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Gardner&Lockman model (2000) and its application in numerical analysis of composite beams

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Abstract

The paper presents analysis of the stress and deflections changes due to creep in statically determinate composite steel-concrete beam. On the basis of the theory of the viscoelastic body of Arutyunian–Trost-Bažant for determining the redistribution of stresses in beam section between concrete plate and steel beam with respect to time „t", two independent Volterra integral equations of the second kind have been derived. The creep functions is suggested by the Gardner&Lockman prediction model for time dependent deformations of concrete and compared with FIP CEB MC90-99 model. The elastic modulus of concrete $E_c(t)$ is assumed to be constant in time „t". Numerical method based on linear approximation of the singular kernal function in the integral equation is presented. Example with the model proposed is investigated.

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Keywords: composite beam, Volterra integral equations, rheology, Gardner&Lockman prediction model, linear approximation, singular kernal function, viscoelastic body

1. Introduction

Steel-concrete composite beams are wide spread form of construction in both buildings and bridges. A reinforced concrete slab is mechanically connected to the top flange of a rolled or fabricated steel beam, thereby forming a composite member that is considerably stronger and stiffer than the steel beam acting on its own. In sagging or positive bending, the concrete slab is most effective forming a wide compressive flange and raising the position of the neutral axis so that most of the steel section is available to carry tension. The time-varying behaviour of composite steel-concrete members under sustained service loads drawn the attention of engineers who were dealing with the problems of their design more than 60 years [1,2]. The solution of structural problems involving creep and shrinkage phenomena in composite steel-concrete beams has been an important task for engineers since the first formulation of the mathematical model of linear viscoelasticity. If on one hand the definition of a suitable formulation of creep laws involved scientists and researchers in past

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decades and many prediction models have been developed, starting from experimental data and from the direct observation of the long term behaviour of concrete structures (Bažant&Baweja -2000, Gardner&Lockman 2001), the development of structural analysis procedures, based on the creep models, is on the other hand, of great interest for engineers who need to investigate the effects of creep and shrinkage on the structures they design.

2. Basic equations for determining the creep coefficient according Garner&Lockman model

The model presented herein corresponds to the last version of the GL2000 model(ACI 2004), including minor modifications to some coefficients and to the strength development with time equation of the original model developed by (Gardner and Lockman in 2001). It is modified Atlanta 97 model 1993, which itself was ifluenced by CEB MC90-99.It present a design-office procedure for calculating the shrinkage and creep of normal-strength concretes, defined as concretes with mean compressive strength less than 82 MPa, that do not experience self desiccation, using the information available at design, namely, the 28-day specified concrete strength, the concrete strength at loading, element size, and relative humidity. According to (Gardner and Lockman 2001), the method can be used regardless of what chemical admixtures or mineral by-products are in the concrete, casting temperature, or curing regime.The predicted values can be improved by simply measuringen concrete strength development with time and modulus of elasticity. Aggregate stiffness is taken into account by using the average of the measured cylinder strength and that back-calculated from the measured modulus of elesticity of concrete. The compliance expression is based on the modulus of elasticity at 28 days, instead of the modulus of elasticity at age of loading. This model includes a term for drying before loading,which applies to both basic and drying creep.

Required parameters: Age of concrete when drying starts, usually taken as the age at the end of moist curing (days); Age of concrete at loading (days); Concrete mean compressive strength at 28 days (MPa or psi); Concrete mean compressive strength at loading (MPa or psi); Modulus of elasticity of concrete at 28 days (MPa or psi); Modulus of elasticity of concrete at loading (MPa or psi); Relative humidity expressed as a decimal; and Volume-surface ratio (mm or in.);

The creep (compliance) function proposed by the (Gardner and Lockkman 2001), is composed of the elastic and creep strains.The elastic strain is reciprocal of the modulus of elasticity at the age of loading E_{cmto} and the creep strain is the 28 day creep coefficient $\phi_{28}(t,t_0)$ devided by the modulus of elasticity at 28 days E_{cm28}.The creep coefficient $\phi_{28}(t,t_0)$ is the ratio of the creep strain to the elastic strain due to the load applied at the age of 28 days.So:

