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Gluon confinement criterion in QCD

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Abstract

We fix exactly and uniquely the infrared structure of the full gluon propagator in QCD, not solving explicitly the corresponding dynamical equation of motion. By construction, this structure is an infinite sum over all possible severe (i.e., more singular than $1/q^2$) infrared singularities. It reflects the zero momentum modes enhancement effect in the true QCD vacuum, which is due to the self-interaction of massless gluons. Its existence automatically exhibits a characteristic mass (the so-called mass gap). It is responsible for the scale of nonperturbative dynamics in the true QCD ground state. The theory of distributions, complemented by the dimensional regularization method, allows one to put severe infrared singularities under firm mathematical control. By an infrared renormalization of a mass gap only, the infrared structure of the full gluon propagator is exactly reduced to the simplest severe infrared singularity, the famous $(q^2)^{-2}$. Thus we have exactly established the interaction between quarks (concerning its pure gluon (i.e., nonlinear) contribution) up to its unimportant perturbative part. This also makes it possible for the first time to formulate the gluon confinement criterion and intrinsically nonperturbative phase in QCD in a manifestly gauge-invariant ways.

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1. Introduction

To say today that QCD is a nonperturbative (NP) theory is almost a tautology. The problem is how to define it exactly, since we know for sure that QCD has a perturbative (PT) phase as well because of asymptotic freedom (AF) [1]. In order to define exactly the NP phase in QCD, let us start with one of the main objects in the Yang–Mills (YM) sector. The two-point Green's function, describing the full gluon propagator, is (using Euclidean signature here and everywhere below)

$$D_{\mu\nu}(q) = i\{T_{\mu\nu}(q)d(q^2,\xi) + \xi L_{\mu\nu}(q)\}\frac{1}{q^2}, \qquad (1.1)$$

where ξ is the gauge fixing parameter ($\xi = 0$—Landau gauge, $\xi = 1$—Feynman gauge) and $T_{\mu\nu}(q) = g_{\mu\nu} - (q_\mu q_\nu/q^2) = g_{\mu\nu} - L_{\mu\nu}(q)$. Evidently, $T_{\mu\nu}(q)$ is the transverse (physical) component of the full gluon propagator, while $L_{\mu\nu}(q)$ is its longitudinal (unphysical) one. The free gluon propagator is obtained by simply setting the full gluon form factor $d(q^2,\xi) = 1$ in Eq. (1.1). The dynamical equation of motion for

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the full gluon propagator (1.1) is the so-called gluon Schwinger–Dyson (SD) equation, which is part of the whole SD system of dynamical equations of motion [1]. The solutions of the gluon SD equation are supposed to reflect the complexity of the quantum structure of the true QCD ground state. Precisely this determines one of the central roles of the full gluon propagator in the SD system of equations. The SD equation for the full gluon propagator is a highly nonlinear system of four-dimensional integrals, containing many different, unknown in general, propagators and vertices, which, in their turn, satisfy too complicated integral equations, containing different scattering amplitudes and kernels, so there is no hope for exact solution(s). However, in any case the solutions of this equation can be distinguished from each other by their behavior in the infrared (IR) limit, describing thus many (several) different types of quantum excitations and fluctuations of gluon field configurations in the QCD vacuum. The ultraviolet (UV) limit of these solutions is uniquely fixed by AF.

The IR asymptotics of the full gluon propagator can be either singular or smooth. However, the smooth behavior of the full gluon propagator (1.1) is possible only in one exceptional covariant gauge—the Landau gauge ($\xi = 0$) [2], i.e., it is a gauge artifact solution in this case. Being thus a gauge artifact, it can be related to none of the physical phenomena such as quark and gluon confinement or dynamical breakdown of chiral symmetry (DBCS), which are, by definition, manifestly gauge-invariant. To our best knowledge, beyond the covariant gauges, other than the Landau gauge, the smooth behavior is not known. Anyway, nobody knows how to relate the smooth asymptotics in any covariant gauge to color confinement, DBCS, etc. For example, it does not provide a linearly rising potential between heavy quarks "seen" by lattice QCD simulations [3]. Hence we will not discuss it in what follows, though a solution with smooth asymptotics may exist as a formal one to the gluon SD equation.

