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A Hybrid Experimental-Numerical Sif Determination Technique

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Abstract

Hybrid methods, wherefore numerical and experimental data are used to calculate a critical parameter, have been used for several years with great success in Experimental Mechanics and, in particular, in Facture Mechanics. This letter reports on the development of a hybrid methodology for the determination of the stress intensity factor (SIF) parameter, which entails combining experimental and numerical procedures to compute the SIF based of linear elastic fracture-mechanics concepts.

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Keywords: digital image correlation; overdeterministic method; stress intensity factor; fatigue testing.

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1. Introduction

In the course of their lifetime, mechanical structures are subjected to adverse changes in their structural properties, driven mainly by fatigue, environmental degradation, wear, errors in design and construction, overloads and unexpected solicitations as can result from earthquakes or abnormal impacts. Modern structural design based on damage tolerance principles requires tight inspection and maintenance plans, which are costly and therefore add up to the total cost of ownership of those structures. Loosening inspection frequencies without compromising safety is therefore highly desirable, in order to keep operational costs at amenable levels throughout service life. This is even more important, as the deformations a mechanical part must endure throughout its lifetime will alter project data and render all predictions based on it highly questionable, if not altogether useless.

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In order to implement this philosophy of tolerance to damage, a precise assessment of structural strength and crack extension is critical, particularly on structures subjected to cyclic stress loads and fatigue damage. This demanding analysis must therefore provide enough information about damage extension and its effect on the strength of the structure, in order to elaborate predictions on crack propagation. Linear Elastic Fracture Mechanics (LEFM) concepts have been used to sustain these analyses, together with crack growth principles and other theories such as Critical Distances. Central to all, is the correct determination of the Stress Intensity Factor (SIF) of a crack. SIF is determined numerically through Finite Element Analysis (FEA) methods, although recent advances in the literature seem to suggest there is a definite benefit if the necessary data for SIF calculation is provided experimentally.

Digital Image Correlation (DIC) methods are able to accurately determine 2D and 3D strain fields and therefore seem well adapted to provide the necessary field data for exact SIF calculations. One of the chief advantages of DIC is that no physical sensor has to be installed. This type of measurement system is flexible, since it allows measuring almost any type of deformation in time and space, giving access to information about strain gradients and their variations in time. The complete damage assessment can thus be improved into a rapid, non-contact, full-field methodology by the joint application of known techniques. Better fatigue-life estimation becomes feasible and reduced maintenance costs can be expected as a result.

A multipoint over-deterministic method is used for SIF calculation, wherefore experimental data collected from optical images is fitted to Muskhelishvili's equations describing the stress field around the crack tip (Muskhelishvili, 1933). The procedure is based on the over-deterministic approach, used previously in fracture mechanics for processing photo-elastic data in experimental determination of SIFs (Sanford, 1979; Pastrama, 2008). The values of the stresses in an unlimited number of points around the crack tip can be used in order to fit a multi-term series expansion of the stress field. A system of equations is obtained in which the coefficients of each term are the unknowns. The number of equations is equal to the number of the considered points while the number of unknowns is equal to the number of terms chosen in the series expansion, which is much lower. This over-deterministic method has the advantage of being able to use an unlimited number of data points, thus minimizing calculation error.

Still, several issues that could hinder this approach remain to be solved: the currently available DIC commercial apparatus are too cumbersome, too expensive and of little use other than laboratorial work; device calibration still has to be perfected; image distortion corrections need to be improved, and; cost of the final sensors needs to be drastically reduced in order to achieve rapid technology transfer. LOME/INEGI - the Optics and Experimental Mechanics Laboratory of the Engineering Faculty of Porto University, has been actively involved on the development of such an hybrid methodology for the determination of SIF parameter for the past few years, which entails combining the aforementioned experimental and numerical procedures to compute the SIF based of linear elastic fracture-mechanics concepts (Pastrama, 2008; Richter-Trummer, 2010; Tavares, Paulo J., 2013). After having developed a successful implementation of the hybrid SIF determination in Mode I testing, the work currently being carried out in Mode II required the validation of the experimental DIC results against the numerical model data, which was done using the Dual Boundary Element Method (DBEM) and reported herein. The DBEM model was devised with the necessary boundary conditions to mimic, as much as possible, the experimental setup. The results obtained so far indicate an excellent agreement between the experimental DIC data, the numerical DBEM model and the theoretical values, as will be shown below.