$J(t,t_0) = \dfrac{1}{E_{cmt0}(t_0)} + \dfrac{\phi_{28}(t,t_0)}{E_{cm28}}$;where: E_{cmt0} is the modulus of elasticity of concrete at the time of loading t_0 E_{cm28} -is the mean modulus of elasticity concrete at 28 days (MPa); $1/E_{cmt0}$ - represents the initial strain per unit stress at loading. $\phi(t,t_0)$ gives the ratio of the creep strain since the start of loading at the age t_0 to the elastic strain due to a constant stress applied at a concrete age of 28 days;

The 28-day creep coefficient $\phi_{28}(t,t_0)$ calculated using the next formulae:

$$\phi_{28}(t,t_o) = \Phi(t_c)\left[2\frac{(t-t_o)^{0,3}}{(t-t_o)^{0,3}+14} + (\frac{7}{t_0})^{0,5}(\frac{(t-t_o)}{(t-t_o)+7})^{0,5} + 2,5(1-1,086h^2)(\frac{(t-t_o)}{(t-t_o)+0,12(V/S)^2})^{0,5}\right]; \quad (1)$$

The creep coefficient includes three terms. The first two terms are required to calculate the basic creep ,and the

third term is for the drying creep. At a relative humidity of 0,96 there is only basic creep .There is no drying creep. $\Phi_{(t_c)}$ is the correction term for the effect of drying before loading.

If $t_o = t_c; \Phi(t_c) = 1$;

When:

$t_0 > t_c \qquad \Phi(t_c) = \left[1 - \left(\frac{(t_0 - t_c)}{(t_0 - t_c) + 0{,}12(V/S)^2}\right)^{0,5}\right]^{0,5}$; t_o = age of concrete at loading, (days); t_c = age of concrete when drying starts at the end of moist curing, (days)

To calculate relaxation, $\Phi(t_c)$ remains constant at the initial value throughout the relaxation period. For creep recovery calculation, $\Phi(t_c)$ remains constant at the value at the age of loading.

If experimental values are not available the modulus of elasticity E_{cmt} at any time t is given by: $E_{cmt} = 3500 + 4300\sqrt{f_{cmt}}$, where the strength development with time can be calculate from the compressive strength using the equation : $f_{cmt} = \beta_e^2 f_{cm28}$ .This equation is a modification of the CEB strength-development relationship. So $\beta_e = \exp\left[\frac{s}{2}(1 - \sqrt{\frac{28}{t}}\right]$; where s =0,4 is CEB(1993) style strength –development parameter, and β_e relates strength development to cement type. Relationship between specified and mean compressive strength of concrete can be estimated from the equation: $f_{cm28} = 1{,}1 f_c^{/} + 5{,}0$ .This equation is a compromise between recommended equation of ACI Committee 209 (1982) and ACI Committee 363 (1992).It can be noted that this equation does not include any effects for aggregate stiffness or concrete density. Instead of making an allowance for density of the concrete,is preferable to measure the modulus of elasticity.

To further justify the present model, it is desirable to clarify the differences from other models, which are presented in this paper. According (Bažant&Baweja 1993, 2000) many basic features of Model GL are questionable on the basis of the current understanding of the mechanics and physics of concrete shrinkage and creep, and violate the guidelines published by a RILEM Committee. Some of them are as follows: disagreement with diffusion theory, the effect of age on creep according this model is far too weak and too short-lived, the creep coefficient for the additional creep due to drying is given in this model by a curve that does not have a bounded final value, the creep recovery curve calculated according to the principle of superposition is violated by the GL model (Bažant&Baweja 2000).