Thus we are left with the IR singular behavior of the full gluon propagator only, which is possible in any gauge (in principle, the free gluon propagator can be also used in any gauge. The Feynman gauge free gluon propagator in the IR has been used by Gribov [4] in order to investigate quark confinement within precisely the SD system of equation approach). The only problem is to decide which type of the IR singularities is to be accounted for. The free gluon propagator (see Eq. (1.1) with $d(q^2) = 1$) has an exact power-type $1/q^2$ IR singularity. So the IR singularities as much singular as $1/q^2$ as $q^2 \to 0$ will be called PT IR singularities. The IR singularities which are more severe than the above-mentioned exact power-type IR singularity of the free gluon propagator will be called NP IR singularities, i.e., they are more severe than $1/q^2$ as $q^2 \to 0$. They should be summarized (accumulated) into the full gluon propagator and described effectively correctly by its structure in the deep IR domain. Let us remind that for a long time from the very beginning of QCD it has been already well known that the QCD vacuum is really beset with severe (or equivalently NP) IR singularities if standard PT is applied [1,5–12]. "But it is to just this violent IR behavior that we must look for the key to the low energy and large distance hadron phenomena. In particular, the absence of quarks and other colored objects can only be understood in terms of the IR divergences in the self-energy of a color bearing objects" [10]. It is worth emphasizing that in Ref. [13] it is explicitly shown how severe IR singularities inevitably appear in the vacuum of QCD, providing thus the basis for the zero momentum modes enhancement (ZMME) effect there. So it is intrinsically peculiar to the true QCD ground state due to the self-interaction of massless gluons. Precisely this effect is reflected by the appearance of severe IR singularities in the gluon propagator.

It is clear also that any deviation of the full gluon propagator from the free one in the IR, automatically requires an introduction of the corresponding mass scale parameter, responsible for the nontrivial dynamics in the IR region, the so-called mass gap (see below). This is important, since there is none explicitly present in the QCD Lagrangian (current quark mass cannot be considered as a mass gap, since it is not the renormalization group invariant). Of course, such gluon field configurations, which are to be described by the severely IR structure of the full gluon propagator, can be only of dynamical origin. The only dynamical mechanism in QCD which can produce such configurations in the vacuum, is the self-interaction of massless gluons in the deep IR domain. So the above-mentioned mass gap appears on dynamical ground. Let us remind that precisely this self-interaction in the UV limit leads to AF.

The main purpose of this Letter is to establish exactly the deep IR structure of the full gluon propagator, not solving the gluon SD equation directly, which is a formidable task, anyway. On this basis we will be able to derive the gluon confinement criterion in a manifestly gauge-invariant way.

2. The general structure of the full gluon propagator

For the above-mentioned purpose, namely, how to define the NP phase in QCD, it is convenient to begin with the exact decomposition of the full gluon form factor in Eq. (1.1) as follows:

$$d(q^2) = d(q^2) - d^{\mathrm{PT}}(q^2) + d^{\mathrm{PT}}(q^2) = d^{\mathrm{NP}}(q^2) + d^{\mathrm{PT}}(q^2), \quad (2.1)$$

where, for simplicity, the dependence on the gauge fixing parameter is omitted. In fact, this formal equation represents one unknown function (the full gluon form factor) as an exact sum of the two other unknown functions, which can be always done. So at this stage there is no approximation made (only exact algebraic manipulations). We would like to let the PT part of this exact decomposition to be responsible for the known UV asymptotics (since it is fixed by AF) of the full gluon propagator, while the NP part is chosen to be responsible for its unknown yet IR asymptotics. It is worth emphasizing that in realistic models of the full gluon propagator, the NP part usually correctly reproduces its deep IR asymptotics, determining thus the strong intrinsic influence of the IR properties of the theory on its NP dynamics. Evidently, the decomposition (2.1) represents an exact subtraction of the PT contribution at the fundamental gluon level, and consequently both terms in the right-hand side of Eq. (2.1) are determined in the whole momentum range, $[0, \infty)$. Let us emphasize that the exact gluon form factor $d(q^2)$ being also NP, nevertheless, is “contaminated” by the PT contributions, while $d^{\mathrm{NP}}(q^2)$ due to the subtraction (2.1) is free of them, i.e., it is truly NP.