2. Analysis

The experimental work was carried out with Middle Test (MT) specimens for Mode I loading as per ASTM E647 standard (ASTM Standard, 2003) and Compact Tension Specimen (CTS) for Mode II as described in Richard and Sander (Sander and Richard 2003). Both specimens were prepared from 3mm thick aluminium alloy AA6082-T6 in L-T direction. A VIC-3D DIC workstation from Correlated Solutions with 4.1 MP CMOSIS sensor, global shutter at 150 Hz and camera-link interface was used, with 80mm focal length and 1:4 aperture ratio lenses from Qioptic. This equipment seems to be quite competent for the analysis at hand.

2.1. Mode I loading

The initial central notch had an approximate 8mm length and the loading specifications consisted on F_{max} = 17000 N and R = 0.1. The typical stochastic pattern for DIC measurements was applied with an aerograph and four image pairs were recorded between 25000 and 40000 cycles at regular 5000 cycle intervals. Each image pair was recorded for minimum and maximum load as per Table 1.

The DIC strain field can be seen in Fig. 1 together with a data line at the crack tip.

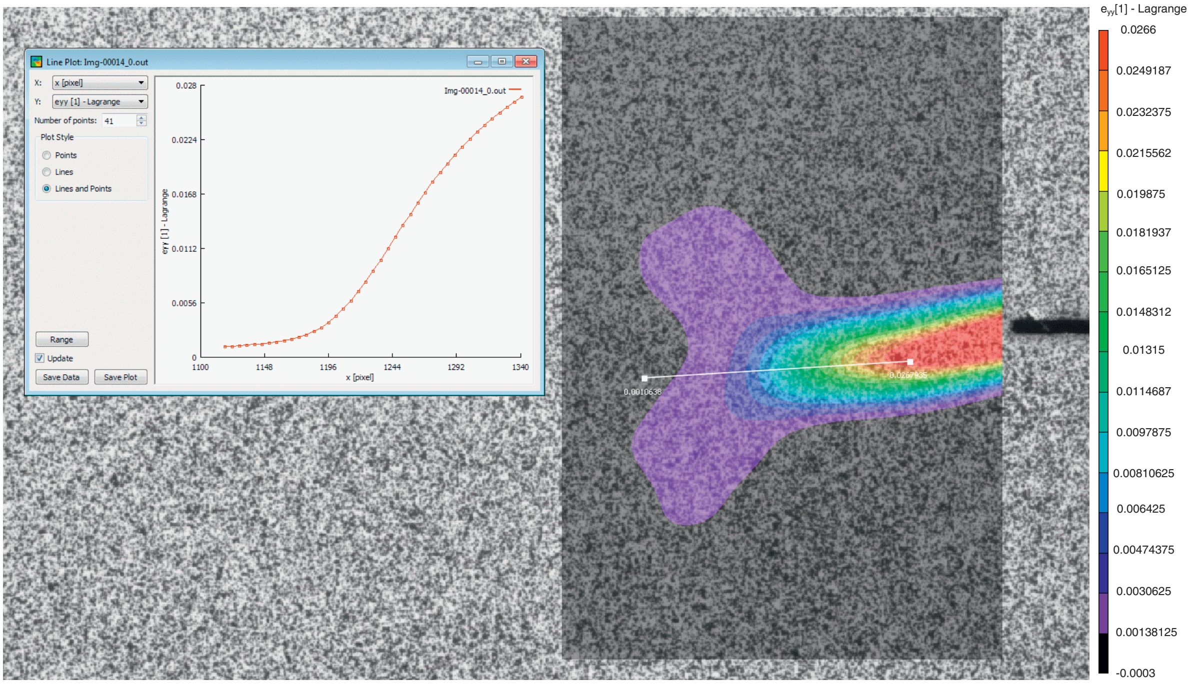


Fig. 1 - DIC strain field on the MT specimen

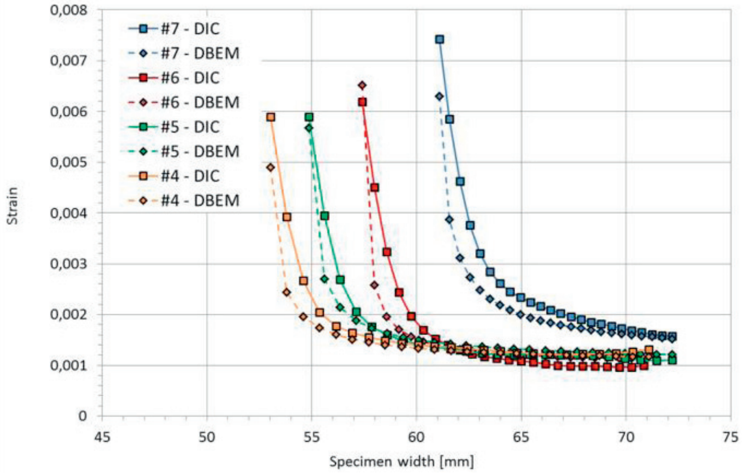


Fig. 2 - KI results at four crack sizes: DIC and DBEM comparison

The results, presented in Fig. 2 above and Table 1, **all** show a good agreement to the numerical data from the DBEM model, a clear improvement from the results published previously, due to the fact high resolution equipment was used to register the images and calculate the experimental strain fields.

Table 1 - Mode I SIF results. The agreement between the numerical and experimentally obtained data is clear

		F		B		K_I (right side) (MPa sqrt(mm))		
#	# cycles	Left	Right	Right	Left	DBEM	DIC	Delta (%)
1	10000	8.92	9.08	8.95	8.96	---	---	---
2	15000	10.87	9.52	10.14	10.14	---	---	---
3	20000	12.17	10.82	11.39	11.53	---	---	---