3. Basic assumption and material constitutive relationship

The hypotheses (essentially based on those introduced in initial studies [1,2], in the elastic analysis of composite steel-concrete sections with stiff (rigid) shear connectors are assumed as following: Bernoulli's concerning plane strain of cross-sections (Preservation of the plane cross section for the two elements considered compositely); No vertical separation between parts, in other words identical vertical displacement at the slab-beam interface is assumed; The connection system is distributed continuously along the axis of the beam.; The cross sections are free to deform (because they belong to statically determinate structures); Concrete is not cracked $\sigma_c \le (0.4 \div 0.5) R_c$; For the service load analysis of these cross sections the stress levels are small and, therefore, linear elastic behavior may be assumed for the steel beam, in another words Hooke's law applies to steel as well as to concrete under short-time loads; Moreover, for the concrete part, if the dependence of strains and stresses upon histories of water content and temperature is disregarded, with the

exclusion of large strain reversals, and under normal environment conditions, the strain can be considered as a linear functional of the previous stress history alone. This linearity implies the principle of superposition, which states that strain response due to stress increments applied at different times may be added; In the range of service ability loads concrete behaves in a way allowing to be treated as a linear viscoelastic body. On the basis of our assumptions for the purpose of structure analysis the total strain for concrete subjected to initial loading at time t_0 with a stress $\sigma(t_0)$ and subjected to subsequent stress variations $\Delta\sigma(t_i)$ at time t_i may be expressed as follows: $\varepsilon_{tot}(t,t_0)-\varepsilon^{sh}(t,t_0)=\sigma(t_0)J(t,t_0)+\int_{t_0}^{t}\frac{d\sigma(\tau)}{d\tau}J(t,\tau)d\tau$, where t is the time elapsed from casting of concrete; $\varepsilon_{tot}(t,t_0)$ - total axial strain; $\varepsilon^{sh}(t,t_0)$ - strain due to shrinkage, i.e. an elastic strain. Then the stress-strain behavior of concrete can be described with sufficient accuracy by the next integral equations by Bolztmann-Volterra (2):

$$\varepsilon_c(t)=\frac{\sigma_c(t_0)}{E_c(t_0)}\left[1+\phi(t-t_0)\right]+\int_{t_0}^{t}\frac{d\sigma_c(\tau)}{d\tau}\frac{1}{E_c(\tau)}\left[1+\phi(t-\tau)\right]d\tau\,. \tag{2}$$

According to Gardner&Lockman model the basic equations of Bolztmann-Volterra yields (3)

$$\varepsilon_c(t)=\frac{\sigma_c(t_0)}{E_c(t_0)}\left[1+\phi(t-t_0)\right]+\int_{t_0}^{t}\frac{d\sigma_c(\tau)}{d\tau}\frac{1}{E_c(\tau)}\left[1+\phi(t-\tau)\right]d\tau\,, \tag{3}$$

where : $\phi(t-\tau)=\Phi(t_c)\beta_c(t-\tau)$; is the so called the creep function. The function $\beta_c(t-\tau)$ (where t is the time interval during which the structure is under observation, τ is the running coordinate of time) characterizes the process of creeping. The so called function of aging $\beta(t_0)=\sqrt{\frac{7}{t_0}}$, which characterized the process of the aging is included in the second term of function of creeping $\beta_c(t-\tau)$. $\Phi(t_c)$ is the correction term for the effect of drying before loading. The constitutive law expressed by (16d), represents the stress-strain-time relationship for the concrete slab according(Gardner & Lockman 2000) model.

$$\varepsilon_c(t)=\frac{\sigma_c(t_0)}{E_c(t_0)}\left[1+\Phi(t_c)\beta(t-t_o)\right]+\int_{t_0}^{t}\frac{d\sigma_c(\tau)}{d\tau}\frac{1}{E_c(\tau)}\left[1+\Phi(t_c)\beta(t-\tau)\right]d\tau\,;\quad \text{where:}$$

$$\phi(t-t_0)=\Phi_c\beta(t-t_0)=\Phi(t_c)\left[2\frac{(t-t_o)^{0,3}}{(t-t_o)^{0,3}+14}+\left(\frac{7}{t_0}\right)^{0,5}\left(\frac{(t-t_o)}{(t-t_o)+7}\right)^{0,5}+2,5(1-1,086h^2)\left(\frac{(t-t_o)}{(t-t_o)+0,12(V/S)^2}\right)^{0,5}\right] \tag{4}$$

