QCD inspired relativistic effective Hamiltonian model for mesons

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ABSTRACT

A QCD inspired relativistic effective Hamiltonian model for mesons has been proposed based on light-front QCD effective Hamiltonian. The squared invariant rest mass operator is used as the effective Hamiltonian. The model has been improved significantly in four major aspects: i) it is proved that in constituent rest frame and in internal Hilbert subspace, the total angular momentum of mesons is conserved, the mass eigen equation can be expressed in total angular momentum representation and in terms of a set of coupled radial eigen equations; ii) a relativistic confining potential is introduced to describe the excited states; iii) an SU(3) flavor mixing interaction is included and a set of coupled mass eigen equations are obtained for different flavor components; iv) the $L$–$S$ coupling and the tensor interaction are taken into full account. The model has been applied to describe the whole meson spectra of about 265 mesons with available data. The meson masses, squared radii, and decay constants are calculated, and the agreement with data is satisfying. For the mesons whose mass data have large experimental uncertainty, the model produces certain mass values for test. For some mesons whose total angular momenta and parities are not assigned experimentally, the model gives a prediction of their spectroscopic configuration $^{2S+1}L_J$. The radial excitation spectra are also analyzed, the discrepancy between the calculated spectra and the data indicates angular momentum effect and higher Fock space effect for some mesons. The relation between our model and the infrared conformal scaling invariance as well as the holographic light front QCD meson model is discussed.

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To study hadronic properties at low energy scales, nonperturbative effects must be taken into account [1]. To describe mesons and baryons, there are several main approaches: the coupled Bethe–Salpeter (BSE) and Dyson–Schwinger (DSE) equation approach [2], the relativistic constituent quark model (CQM) based on Bethe–Salpeter equation [3,4], and the relativistic string Hamiltonian approach [5].

The light-front formalism [6] provides a convenient nonperturbative framework for the relativistic description of hadrons in terms of quark and gluon degrees of freedom [7], some nonperturbative light-front QCD approaches are available, such as light-front Bethe–Salpeter approach [8], holographic light-front QCD model based on AdS/CFT correspondence [9].

The light-front QCD effective Hamiltonian theory proposed by Brodsky and Pauli [10] is an attempt to describe hadron structures as bound constituent quark systems in terms of Fock-space for light-front wave functions. Within the framework of discretized light-front QCD, Pauli et al. have derived nonperturbatively an effective light-front Hamiltonian of mesons on $qar{q}$ sector [11]. The mass eigen equations of mesons are formulated in momentum-helicity (or momentum-spin) representation which hinders its solution in total angular momentum representation. Besides, confining potentials and flavor mixing interactions are lacking, excited states of mesons and flavor diagonal mesons cannot be treated properly [12].

In order to apply this approach to describe mesons in whole $qar{q}$ sector, significant improvements are needed. First, we have proved that in constituent rest frame where the total momentum of the system is zero and in internal coordinate Hilbert subspace, the total angular momentum of the meson system is conserved [13,14]. Since the effective Hamiltonian is the squared rest mass operator and its eigen equation is frame-independent [10], we can work in the constituent rest frame and in internal coordinate Hilbert subspace, and make the following three improvements on the model: (1) transforming mass eigen equations from momentum-spin representation to total angular momentum representation and establishing a set of coupled radial mass eigen equations for each total angular momentum; (2) introducing a relativistic confining potential into the effective meson interaction phenomenologically based on lattice QCD calculation; (3) including an SU(3) flavor-mixing interaction. After having done above, we have a complete QCD

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inspired bound state model for mesons on the $q\bar{q}$ sector. This model has been applied to about 265 mesons with available data and with total angular momentum $J = 0$ to 6. The mass spectra, squared radii, and decay constants are calculated, and the agreement with the data is satisfying. For some light mesons and some radial excitation spectra, systematic discrepancy between calculated spectra and data implies that some physics ingredients (such as angular momentum effect and higher Fock space effect) are lacking in the present model as will be discussed below.