Substituting the exact decomposition (2.1) into the full gluon propagator (1.1), one obtains

$$D_{\mu\nu}(q) = D_{\mu\nu}^{\mathrm{INP}}(q) + D_{\mu\nu}^{\mathrm{PT}}(q), \quad (2.2)$$

where

$$D_{\mu\nu}^{\mathrm{INP}}(q) = iT_{\mu\nu}(q)d^{\mathrm{NP}}(q^2)\frac{1}{q^2} = iT_{\mu\nu}(q)d^{\mathrm{INP}}(q^2), \quad (2.3)$$

and $D_{\mu\nu}^{\mathrm{PT}}(q) = i\{T_{\mu\nu}(q)d^{\mathrm{PT}}(q^2) + \xi L_{\mu\nu}(q)\}(1/q^2)$. Here the superscript “INP” is the shorthand notation for intrinsically NP. Its definition will be given below. The exact decomposition (2.2) has a remarkable feature. The explicit gauge dependence of the full gluon propagator is exactly shifted from its INP part to its PT part. In other words, we want the INP part to be always transverse, while leaving the PT part to be of arbitrary gauge. This exact separation will have also a dynamical ground. It is clear also that the PT part of the full gluon propagator is, by definition, as much singular as the free gluon propagator’s exact power-type IR singularity. This is the first reason why the longitudinal part of the full gluon propagator, which has the same exact IR singularity, has been shifted to its PT part, and the existence of which is determined by AF.

As was mentioned above, we want the INP gluon form factor $d^{\mathrm{INP}}(q^2)$ to be responsible for the deep IR structure of the full gluon propagator, which is saturated by severe IR singularities. So there is a problem how to take them into account analytically in terms of the full gluon propagator. For this aim, it is convenient to introduce the auxiliary INP gluon form factor as follows:

$$d_{\lambda_k}^{\mathrm{INP}}(q^2, \Delta^2) = (\Delta^2)^{-\lambda_k - 1}(q^2)^{\lambda_k} f_{\lambda_k}(q^2), \quad (2.4)$$

where the exponent λ_k is, in general, an arbitrary complex number with $\mathrm{Re}\,\lambda_k < 0$ (see below). The mass squared parameter Δ^2 (the above-mentioned mass gap) is responsible for the scale of NP dynamics in the IR region in our approach. The functions $f_{\lambda_k}(q^2)$ are dimensionless, regular at zero, by definition, and otherwise remaining arbitrary, but preserving AF in the UV limit. And finally the number k is a positive integer, i.e., $k = 0, 1, 2, 3, \ldots$ (see below). Evidently, a real INP gluon form factor $d^{\mathrm{INP}}(q^2)$, which now should depend on the mass gap as well, i.e., $d^{\mathrm{INP}}(q^2) \equiv d^{\mathrm{INP}}(q^2, \Delta^2)$, is a sum over all $d_{\lambda_k}^{\mathrm{INP}}(q^2, \Delta^2)$.

However, this is not the whole story yet. Since we are especially interested in the deep IR structure of the full gluon propagator, the arbitrary functions $f_{\lambda_k}(q^2)$ should be also expanded around zero in the form of the

Taylor series in powers of q^2, i.e.,

$$f_{\lambda_k}(q^2) = \sum_{m=0}^{[-\lambda_k]-2} \frac{(q^2)^m}{m!} f_{\lambda_k}^{(m)}(0) + \sum_{m=[-\lambda_k]-1}^{\infty} \frac{(q^2)^m}{m!} f_{\lambda_k}^{(m)}(0), \quad (2.5)$$

where $[-\lambda_k]$ denotes its integer number. Also

$$f_{\lambda_k}^{(m)}(0) = \left(\frac{d^m f_{\lambda_k}(q^2)}{d(q^2)^m} \right)_{q^2=0}. \quad (2.6)$$