4	25000	13.77	12.58	13.00	13.10	489	544	**-11.3**
5	30000	15.64	14.41	14.84	15.13	535	528	**1.3**
6	35000	18.56	16.87	17.38	17.82	584	569	**2.5**
7	40000	22.34	20.48	21.08	21.54	716	677	**5.4**

2.2. Mode II loading

Cyclical load of a CTS specimen in pure Mode II can be very challenging, due to the fact that any small asymmetry will rapidly transport the experiment into the realm of mixed mode loading. Moreover, in order to obtain a displacement field upon pure Mode II loading conditions on a fatigue crack, and unless severe plastic damage has been inflicted to the specimen such that a regular fatigue crack growth became seriously compromised and Linear Elastic Fracture Mechanics (LEFM) principles no longer apply, the past history of that growth should not influence, in any respect, the deformation field obtained upon Mode II. As such, the specimen can be loaded in Mode I to obtain a crack of a certain size and then loaded in pure Mode II as approximately as possible for image capture and displacement field registration. This reasoning was applied in the experiment described below and a total of twelve measurements performed before the specimen started deforming catastrophically.

According to Richard and Sander (Sander and Richard 2003), the SIF solutions for the CTS geometry and different loading angles to the original crack orientation are given by:

$$K_I = \frac{F}{wt}\sqrt{\pi a}\,\frac{\cos\alpha}{1 - {}^{a}/_{w}}\sqrt{\frac{0.26 + 2.65\left({}^{a}/_{(w-a)}\right)}{1 + 0.55\left({}^{a}/_{(w-a)}\right) - 0.08\left({}^{a}/_{(w-a)}\right)^2}} \tag{1}$$

and

$$K_{II} = \frac{F}{wt}\sqrt{\pi a}\,\frac{\sin\alpha}{1 - {}^{a}/_{w}}\sqrt{\frac{-0.23 + 1.40\left({}^{a}/_{(w-a)}\right)}{1 + 0.67\left({}^{a}/_{(w-a)}\right) + 2.08\left({}^{a}/_{(w-a)}\right)^2}} \tag{2}$$

where F is the applied force, W is the width of the specimen, t is the thickness of the specimen, a is the crack length and α is the angle of loading direction with respect to the crack plane.