In constituent rest frame and in internal coordinate Hilbert subspace [13,14], the mass eigen equation of mesons on $q\bar{q}$ sector according to Pauli [15], reads

$$
[M_0^2 - (E_1(k) + E_2(k))^2] \psi_{s_1s_2}(k) = \sum_{s_1's_2'} \int d^3k' U_{s_1s_2; s'_1s'_2}(k; k') \psi_{s'_1s'_2}(k').
$$

(1)

Since the effective Hamiltonian is the squared rest mass operator of the meson, the above mass eigen equation is frame-independent and written in momentum-spin representation where spin singlet and triplet are mixed. However, in momentum-spin representation, the momentum-spin plane wave contains all possible angular-spin partial waves, the total angular momentum is thus not conserved. Nevertheless, it has been pointed out that in rest constituent frame and in internal coordinate Hilbert subspace, the total angular momentum $J$ of the meson is conserved, and the mass eigen equation can thus be expressed in total angular momentum representation [13,14]. Now we transform the mass eigen equation (1) from momentum-spin representation to total angular momentum representation and establish the mass eigen equation for each $J$. Expanding the momentum-spin plane wave function in terms of the spin-spherical harmonic wave functions labeled with $JS\bar{M}$ and projecting out the spin and angular part of the wave function in $|JS\bar{M}\rangle$ subspace, we obtain the mass eigen equations for radial wave functions $R_{J\bar{M}}(k)$ [14],

$$
[M_0^2 - (E_1(k) + E_2(k))^2] R_{J\bar{M}}(k) = \sum_{J'=J-1}^{J+1} \sum_{s'=0}^{1} \int d^3k' U_{J\bar{M}; J'\bar{M}'}(k; k') R_{J'\bar{M}'}(k').
$$

(2)

This is a set of coupled equations for radial wave functions $R_{J\bar{M}}(k)$ with different partial waves as well as spin singlet and triplets, coupled by the relativistic spin–orbital potentials and tensor potential. The radial eigen wave function $R_{J\bar{M}}(k)$ has the conventional definition and physical meaning. The bound states of mesons can be described briefly by the spectroscopic symbol of $^{2S+1}L_{J}$, where $J$, $S$, and $L$ denote total angular momentum, total spin, and total orbital angular momentum, respectively. Parity and $C$ parity are: $P = (-1)^{J+1}$ and $C = (-1)^{J+\frac{1}{2}}$

The kernel $U_{J\bar{M}; J'\bar{M}'}(k; k')$ can be written as

$$
U_{J\bar{M}; J'\bar{M}'}(k; k') = \sum_{m \bar{m}_{1} s_{1} s_{2}} \sum_{\bar{s}_{1} \bar{s}_{2}} \int d\Omega_{k} d\Omega' \langle m \bar{m}_{1} s_{1} s_{2} | J \bar{M} | \bar{s}_{1} \bar{s}_{2} \rangle Y_{m}(\Omega_{k}) Y_{m'}^{\dagger}(\Omega'_{k})
\times \langle l m' s' \bar{m}' | J M | l m s \rangle
\times \left\{ \frac{1}{2} s_{1} 1 \bar{s}_{1} \bar{s}_{2} \right\}.
$$

(3)

The above kernel $U_{J\bar{M}; J'\bar{M}'}(k; k')$ contains different kinds of central potentials, relativistic spin–orbit couplings, and tensor potentials changing $l$ by $\Delta l = \pm 2$ and mixing spin singlet and triplets [14].

Numerical results showed that the above model without confining potential can well describe the ground states, but cannot apply to the radial excited states of mesons. To remedy this shortcoming, a confining potential must be included in the model [12]. In the present Letter, the relativistic confining potential in momentum space is taken as [16]:

$$
V_{\text{conf}}(Q) = \lim_{r \to 0} \frac{1}{2} m^{2} |Q|^{2} \left[ \frac{1}{1 + \frac{Q^2}{m^2}} \right] \quad \text{with} \quad (Q^2 = (k - k')^2 + m^2) \quad \text{and} \quad m^2 = (E_1 - E_2)(E_2 - E_1/2).
$$

As the relativistic confining potential $V_{\text{conf}}(Q)$ is included in the interaction, one has the new kernel $U_{J\bar{M}; J'\bar{M}'}(k; k')$ containing the vector interaction $V_{\gamma} = -\frac{\sigma(Q)}{Q}$ and the scalar interaction $V_{S} = -\frac{1}{4} V_{\text{conf}}(Q)$ [14].