As a result, we will be left with the finite sum of power terms with an exponent decreasing by unity starting from $-\lambda_k$. All other remaining terms from the Taylor expansion (2.5), starting from the term having already a PT IR singularity (the second sum in Eq. (2.5)), should be shifted to the PT part of the full gluon propagator in Eq. (2.2). So the INP part of the full gluon form factor becomes

$$d^{\mathrm{INP}}(q^2, \Delta^2) = \sum_{k=0}^{\infty} d_k^{\mathrm{INP}}(q^2, \Delta^2) = \sum_{k=0}^{\infty} (\Delta^2)^{1+k} (q^2)^{-2-k} \sum_{m=0}^{k} \frac{(q^2)^m}{m!} f_k^{(m)}(0), \quad (2.7)$$

and $f_k^{(0)}(0) \equiv f_k(0)$, while the piece which is to be shifted to the PT part of the full gluon propagator (2.2) can be shown to have only the PT IR singularities with respect to the gluon momentum, indeed [13]. In this case the subscript "λ_k" should be replaced by the subscript "k", since $\lambda_k = -2 - k,\ k = 0, 1, 2, 3, \ldots$ in four-dimensional QCD, i.e., QCD itself [13]. So the simplest power-type NP IR singularity in QCD is $(q^2)^{-2}$. Thus $d^{\mathrm{INP}}(q^2, \Delta^2)$ describes the true (physical) NP vacuum of QCD, while $d_k^{\mathrm{INP}}(q^2, \Delta^2)$ describe the auxiliary ones, and the former is an infinite sum of the latter ones. The expansion (2.7) is obviously the Laurent expansion in the inverse powers of the gluon momentum squared, which every term ends at the simplest NP IR singularity $(q^2)^{-2}$ (see below). The only physical quantity (apart from the mass gap, of course) which can appear in this expansion is the coupling constant squared in the corresponding powers. In QCD it is dimensionless and is evidently included into the f_k functions. Let us note in advance that all the finite numerical factors and constants (for example, the coupling constant) play no independent role in the presence of a mass gap.

It is instructive to show explicitly expansions for a few first different $d_k^{\mathrm{INP}}(q^2, \Delta^2)$, namely

$$\begin{aligned} d_0^{\mathrm{INP}}(q^2, \Delta^2) &= \Delta^2 f_0(0) (q^2)^{-2}, \\ d_1^{\mathrm{INP}}(q^2, \Delta^2) &= (\Delta^2)^2 f_1(0) (q^2)^{-3} + (\Delta^2)^2 f_1^{(1)}(0) (q^2)^{-2}, \\ d_2^{\mathrm{INP}}(q^2, \Delta^2) &= (\Delta^2)^3 f_2(0) (q^2)^{-4} + (\Delta^2)^3 f_2^{(1)}(0) (q^2)^{-3} \\ &\quad + \frac{1}{2} (\Delta^2)^3 f_2^{(2)}(0) (q^2)^{-2}, \end{aligned} \quad (2.8)$$

and so on. Apparently, there is no way that such kind of an infinite series could be summed up into the finite functions, i.e., functions which could be regular at zero. That is why the above-mentioned smooth gluon propagator is, in general, very unlikely to exist. Let also note in advance that the simplest NP IR singularity $(q^2)^{-2}$ is present in each expansion, which emphasizes its special and important role (see below).

The expansion (2.7), on account of the relations (2.8), can be equivalently written down as follows:

$$\begin{aligned} d^{\mathrm{INP}}(q^2, \Delta^2) &= \sum_{k=0}^{\infty} (q^2)^{-2-k} \sum_{m=0}^{\infty} \frac{1}{m!} (\Delta^2)^{k+m+1} f_{k+m}^{(m)}(0) \\ &= \sum_{k=0}^{\infty} (q^2)^{-2-k} (\Delta^2)^{k+1} \sum_{m=0}^{\infty} \frac{1}{m!} \varphi_{k,m}(0), \end{aligned} \quad (2.9)$$

where we use the relation $f_{k+m}^{(m)}(0) = (\Delta^2)^{-m} \varphi_{k,m}(0)$, which obviously follows from the relation (2.6), since all $f_k^{(m)}(0)$ have the dimensions of the inverse mass squared in powers of m, i.e., $[f_k^{(m)}(0)] = [\Delta^{-2}]^m = [\Delta^2]^{-m}$. Here $\varphi_{k,m}(0)$ are dimensionless quantities, by definition. This expansion explicitly shows that the coefficient at each NP IR singularity is an infinite series itself. It also shows that we can analyze the IR properties of the INP part of the full gluon form factor in terms of the mass gap Δ^2 and the dimensionless quantities $\varphi_{k,m}(0)$ only, which is very convenient from a technical point of view.