which for the kernel accepts the following form (4):

$$\phi(t-\tau)=\Phi_c\beta(t-\tau)=\Phi(t_c)\left[2\frac{(t-\tau)^{0,3}}{(t-\tau)^{0,3}+14}+\left(\frac{7}{\tau}\right)^{0,5}\left(\frac{(t-\tau)}{(t-\tau)+7}\right)^{0,5}+2,5(1-1,086h^2)\left(\frac{(t-\tau)}{(t-\tau)+0,12(V/S)^2}\right)^{0,5}\right] \tag{5}$$

The secant modulus of elasticity of concrete E_{cmt0} is invariant in time t. The variable modulus at any time t_0 of loading is given by $E_{cmt}=3500+4300\sqrt{f_{cmt}}$ in SI units; where the strength development with time can be calculate from the compressive strength using the equation : $f_{cmt}=\beta_e^{\,2}f_{cm28}$, where $\beta_e=\exp\left[\frac{s}{2}(1-\sqrt{\frac{28}{t}}\right]$ and

s = 0,4 is CEB(1993) style strength –development parameter, and β_e relates strength development to cement type. Relationship between specified and mean compressive strength of concrete can be estimated from the equation: $f_{cm28} = 1{,}1 f_c^{/} + 5{,}0$. According to our proposal, the influence of the development of the bending moment $M_{c,r}(t)$ in the concrete member, upon the redistribution of the normal force of concrete $N_{c,r}(t)$ can be neglected. For the service load analysis no slip and uplift effects occurs between the steel and concrete. A single theory of interaction ignoring shear lag effects is considered. With another word we can say that shear lag phenomenon of the deck slab is considered by using the appropriate effective slab width (Křistek&Bažant 1987, Křistek&Škaloud 1991).

4. Basic equation of equilibrium

Let us denote both the normal forces and the bending moments in the cross-section of the plate and the girder after the loading in the time t = 0 with $N_{c,0}$, $M_{c,0}$, $N_{a,0}$, $M_{a,0}$ and with $N_{c,r}(t)$, $M_{c,r}(t)$, $N_{a,r}(t)$, $M_{a,r}(t)$ a new group of normal forces and bending moments, arising due to creep and shrinkage of concrete. For a composite bridge girder with $J_c = \dfrac{A_c(nI_c)n}{A_s I_s} \le 0{,}2$ according to the suggestion of (Sonntag 1951) we can write the equilibrium conditions in time t as follows:

$$N(t) = 0; \quad N_{c,r}(t) = N_{a,r}(t); \qquad \sum M(t) = 0; \quad M_{c,r}(t) + N_{c,r}(t) r = M_{a,r}(t). \tag{6}$$

Due to the fact that the problem is a twice internally statically indeterminate system, the equilibrium equations are not sufficient to solve it. It is necessary to produce two additional equations in the sense of compatibility of deformations of both steel girder and concrete slab in time t.

After integrating the two equations by parts transforming the integrals into Riemann ones and using the (6) for assessment of normal forces $N_{c,r}(t)$ and bending moment $M_{c,r}(t)$ two linear integral Volterra equations of the second kind are derived.

$$N_{c,r}(t) = \lambda_N \int_{t_0}^{t} N_{c,r}(\tau) \frac{d}{d\tau}\left[1 + \Phi(t_c)\beta_c(t-\tau)\right] d\tau + \lambda_N N_{c,0} \Phi(t_c) \beta_c(t - t_0) \tag{7}$$

$$M_{c,r}(t) = \lambda_M \int_{t_0}^{t} M_{c,r}(\tau) \frac{d}{d\tau}\left[1 + \Phi(t_c)\beta_c(t-\tau)\right] d\tau + \lambda_M M_{c,0} \Phi(t_c) \beta_c(t - t_0) - \\ -\lambda_M \frac{E_c I_c}{E_a I_a} N_{c,r}(t) r, \tag{8}$$

