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Weak signal detection based on adaptive cascaded bistable stochastic resonance system

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Abstract

Stochastic resonance system is an effective method to extract weak signal, however, system output is directly influenced by system parameters. Aiming to this, a method about weak periodic signal extraction was developed based on adaptive stochastic resonance. Firstly cascaded stochastic resonance system was established in order to achieve better low-pass filtering effect. And then, variance of zero point distance was chosen as measurement index of cascade system. It's able to overcome the shortage that traditional adaptive stochastic resonance system needs to know the signal frequency beforehand. Also, it could obtain optimum system parameters adaptively. Basing on these parameters, input signal will be handled, and optimum output could be obtained. Furthermore, different periodic signal have been recognized, and finally the validity of the method is verified through simulation experiments.

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Keywords: adaptive cascaded bistable stochastic resonance, measurement index, signal extraction, signal recognition;

1. Introduction[i]

In mechanical, electronic, chemical, communications and many other fields, many useful signals are often submerged in strong noise. How to extract and recognize these signals is a hot spot of industry attention. Currently, the main way to improve the signal to noise ratio (SNR) is to suppress the noise [1-2]. But when the noise frequency and the signal frequency is closed or coincide, with the elimination of the noise of the band, useful signal is always damaged. So it is not benefit to the weak signal detection.

Stochastic resonance was first proposed by the Italian scholar Bentz [3], et al. Stochastic resonance uses non-linear system to produce synergy between input signal and noise, so as to achieve the purpose of the detection signal. The synergy is similar to the resonant in mechanics. Comparing with traditional methods, stochastic resonance does not require prior knowledge. It is a non-correlation detection method. More specifically, stochastic resonance can take advantage of noise to enhance the weak signal. Some noise energy transfers to the characteristic signal. Thus effective detection of weak signal can be achieved through the stochastic resonance. It is often used for the detection of periodic signal. Since the emergence of adaptive stochastic resonance, people continually search for the way to optimize stochastic resonance parameters. Signal to noise ratio and other indicators [4-5] are often used as a system measure index. But because of the absence of prior knowledge, the problem of parameter selection is difficult to solve.

Firstly, this paper analyzes the cascaded stochastic resonance system, selects zero point distant variance as the measure index. Then build a periodic signal extraction model based on adaptive stochastic resonance. At the same time, periodic signal types can be effectively distinguished by using the mathematical properties of kurtosis index. So the periodic signal recognition is realized. Finally, simulation experiment analysis is conducted.

2. Cascaded Stochastic Resonance

2.1. Basic theory of stochastic resonance

Bistable stochastic resonance system is described by Langevin equation. The mathematical model [6] is:

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$$\dot{x} = -\dot{V}(x) + u(t) + \xi(t) \quad (1)$$

Where x is the system output, $u(t)$ is the input periodic signal, for example sinusoidal signal $A\sin(2\pi ft+\varphi)$, $\xi(t)$ is the additional noise, $V(x)$ is nonlinear potential function, expressed as:

$$V(x) = -\frac{a}{2}x^2 + \frac{b}{4}x^4 \quad (2)$$

Where a, b are structural parameters of nonlinear bistable system, greater than 0. When no input signal is applied, potential function takes the minimum in the potential well ($x_m = \pm\sqrt{a/b}$), the maximum in the potential barrier($x=0$).

When the periodic signal without noise is input, if the input periodic signal system meets the system static triggering condition, $|A|>A_0$($A_0 = \sqrt{4a^3/27b}$), where A is the maximum amplitude of the input signal, A_0 is the static triggering threshold for the system. At this time, bistable potential function tilts periodically, as shown in Fig. 1, two potential wells alternately rise and fall. When periodic and noise enter at the same time, periodic changes of system potential well is brought by periodic signal, which is effective to synchronize the switch caused by noise. Thus noise energy in the system output is suppressed. So that the periodic component of the system output has been enhanced, the SNR of the output is improved. This phenomenon is essentially a synergistic effect of signal and noise in nonlinear bistable system, which is called stochastic resonance. Of course, if input signal and noise do not satisfy the system static triggering condition, system structure parameters a, b can be changed to adjust the height of the potential barrier. So that the mixed signal input to the system has sufficient energy to support particle to cross the barrier, then the system can occur stochastic resonance phenomenon.

Tradition stochastic resonance is only applicable to small signal detection, with great restriction on the application. Leng of Tianjin University puts forward a variable step size stochastic resonance. It is a method applicable to large parameter signal detection. The theory is that step length h does not take the reciprocal of the sampling frequency, and make $h>1/f_s$. The experience range of h is 0.1 to 1. By changing the system structure parameter a, b and the step h, signal with large parameters can be detected.