It is time now to emphasize the distribution nature of severe IR singularities, i.e., they are to be correctly treated by the distribution theory (DT) [14], complemented by the dimensional regularization (DR) method [15]. The regularization of the NP IR singularities in QCD is determined by the corresponding Laurent expansion in powers of ϵ as follows [13,14]:

$$\left(q^2\right)^{-2-k} = \frac{1}{\epsilon} a(k)\left[\delta^4(q)\right]^{(k)} + \text{f.t.}, \quad \epsilon \to 0^+, \tag{2.10}$$

where $a(k)$ is a finite constant depending only on k and $[\delta^4(q)]^{(k)}$ represents the kth derivative of the δ-function. Here ϵ is the IR regularization parameter, defined as $D = 4 + 2\epsilon$ within a gauge-invariant DR method [15]. It should go to zero at the end of the computations. By f.t. is denoted the regular part of the Laurent expansion. It plays no role in future analysis [13,14]. We point out that after introducing this expansion everywhere one can fix the number of dimensions, indeed, i.e., put $D = n = 4$ for QCD without any further problems, since there will be no other severe IR singularities with respect to ϵ as it goes to zero, but those explicitly shown in this expansion. Thus, as it follows from the Laurent expansion (2.10) that is dimensionally regularized, any power-type NP IR singularity, including the simplest one, scales as $1/\epsilon$ as it goes to zero. Just this plays a crucial role in the IR renormalization of the theory within our approach. Concluding, it worth underling that the structure of the NP IR singularities in configuration space with Minkowski signature is much more complicated that in momentum space with Euclidean signature [14]. We also prefer to work in the covariant gauges in order to avoid pecularities of the noncovariant gauges [1,16], for example how to untangle the gauge poles from the dynamical poles, the only ones which are important for the calculation of any physical observable.

3. IR renormalization of a mass gap

In the presence of such severe IR singularities (2.10), all the "bare" parameters (dimensional or not, does not matter) should, in principle, depend on ϵ as well, i.e., they become IR regularized. Let us thus introduce the following relation

$$\Delta^2 = X_\Delta(\epsilon)\bar{\Delta}^2, \quad \epsilon \to 0^+, \tag{3.1}$$

where $X_\Delta(\epsilon)$ is the corresponding IR multiplicative renormalization (IRMR) constant. Here and below, the quantities with an overbar are IR renormalized, by definition, i.e., they exist as ϵ goes to zero. However, in the above-mentioned paper [13] it has been proven that neither the QCD coupling constant squared nor the gauge fixing parameter are to be IR renormalized, i.e., they are IR finite from the very beginning. As was mentioned above, they can appear only in $\varphi_{k,m}(0)$ quantities. So these quantities also are IR finite from the very beginning, which means that we can put $\varphi_{k,m}(0) \equiv \bar{\varphi}_{k,m}(0)$. This is so indeed, since the rest in these quantities is simply the product of the numerical factors like π's in negative powers, eigenvalues of the color group generators (we are not considering the numbers of different colors and flavors as free parameters of the theory), etc.

We already know that all the NP IR singularities, which can appear in the full gluon propagator scale as $1/\epsilon$ with respect to ϵ (see Eq. (2.10)). So let us introduce the so-called IR convergence condition as follows:

$$X_\Delta^{1+k}(\epsilon) = \epsilon \tilde{A}_k, \quad \epsilon \to 0^+, \tag{3.2}$$

where we put $\tilde{A}_k = \bar{A}_k / \sum_{m=0}^{\infty}(1/m!)\bar{\varphi}_{k,m}(0)$, for convenience. Then the cancellation of the NP IR singularities with respect to ϵ will be guaranteed in Eq. (2.9), and one obtains the finite (nonzero) result in the $\epsilon \to 0^+$ limit for every $k = 0, 1, 2, 3, \ldots$. Here $\bar{A}_k$ and $\tilde{A}_k$ are some arbitrary, but finite constants, not depending on ϵ as it goes to zero.