It is extremely difficult to derive a flavor mixing interaction within the framework of light-front QCD. However, without this interaction, one cannot deal with the flavor diagonal mesons such as $\pi^0$, $\rho^0$, etc. For simplicity, we introduced a simple flavor mixing interaction phenomenon at the probabilistically producing two extra parameters,

$$
V_{\gamma} = y_{0} \left[ T_{uu}(1) T_{dd}(2) + T_{ud}(1) T_{du}(2) \right]
+ \delta_{0} \left[ T_{us}(1) T_{ds}(2) + T_{us}(1) T_{ds}(2) \right]
+ T_{ds}(1) T_{ds}(2) + T_{ds}(1) T_{ds}(2) \right].
$$

(4)

where $y_{0}$ and $\delta_{0}$ are the strengths of the flavor-mixing interaction, the index 1 and 2 label the quark and antiquark in mesons, respectively. The flavor SU(3) generators $T_{pp}$ with $p, p' = u, d, s$ and its operations on flavor wave functions $|p\rangle$ and $|p'\rangle$ are defined as usual [14]. The action of the flavor mixing interaction on momentum wave functions is as follows: $V_{\gamma} |uu\rangle = y_{0} |dd\rangle + \delta_{0}|ss\rangle$, $V_{\gamma} |dd\rangle = y_{0}|uu\rangle + \delta_{0}|ss\rangle$, $V_{\gamma} |ss\rangle = \delta_{0}|dd\rangle + y_{0}|uu\rangle$.

Combining this interaction with previous one in Eq. (2), we have a set of radial mass eigen equations coupled among different partial waves and flavor components,

$$
[M_0^2 - (E_1(k) + E_2(k))^2] R_{J\bar{M}}^{p_1 p_2}(k) = \sum_{J'=J-1}^{J+1} \sum_{s'=0}^{1} \int d^3k' U_{J\bar{M}; J'\bar{M}}^{p_1 p_2}(k; k') R_{J'\bar{M}'}^{p_1 p_2}(k').
$$

(5)

Finally, the interaction kernel including both the confining potential and the flavor-mixing interaction is

$$
U_{J\bar{M}; J'\bar{M}'}^{p_1 p_2}(k; k') = \sum_{mm' \mu \bar{\mu} s_1 s_2 \bar{s}_1 \bar{s}_2} \int d\Omega_{k} d\Omega' \langle m \bar{m}_{1} s_{1} s_{2} | J \bar{M} | \bar{s}_{1} \bar{s}_{2} \rangle Y_{m}(\Omega_{k})
\times Y_{m'}^{\dagger}(\Omega'_{k}) W_{s_1 s_2; \bar{s}_1 \bar{s}_2}^{p_1 p_2}(k, k') Y_{m'}(\Omega'_{k})
\times \langle l m' s' \bar{m}' | J M | l m s \rangle
\times \left\{ \frac{1}{2} s_{1} 1 \bar{s}_{1} \bar{s}_{2} \right\}.
$$

(6)