The initial crack size and the maximum crack extension were determined such that the uncracked ligament was larger than the maximum allowable SIF. The resulting K_{II} was used to validate the numerical DBEM model.

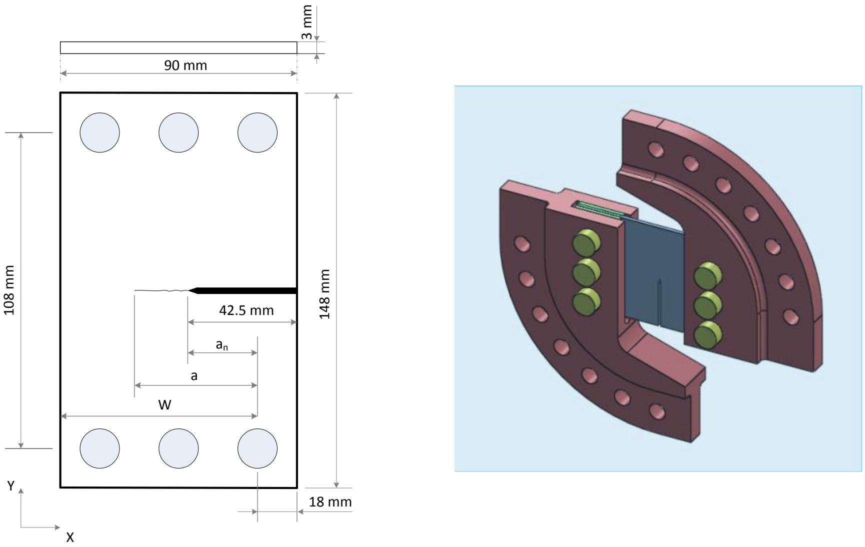


Fig. 3 - The CTS specimen schematic and the loading device

An initial crack of 7.87mm, corresponding to a K_I value of 315.6 N/mm$^{3/2}$, was obtained loading the specimen in Mode I after 120000 cycles as per the conditions in Table 2-a and loaded in Mode II as per the conditions in Table 2-b at each measurement step. The SIFs from numerical DBEM for this crack length, 295.9 N/mm$^{3/2}$, and the Sanders equation above, 288.9 N/mm$^{3/2}$, showed a fair agreement as expected.

Table 2-a. Mode I loading conditions

Property	Value
F_{mean}	1100 N
$F_{amplitude}$	1800 N
R-ratio	0.1
frequency	10 Hz

Table 2-b. Mode II loading conditions

Property	Value
F_{mean}	2750 N
$F_{amplitude}$	4500 N
R-ratio	0.1
frequency	---

Two images were then taken at minimum and maximum load and the displacement field calculated in DIC. The resulting displacement field can be seen in Fig. 3.

The experimental data was compared with the numerical DBEM results along a straight data line ahead of the crack tip. The results can be seen in Fig. 5, showing an excellent agreement.