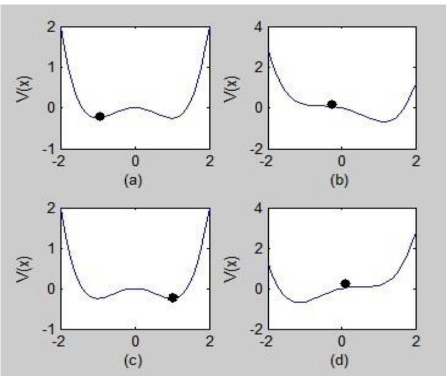


Fig.1. Moving particle in double-well potential

2.2. Bistable stochastic resonance system

Through the mathematical model of stochastic resonance, the corresponding bistable system structure diagram can be obtained, shown in Fig. 2.

Putting two bistable systems shown in Fig. 2 in series (input of the previous system corresponds to output of the next system) can constitute a bistable stochastic resonance system [7] shown in Fig. 3.

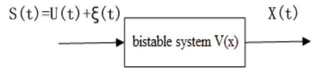


Fig.2. Structure of bistable SR system

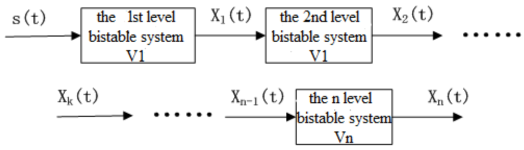


Fig.3. Structure of cascaded bistable SR system

Cascaded stochastic resonance system can improve the signal to noise ratio of the output signal. That will help to extract the useful signal and reduce noise. The mechanism [8-9] is that the output signal spectrum distributes by Lorentz distribution. That means the output signal energy is concentrated in the low frequency region, while reducing the energy of high frequency region. Thus, the noise strengthened the signal intensity instead of weakening that. By cascade stochastic resonance, energy of high-frequency signals shifts to low frequency constantly, so the energy of low-frequency useful signal increases, that of high-frequency noise components decreases. It is equivalent to filter out the high frequency ingredients. Therefore, cascaded stochastic resonance plays a role of the low-pass filter. However, after the traditional filter filtering out signal in useless band, the output signal is always becomes small. Even if the output signal is amplified, noise is also larger, so it is not conducive to the subsequent processing of signal. The cascaded stochastic resonance just does not have this defect, which can increase the output signal while weaken noise. Fig. 4 shows the simulation results of the sinusoidal signal through a bistable system processing. Taking a=1, b=1.5, A=0.5, f=20Hz, D=0.3, f_s=1000Hz during analysis. Runge-Kutta algorithm is used to calculate variable step size stochastic resonance, where calculated step size h=0.5. As shown in Fig. 4, output($x2$) waveform of two levels stochastic resonance system is smoother than output($x1$) waveform of one level stochastic resonance system. That is because high-frequency signal of $x1$ is almost completely filtered out. In a certain sense, the increase of series will help energy of high-frequency signal transfer to low-frequency signal. The resulting result is that the proportion of low-frequency characteristic components in the total signal increases continuously. So the cascaded stochastic resonance system not only plays the role of a low-pass filter, but also improves SNR. But with the number of stages increasing, the system operation time becomes longer, two levels or three levels stochastic resonance system is typically selected.

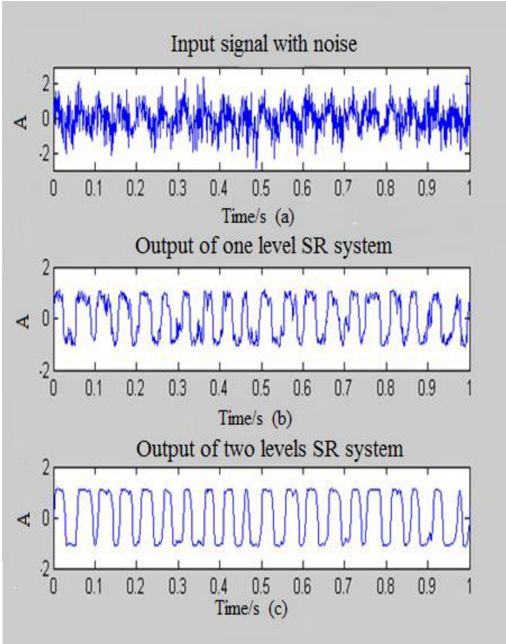


Fig.4. Output of two levels cascade SR system

3. Adaptive periodic signal extraction model

For adaptive periodic signal extraction, in addition to establish variable step cascaded stochastic resonance system model, the most critical is to determine a good measurement index. Comparing with the measurement index, the optimal value of the model parameters (including the structure parameters *a*, *b* of stochastic resonance system, and the calculation step *h*) is determined.