In which: $\lambda_N = \left[1 + \dfrac{E_c A_c}{E_a A_a}\left(1 + \dfrac{A_a r^2}{I_a}\right)\right]^{-1}$; $\lambda_M = \left[1 + \dfrac{E_c I_c}{E_a I_a}\right]^{-1}$

5. Numerical example

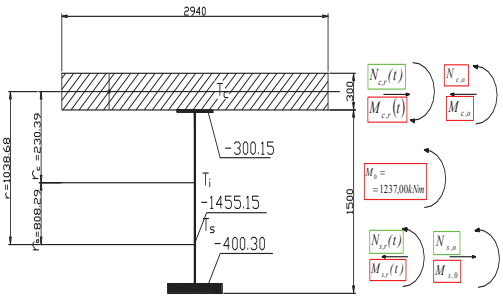
The method presented in the previous paragraph is now applied to a simply supported beam, subjected to a uniform load, whose cross section is shown in fig. 2. On the base of numerous solved examples the optimal step of one day for solving the integral equations (7) and (8) is found. According to Gardner&Lockman model we have respectively: $RH = 80\,\%$ (humidity).

$$E_c = 2,8014.10^4\ MPa,\ E_a = 2,1.10^5\ MPa,\ A_c = 8820\,cm^2,\ A_a = 383,25\,cm^2,\ n = \frac{E_a}{E_c} = 7,496$$

$$I_c = 661500\,cm^4,\ I_a = 1207963,7\,cm^4,\ r_c = 25,50\,cm,\ r_a = 78,37\,cm,\ r = 103,870\,cm,$$

$$A_i = 1560,82\,cm^2,\ I_i = 4415813,859\ cm^4, M_0 = 1237\,kNm,\ N_{c,o} = 840,50kN,$$

$$M_{c,o} = 24,7206\,kNm\ ,\ M_{a,o} = 338,386kNm,\ \lambda_N = 0,068593645,\ \lambda_M = 0,931921295$$



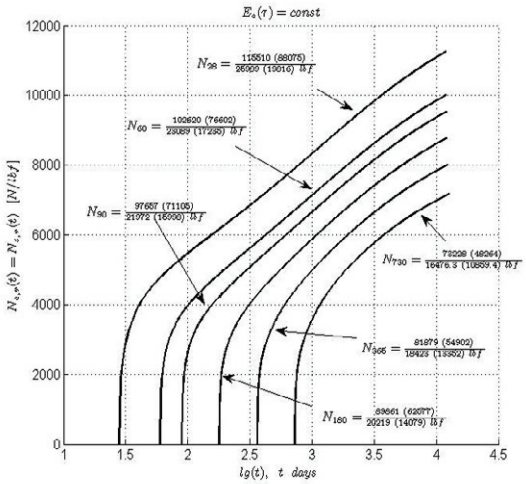


Fig 1. Composite beam with cross-section characteristic; Fig. 2. Values of normal forces $N_{c,r}(t) = N_{a,r}(t)$ in time t when loading is applied in time t_0=28,60,90,180,365 and 730 days

6. Conclusion

It is developed a numerical method for time-dependent analysis of composite steel-concrete sections according to GL2000 model. The obtained results are compared with FIP CEB MC90-99 model [1,2], written in round brackets. The results in stresses obtained by these numerical methods according to GL2000 model and CEB MC90-99 provision are comparable each other, excluding the results in the stresses of upper fibres in steel beam.

References

[1] Partov, D., Kantchev, V. : Level of Creep Sensitivity in composite Steel -Concrete Beams according to ACI 209R-92 model, comparison with Eurocode-4(CEB MC90-99), *Engineering MECHANICS*, **18**(4), 2011, p. 91-116.

[2] Partov, D., Kantchev V.:, Numerical analysis of composite Steel-Concrete Section using integral equation of Volterra, *Central European Journal of Engineering*, **1**(3), 2011, 313-331.