Here $W_{s_1 s_2; \bar{s}_1 \bar{s}_2}^{p_1 p_2}(k, k')$ is defined as [14].
The parameters of the model are determined from best fit to experimental data. Reproducing the masses of $\pi^0$, $\pi^\pm$, and $\pi^0$ (1300), we can determine $\hat{a}$, $\lambda$, and the masses of up and down quarks. Then by reproducing the masses of $K^\pm$, $D^\pm$, and $B^\pm$, the mass parameters of strange, charm, and bottom quarks are obtained. The parameters of flavor mixing interaction are determined by the best fit to the data of flavor diagonal mesons. From all the available data of mesons we have obtained an appropriate set of parameters for flavor off-diagonal mesons: $\hat{a} = 0.2574$, $\lambda = 0.92 \times 10^6$ MeV$^2$, $m_{ud} = 0.297$ GeV, $m_s = 0.418$ GeV, $m_t = 1.353$ GeV, $m_b = 4.447$ GeV, and for the flavor diagonal mesons: $\gamma_0 = 0.1$ and $\delta_0 = 0.1$. The number of the model parameters is minimal for this kind of semi-phenomenological models and comparable to BSE and CQM. The masses and wave functions of scalar and pseudoscalar, vector and axial-vector, tensor and pseudotensor mesons, and others with $J = 3–6$ have been calculated and compared with the experimental data in the table (including 265 mesons and anti-mesons: 123 $(u, d)$-light mesons, 50 $(s, u)$-$K$ mesons, 24 $(c, u)$-$D$ mesons, 14 $(s, c)$-$D$ mesons, 12 $(b, u)$-$B$ mesons, 10 $(s, b)$-$B$ mesons, 2 $(c, b)$-$B$ mesons, 16 $(c, c)$ mesons, and 14 $(b, b)$ mesons). It is remarkable that among 265 mesons, 259 mesons are described by this model within mass deviation less than 23%. In addition, the mean squared radii and decay constants for some pseudoscalar mesons are also calculated and compared with the data in parentheses: $(r^2)_{\pi^0} = 0.385 (0.452)$ fm$^2$, $(r^2)_{K^\pm} = 0.253 (0.314)$ fm$^2$, $(r^2)_{\pi^\pm} = 0.235$ (nodata) fm$^2$, and $f_{\pi^\pm} = 135.2 (130.4)$ MeV, $f_{K^\pm} = 210.7 (155.5)$ MeV, $f_{\pi^\pm} = 189.2 (205.8)$ MeV.

In the above calculations, only one set of parameters are used, which deserves discussion. The effective coupling strength or running coupling constant $\hat{a}$ and the related constituent quark masses have a great influence on ground state of light mesons, such as $\pi$. The confining potential strength $\lambda$ governs the quark confinement at large distance and has strong influence on the excited states of light mesons and also on the spectra of heavy mesons. From the recent experiments of hadron physics, we know that the running QCD coupling $\alpha(Q^2)$ becomes large constant but not singular in the low momentum limit, which is called infrared conformal invariance. This experimental fact explains why our model with a set of constant parameters works well to describe the meson structures in the energy region of 0.14 GeV $\rightarrow$ 10 GeV, and our results may be thought of confirming the infrared conformal invariance feature of QCD on meson sector.

The comparison between experimental data and the corresponding model calculations is presented in two ways: 1) Numerical representation: experimental data and the corresponding theoretical results are listed in Table 1 for 265 mesons where the mass deviation errors are given in percentages with respect to experimental values. The advantage of this kind of representation is that direct numerical comparison of meson mass spectra can be seen, which gives people an overall evaluation of the results. The disadvantage of the above representation is that radial excitation spectra for each of species of mesons are not presented explicitly so that an evaluation of the model on QCD effect is not easy to obtained. 2) Plot representation: to remedy the shortcoming of the above numerical representation, radial excitation spectra for different meson species are given in 10 figures where the experimental data of radial excitation spectra of 8 species of mesons are compared directly with the model calculations.

First we discuss the numerical representation in Table 1 in detail. For light scalar mesons such as $a_0$, $K_0^*$, etc., although the structure of the scalar mesons remains a challenging puzzle, our model still describes $a_0(980)$, $a_0(1450)$, $K_0^*(800)$, etc. quite well. For heavy mesons, because of the large masses of heavy quarks, the effective double-gluon-exchange interactions for off-diagonal heavy mesons are weak, which makes the model applicable to them. Therefore, the calculated mass spectra for the mesons of ud, u/ds, u/dc, sc, cc, ub, u/db, sb, and bb are in good agreement with the data. However the meson $K^*(892)$ on u/ds sector with larger error of 50.1% needs special investigation (see below).

It should be noted that the $J$ and $P$ of $D^{\pm\pm}$ are not identified by experiments, but their width and decay modes are observed and consistent with the $1^-$ state. Nevertheless, our model provides a definite assignment of $J = 1$ and $P = -1$ for $D^{\pm\pm}$. Similar predictions of the unidentified $J$ and $P$ are also made for other 8 mesons: $X(1835)$, $D_1(2420)^\pm$, $K^{*}(1630)$, $D^*(2640)$, $B^*_s(2573)^0$, $B_0^*(5732)$, $B_0^{*+}(5850)^0$, and $f_1(2220)$.