3.1. Tectonic of periodic signal measure index

3.1.1 Zero distance variance

Zero distance variance refers to the variance of distance between two adjacent intersections caused by the signal and the horizontal axis. The smaller zero distance variance is, the smaller the fluctuation of zero point (intersection of signal and the horizontal axis) distant is, the better the signal periodicity is. Considering the discrete signal is difficult to ensure having the intersections with the horizontal axis, so zero crossing point is needed to find. Variable step cascaded system output is $x(t)$, zero crossing point is：

$$O(i)=\{x(t_i);x(t_{i+1})\} \quad 1\le i\le N-1 \tag{3}$$

When $x(t_i)\geq 0(\leq 0)$, there must be $x(t_{i+1})< 0(>0)$, then $x(t_i)$ as zero point to handle.

It could be assumed that total M+2 zero points is obtained. But the initial state is taken into account, the particle locates in the barrier, so eliminate the interference of the first zero point. Then zero distance can be obtained:

$$L_O(j)=t_{j+2}-t_{j+1} \qquad j=1,2,\ldots,M \tag{4}$$

Zero distance variance is:

$$FL=\frac{1}{M}\sum_{i=1}^{M}(L_O(j)-\overline{L_O})^2 \tag{5}$$

As shown in the definition of variance above, the smaller zero distance variance is, the better the signal periodicity is. So FL can be used to measure the periodic state of signal. When FL is less than the set threshold, it is indicated the existence of periodic signal in the mixed input signal. Thus the extraction of periodic signal is realized.

It should be pointed out that when the model parameter is unreasonable, the situation like that in Fig. 5(c) may appear. In this situation, there are only one or two zero points. That is to say, it’s a stochastic resonance system with only one potential well, and no transition exists. This happens because the barrier is too high, the signal is difficult to jump over the barrier, only can fluctuate within a single potential well. Better result would be obtained with resonance between wells when bistable system is used for periodic signal extraction. Hence, if there are only one or two zero points, model parameters should be re-selected.

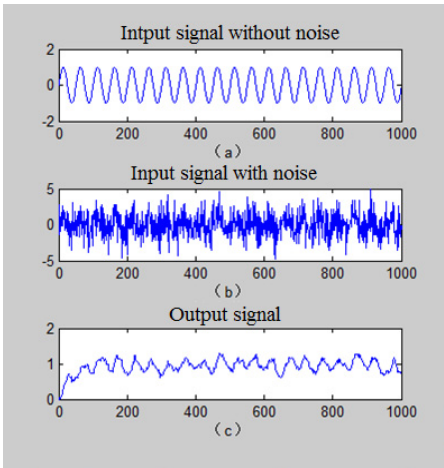


Fig.5. Output of Single-well potential system

3.1.2 Zero distance variance

From the analysis above, zero distance variance FL presents the state of the periodic signal. When FL is less the set threshold, it is indicated the existence of periodic signal in the mixed input signal, and the extraction of periodic signal is realized. Therefore, zero distance variance is selected as measurement index of the adaptive periodic signal extraction model.

As shown in formula (5), the smaller the zero distance variance is, the better quality the periodic signal waveform is.

So the optimal measurement criteria of the model parameters are:

$$\min(FL(a,b,j)) \tag{6}$$

Through the minimization criterion equation (6), the optimal model parameters are determined adaptively. And the optimal output is achieved.

3.2. Process of model

The study led to the conclusion that *a*, *b*, *h* are the model parameters. And FL is selected as the measurement index.

The realization process is as follows:

1) Particle swarm initialization. The position of particle *i* is randomly initialized as (a_i, b_i, h_i). And the active area of the particles is restricted.
2) Numerical simulation is taken for the system corresponding to each particle by using the Runge-Kutta algorithm. Then the system output signal corresponding to each particle is obtained.
3) Calculate the index FL. Calculate the fitness of each particle, as well as individual extremum and global extremum.
4) Update the position and velocity of each particle, and obtained a new model parameters (a_i, b_i, h_i).
5) Determine whether the particles are within the setting region. If the particle is still in the setting area, then return to the second step to continue the iterative optimization. Stop iterating until the error criterion or the maximum generation is met. Finally the optimal parameters *a*, *b*, *h* is obtained. And then the best output is achieved.