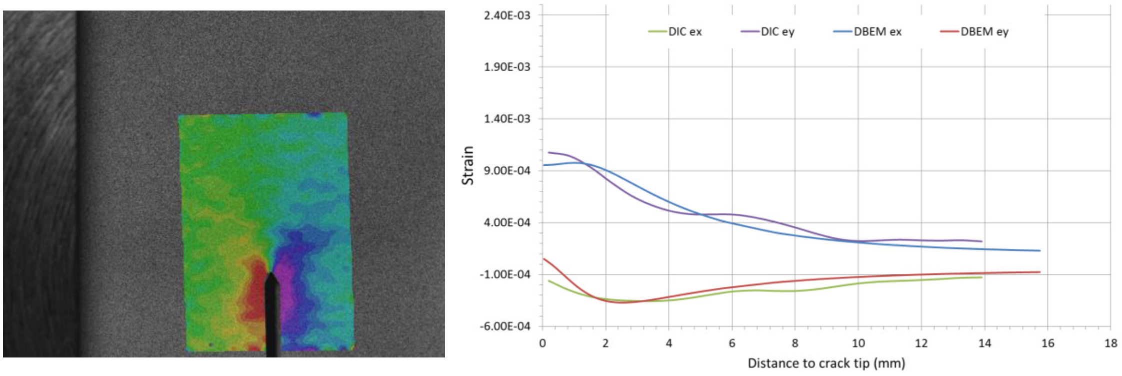
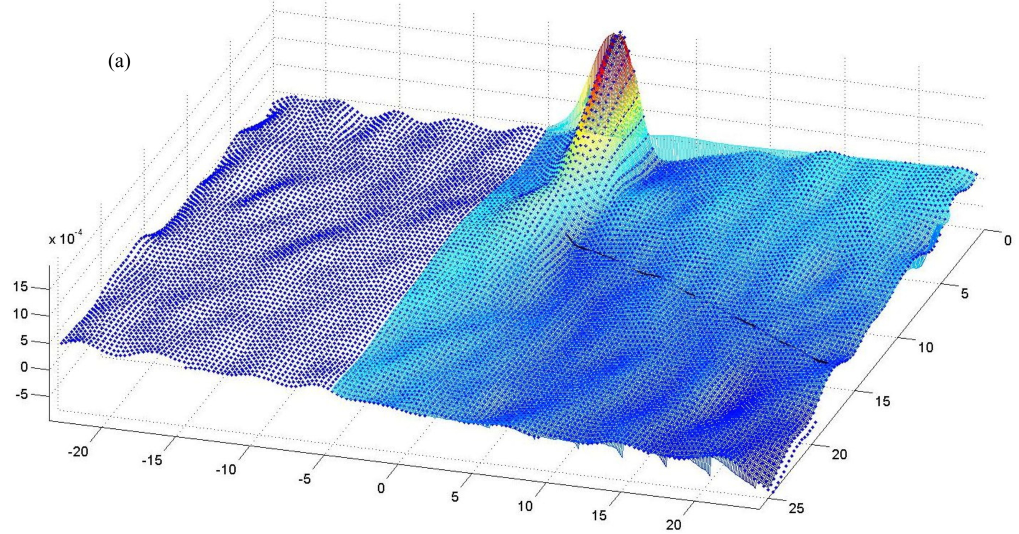


Fig. 4 - Experimental strain results in Mode II fatigue crack testing: (a) DIC strain map in ε_{yy}; (b) DIC and DBEM comparison

Further validation of the obtained results was conducted by interpolating the DIC strain field along a data at 70.5º in front of the crack tip for both ε_{xx} and ε_{yy}, as can be seen in Fig. 5. The results were compared with both the data obtained with DBEM for the same data line and the predicted results from Rice's equations (3) below for pure Mode II in polar coordinates (Rice, 1968), and can be seen in Fig. 5. Some fluctuations in the DIC data can still be low-pass filtered for better readability.

$$\sigma_{xx} = \frac{-K_{II}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\left[2 + \cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right]$$



$$\sigma_{yy} = \frac{K_{II}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \tag{3}$$