The 6 mesons with mass deviations larger than 23% provide some information. For the vector mesons of $\eta$, $\eta'$ (985), $\rho(770)^0$, $\phi(1020)$, and $\omega(782)$ on u/d sector, and $K^*(892)$ on (u/d)s sector, the large discrepancy indicates that the structures of these mesons are special than others and need a different set of parameters; indeed, as the set of parameters are re-adjusted to the set of $(\alpha = 0.4594, \gamma_0 = 0.58, \delta_0 = 0.74)$ and with the others the same, a better fit is found with the deviations less than 23%. Increase of the effective interaction strengths implies that these vector mesons may have strong coupling between $qq$ and $qqqq$ subspaces and among different flavor components.

Now let us discuss the plot representation on radial excitation spectra for 8 kinds of mesons in Figs. 1–10. For all the radial spectra, the ground states are fitted better than the excited states: among 52 radial excitation spectra, about 40 radial ground states are fitted better than excited states. Among 10 shifted-down spectra (most of them are light mesons and strange mesons), 6 of them are with higher total angular momentum ($J = 3–5$), indicating the importance of angular momentum effect which is in turn related to the effective quark mass $m_q$. To get a better fit to these shifted-down radial spectra, an extra orbital angular momentum dependent potential such as $DL^2$ may be useful or the constituent quark masses $m_q$ should be readjusted (especially for u/d quarks and s quark). Among 12 shifted-up spectra, 8 of them are $1^-$ states which need special investigation. The band heads of the 12 spectra are larger than experimental values, indicating that the higher Fock spaces (for instance 4-quark sector) have important effect on the radial excitations. According to quantum mechanics, larger Hilbert space will produce lower ground state energy and help to shift down the spectra. If one still keeps the quark-antiquark scenario for mesons, equivalently one should readjust the parameters for a better fit. In short words, as a semi-phenomenological model, the universal 8 parameters and quark-antiquark sector are not enough to produce a good fit and a detailed description of the whole radial excitation spectra of mesons. However, most of ground states and the overall sequence of radial excitation spectra are reproduced by the model. If one gives up the universal parameter description and readjusts the parameters for different kinds of mesons, the model would have a better fit to radial excitations spectra. The universal parameter description can thus be considered as a first order approximation to the meson spectra.

From the above two-fold comparison, we can see that our model can reproduce the mass spectra with the mean square root deviation of 14%, and that a gross and overall fit to radial excitation spectra of 8 kinds of mesons can be obtained. However, some discrepancies of radial excitation spectra need further investigation and the model should be improved further to include the missing physics ingredients.
100 MeV are not showed). The mesons with question mark '?' are those whose total angular momenta and parities are not identified experimentally.

Table 1
The mesons mass spectra (in MeV). The experimental data, the corresponding theoretical results and their mass deviation errors (ε) are listed (the data error less than 100 MeV are not showed). The mesons with question mark '?' are those whose total angular momenta and parities are not identified experimentally.

<table>
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<tr>
<th>Name</th>
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<th>Expt</th>
<th>Our's</th>
<th>err</th>
<th>Name</th>
<th>$f^c$</th>
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Light mesons: u/d quarks

$p^-$

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Strange mesons (kaons): s, u/d quarks

$k^0$

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Charmed mesons: c, u/d quarks

$D^0_c$

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Bottomed mesons: b, u/d quarks

$B^0_b$

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Bottomed strange mesons: b, s quarks

$B^0_{sb}$

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Charmonium mesons: cc

$\eta_c(1S)$

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Fig. 1. Mesons ($u/d, J \leqslant 1$).

Fig. 2. Mesons ($u/d, J \geqslant 2$).

Fig. 3. Mesons ($s, u/d, J \leqslant 2$).

Fig. 4. Mesons ($s, u/d, J \geqslant 2$).