4. Periodic signal type identification

Periodic signal can be extracted effectively based on adaptive stochastic resonance extraction model, but the types of periodic signal cannot be determined. Sine, square wave, triangle wave, sawtooth wave, etc are common types of periodic signal. Under the background of strong noise, according to the type of periodic signal, particularly in the field of diagnosis, fault type can be identified effectively. Thus it is facilitated for people to diagnose the fault and carry out equipment maintenance work.

Therefore, an index is needed to be selected to distinguish types of periodic signal correctly. Kurtosis index *K* is the numerical statistics reflecting signal distribution characteristics, defined as the ratio of signal four moments and two moments square[10].

$$K=\frac{E(x_4)}{E^2(x^2)}=\frac{\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^4}{[\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^2]} \tag{7}$$

Where *N* is the length of signal.

Kurtosis index has the special characteristics of mathematical statistics. Fig. 6 shows the signal waveform with different frequencies and different types. According to the formula (7), the corresponding kurtosis value K is calculated. Results are shown in Table 1.

As shown in Table 1, different types of periodic signal correspond to different kurtosis value, and kurtosis value does not change along with the frequency and the amplitude of the signal.

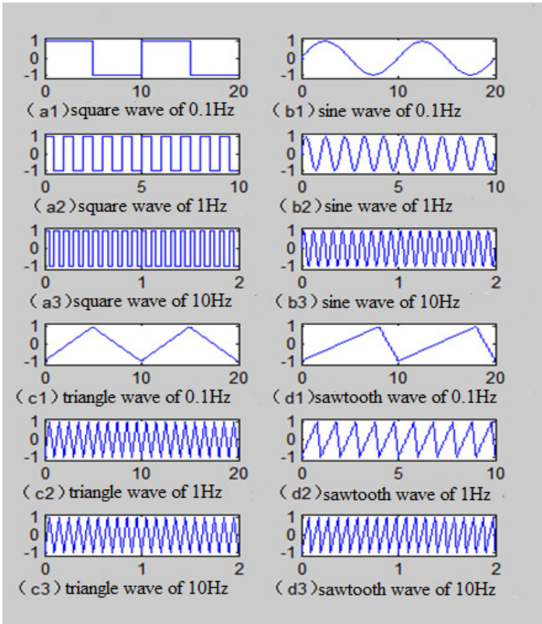


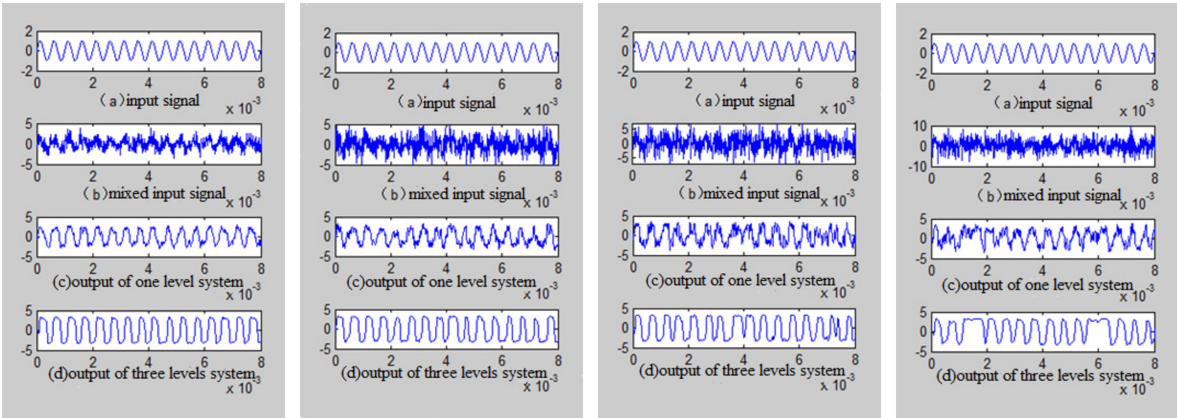
Fig.6. Various periodic waveforms of different frequencies

Table.1. Kurtosis values corresponding to periodic signals

	square	sine	triangle	sawtooth
K	1	1.5	1.8	1.8

5. Simulation analysis

Assume that the signal to be measured is periodic sinusoidal signal, as shown in the first line of Fig. 7. Where A=1, f=2kHz, f_s=100kHz. Image of mixed signal with noise is shown in the second line of Fig. 7. According to the periodic signal extraction based on adaptive stochastic resonance proposed above, three levels stochastic resonance system is chosen, and the search region of *a*, *b* and *h* are [0.1, 10], [0.1, 10], [0.1, 1.0]. The output of one level and three levels system are shown in the third and fourth lines of the Fig. 7 respectively. Under different noise intensity *D*, the model parameters, measurement index, kurtosis values and corresponding SNR were shown in Table 2. By comparing waveform of output signal under different noise intensity in Fig. 7, it shows that the model can extract the high-frequency periodic signal when the noise intensity *D* is less than 8. From the foregoing, this model can extract high-frequency sinusoidal signal with SNR greater than -10.79dB.