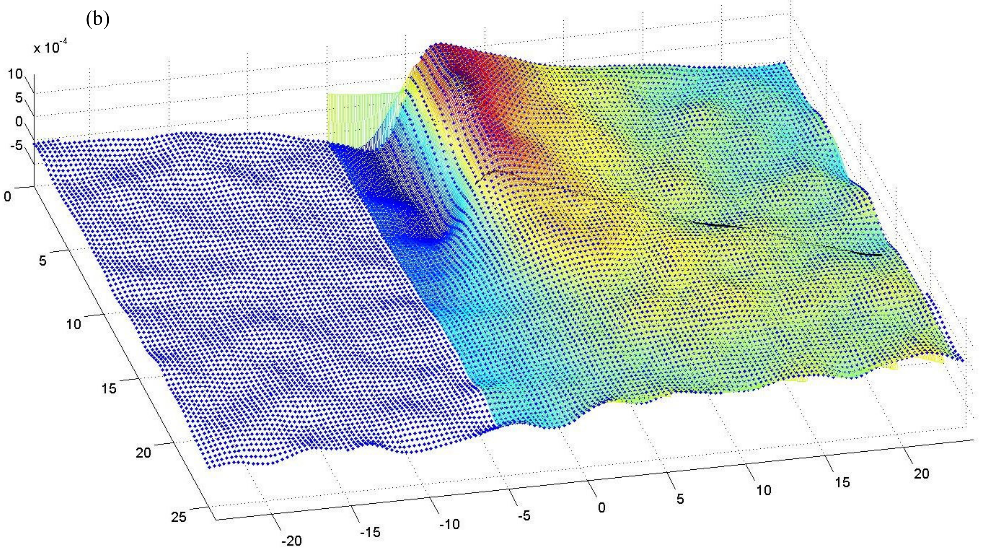


Fig. 5 - Experimental strain results from Mode II interpolated along a line at 70.5º: (a) ε_{xx}; (b) ε_{yy}

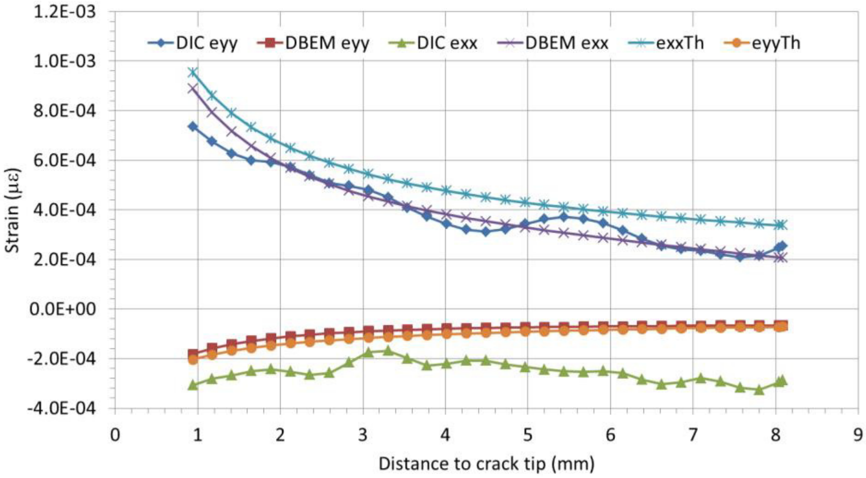


Fig. 6 - Experimental, numerical (DBEM) and theoretical strain results from Mode II along a line at 70.5º for ε_{xx} and ε_{yy}

4. Conclusions

An extensive test campaign for both Modes I and II has been prepared in order to establish the validity of the SIF calculation with a hybrid experimental-numerical methodology. The results presented in this note summarize the work currently ongoing at the Optics and Experimental Mechanics Laboratory on this issue.

The strains measured with the VIC-3D system are first converted into stresses, using the well-known Hooke's equations. These equations of the stress field around the crack tip can then be used to fit the experimental data, thus obtaining an over determined system of equations in which the coefficients of the series expansion are the unknowns. The over deterministic algorithm is used to solve the system and obtain the stress intensity factor, now in both modes I and II

The results obtained so far are quite satisfactory in that similar values have been obtained for the measured and calculated strain fields, raising the confidence in the optical strain measurement system. Additionally, the experimental results for the SIF in both modes show a very good agreement, therefore validating the technique for real life structures structural integrity assessment.

Up to the moment, it is very difficult to precisely find the crack tip on the specimens, in particular when using digital images. This information is however fundamental for good results since the SIF strongly depends on this information. Therefore, a method still has to be developed that guarantees different human operators can easily identify the crack tip.

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