In conclusion, we have formulated the QCD inspired relativistic bound state model for mesons and derived its mass eigen equations in total angular momentum representation. Moreover, the model has been improved significantly by introducing both a relativistic confining potential and an SU(3) flavor mixing interaction. The resulting radial mass eigen equations are solved for 265 mesons. The calculated results are in agreement with the data with the mean square root mass deviation of 14\%, and with an overall and gross fit to radial excitation spectra. Some discrepancies of radial excitation spectra need further investigation and the model needs improvement. In view that the structure of the light scalar mesons is still a subject of controversy [19], and the internal dynamics of heavy–light mesons in the static limit is far more complicated than that of the heavy–heavy ones [20], our model can be thought to be of preliminary success to describe a whole body of mesons.

Finally, the relation of our model with the holographic light-front QCD model and its physical implication deserve further discussion. The holographic light-front QCD approach by S.J. Brodsky and G.F. de Teramond et al. [9] is based on light-front QCD and AdS/CFT correspondence. The AdS/CFT correspondence between string theory in AdS space and conformal field theories in physical space–time leads to an analytic, semi-classical model for strongly-coupled QCD, which has scale invariance and dimensional counting at short distance and color confinement at large distance. This correspondence also provides AdS/CFT or holographic QCD predictions for the analytic form of the frame-independent light-front wave functions and masses of mesons and baryons. The
Fig. 5. Mesons (c, u/d).

Fig. 6. Mesons (c, s).

Fig. 7. Mesons (c, ¯c).

Fig. 8. Mesons (b, s).

Fig. 9. Mesons (b, u/d).

Fig. 10. Mesons (b, ¯b).
authors have found that the transverse separation of quarks within hadron is related to holographic coordinate (the fifth dimensional \( z \)-coordinate) in AdS/CFT correspondence, the mass eigen equation of meson in light-front effective Hamiltonian approach corresponds to the equation of motion for the holographic field of effective gravity field of super string in AdS space at low energy limit. Recently, they have modified the gravitation background by using a positive-sign dilaton metric to generate confinement and break conformal symmetry. In the meanwhile, the chiral symmetry is broken and a mass scale is introduced to simulate the effect. Based on AdS/CFT correspondence, the holographic light-front QCD model yields a first order description of some hadronic spectra (for both mesons and baryons). This model is quite appealing and promising, since it has established a profound relationship between super string theory and QCD in low energy limit. In this model, very few parameters (only cutoff parameter \( \Lambda \)) are used to obtain the spectra for both mesons and baryons, such as \( \pi \), \( \rho \), and \( \Delta \), etc., which fit the experimental data well \[9\]. However, for the large body of mesons, only a few of them are described properly and a large part of mesons are still left over.

With restriction to meson sector, the two meson models, the holographic QCD model of Brodsky et al. \[9\] and our model are compared as follows: 1) In the effective Hamiltonian (the squared rest mass operator) of mesons, the kinematical energy operator is identical for both models, but the interaction terms are different. 2) Holographic QCD model of Brodsky et al. does not specify the effective interactions in detail, but just simulates confining potential by boundary condition (or harmonic oscillator potential), or recently by a positive-sign dilaton metric to generate confinement and break conformal symmetry; instead, our model provides a detailed semi-phenomenological effective interaction where the spinor structure is derived from light-front QCD and the confining potential as well as the flavor mixing interaction are introduced at a phenomenological level. 3) Holographic QCD model of Brodsky et al. in its present form does not include the spin–spin, spin–orbital, and tensor interactions, thus the total angular momentum conservation is not treated properly. However in this model \( L_z \), \( S_z \), and \( J_z \) are conserved quantum numbers to label mass eigen states, which renders it the potential to describe spin splittings; in the contrary, our model specifies the spin–orbital interactions, thus spin dynamics and total angular conservation are treated properly. 4) Finally, our model has been applied to a whole body of mesons (about 265 mesons identified experimentally) with higher precision than those of holographic QCD model. In the above respects, our model has provided a tentative and effective solution to the above listed problems of holographic QCD model of Brodsky et al. Therefore, in this sense, our model can be considered to be of complementarity to and refinement of the holographic light front QCD meson model.

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