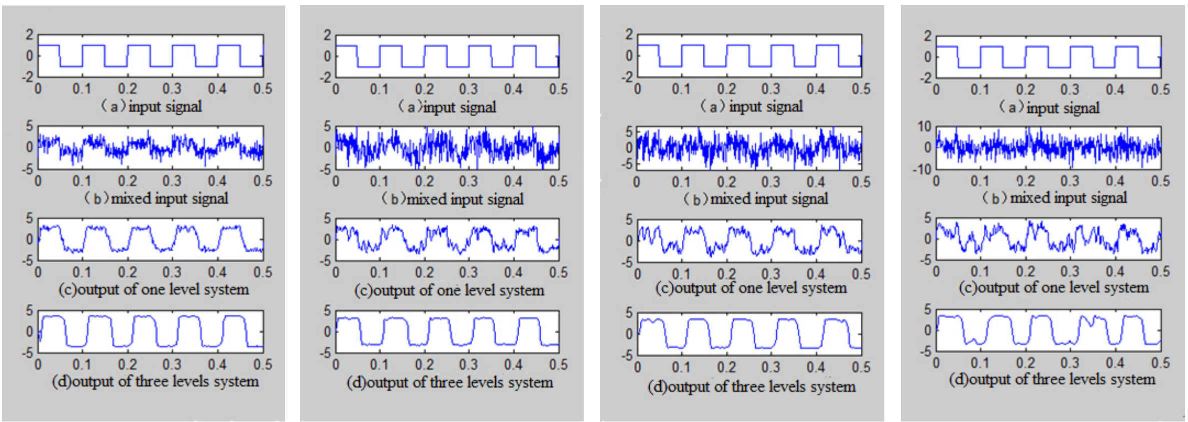
(a) D =1 (b) D =3 (c) D =6 (d) D =8

Fig.7. Input and output of sinusoidal signal extraction model under different noise intensity

Table.2. Optimal parameters of sinusoidal signal extraction model, measurement indicators, kurtosis values and SNR

D	1	3	6	8
a,b,h	0.1,0.1,0.5	0.4,0.1,0.3	0.1,0.2,0.5	0.2,0.1,0.3
FL	3.3844e-006	7.3033e-006	1.9827e-005	3.7044e-005
K	1.3431	1.3706	1.3668	1.4246
SNR(dB)	-3.01	-7.78	-10.79	-12.04

Assume that the signal to be measured is periodic square wave signal, as shown in the first line of Fig. 8. Where A=1, f=10Hz, f_s=500Hz. Image of mixed signal with noise is shown in the second line of Fig. 8. Like the above, three stages stochastic resonance system is chosen. The search region of a, b and h are [0.1, 10], [0.1, 10], [0.1, 1.0]. The output of one level and three levels system are shown in the third and fourth lines of the Fig. 8 respectively. Under different noise intensity D, the model parameters, measurement index, kurtosis values and corresponding signal to noise were shown in Table 3. By comparing waveform of output signal under different noise intensity in Fig. 8, it shows that the model can extract the periodic signal when the noise intensity D is less than 10. From the foregoing, this model can extract square wave signal with SNR greater than -7.78dB.



(a) D =1 (b) D =3 (c) D =6 (d) D =10

Fig.8. Input and output of extraction model under different noise intensity

Table.3. Optimal parameters of square wave signal extraction model, measurement indicators, kurtosis values and SNR

D	1	3	6	10
A,b,h	0.3,0.1,0.5	0.4,0.1,0.3	0.1,0.1,0.5	0.2,0.1,0.3
FL	0.0516	0.1167	0.3199	1.3388
K	1.1166	1.1150	1.1312	1.1544
SNR（*dB*）	0.00	-4.77	-7.78	-10.00

6. Conclusion

This paper presents a weak periodic signal extraction method based on cascaded stochastic resonance. By using the variable step size adaptive stochastic resonance, the weak periodic signals extraction under the condition of large parameters is realized. Cascaded stochastic resonance system has the function of reducing noise and shaping waveform. So the smoothness of the output signal is realized. Zero distance variance is selected as model measurement index, which overcomes the defect that traditional adaptive stochastic resonance algorithm needs prior knowledge of frequency. Finally, the feasibility of the method is verified by simulation.

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