep{12}. An important observation specific to the CA presented in this work is that also the variability of the empirical FD is captured with good precision; see also Figures 2(c), 2(d). Overall, the simulated
standard deviation is, per density rate, in the right order of magnitude. It should, however, be acknowledged that the data used here is non-stationary, meaning that dynamic effects and stationary variability are confounded.

Going beyond the stationary analysis, Figures 2(e), 2(f) compare speed and density over time of the proposed CA model with the measurements of one setup in the laboratory experiment. In the setup considered, the channel width is 3.6 m and bottlenecks at the entrances are 2.0 m (cf. Figure 1(right)). 143 pedestrians walk from left to right and 166 pedestrians walk from right to left.

The experiment took 1490 ms to simulate on a laptop computer. Simulation startup (loading input data etc.) consumed 756 ms and the actual simulation took 734 ms. Assuming the former as constant and taking the fact that the event queue’s complexity is $O(1)$ the run time will increase linearly by about 2.38 ms for every additional pedestrian. Thus, real time simulations in the range of 30k pedestrians seem to be realistic even without parallelization.

5. Conclusion

A dynamic event-based CA model is proposed. The model is built on the macroscopic and stationary model of bidirectional pedestrian flow. The contribution of this work is the implementation of the theoretical findings in a dynamic and microscopic context. It is shown that the microscopic model reproduces the empirically derived fundamental diagram and dynamic features (time series) as they are observed in laboratory experiments adequately. A generalization of the approach from bi- to omnidirectional flows would then also enable the more complex analyzes of, for instance, large tourist areas or festivals.

References