It makes sense to emphasize now that this IR convergence condition should be valid at any k, in particular at $k = 0$, so from Eq. (3.2) it follows $X_\Delta(\epsilon) = \epsilon \tilde{A}_0$, $\epsilon \to 0^+$, which means that in this case one is able to establish an explicit solution for the mass gap's IRMR constant. Thus the mass gap is IR renormalized as follows:

$$\Delta^2 = \epsilon \bar{\Delta}^2, \quad \epsilon \to 0^+, \tag{3.3}$$

where we include an arbitrary but finite constant $\tilde{A}_0$ into the IR renormalized mass gap $\bar{\Delta}^2$, and retaining the same notation, for simplicity. This means that in what follows we can put it to unity, not losing generality, i.e., $\tilde{A}_0 = 1$.

4. ZMME quantum model of the QCD ground state

The true QCD ground state is a very complicated confining medium, containing many types of gluon field configurations, components, ingredients and objects of different nature. Its dynamical and topological complexity means that its structure can be organized at both the quantum and classical levels [1,17]. Our quantum, dynamical model of the QCD true ground state is based on the existence and the importance of such kind of the NP excitations and fluctuations of gluon field configurations which are due to the self-interaction of massless gluons only, without explicitly involving some extra degrees of freedom. They are to be summarized (accumulated) into the INP part of the full gluon propagator, and are to be effectively correctly described by its strongly singular structure in the deep IR domain (for simplicity, we will call them as singular gluon field configurations). At this stage, it is difficult to identify actually which type of gauge field configurations can be behind the singular gluon field configurations in the QCD ground state (i.e., to identify relevant field configurations: chromomagnetic, self-dual, stochastic, etc.). However, if these gauge field configurations can be absorbed into the gluon propagator (i.e., if they can be considered as solutions to the corresponding SD equations), then its severe IR singular behavior is a common feature for all of them. Being thus a general phenomenon, the existence and the importance of quantum excitations and fluctuations of the severely IR degrees of freedom inevitably lead to the ZMME effect in the QCD ground state. That is why we call our model of the QCD ground state as the ZMME quantum model, or simply zero modes enhancement (ZME, since we work always in the momentum space). For preliminary investigation of this model see our papers [18,19] and references therein.

Our approach to the true QCD ground state, based on the ZMME phenomenon there, in terms of the gluon propagator, can be analytically formulated as in Eq. (2.2), but where now

$$D^{\mathrm{INP}}_{\mu\nu}(q,\Delta) = iT_{\mu\nu}(q)d^{\mathrm{INP}}\big(q^2,\Delta^2\big) = iT_{\mu\nu}(q)\left[\Delta^2\bar{A}_0\big(q^2\big)^{-2} + \sum_{k=1}^{\infty}\big(\Delta^2\big)^{1+k}a_k\big(q^2\big)^{-2-k}\right], \qquad (4.1)$$

where $a_k = (\bar{A}_k/\tilde{A}_k)$. In fact, Eq. (4.1) is already partially IR renormalized with respect to the coefficients, but not with respect to the mass gap Δ^2 and the NP IR singularities $(q^2)^{-2-k}$. This has been done for further convenience. After the IR renormalization it effectively becomes

$$D^{\mathrm{INP}}_{\mu\nu}(q,\Delta) = iT_{\mu\nu}(q)d^{\mathrm{INP}}\big(q^2,\Delta^2\big) = iT_{\mu\nu}(q)\Delta^2\big(q^2\big)^{-2}, \qquad (4.2)$$

since only the first term in Eq. (4.1) survives in the $\epsilon \to 0^+$ limit. An arbitrary but finite constant $\bar{A}_0$ has been included into the mass gap with retaining the same notation, for convenience. In fact, it includes the whole expansion in all powers of the coupling constant squared and different combination of the finite numerical factors only [13] (for brief explanation see text after Eq. (3.2) as well). Evidently, no other terms, explicitly shown in the expansion (4.1) as the second sum, will survive in the $\epsilon \to 0^+$ limit. They become terms of the order of ϵ, at least, in this limit (they start from $(\Delta^2)^2 \sim \epsilon^2$, while $(q^2)^{-2-k}$ always scales as $1/\epsilon$). This is also true for the above-mentioned regular part of the Laurent expansion (2.10).

4.1. Confinement criterion for gluons

It is worth discussing the properties of the obtained solution for the full gluon propagator in more detail. It is already clear that by the IR renormalization of the mass gap only, we can render the full gluon propagator IR finite from the very beginning, i.e., to put $D(q) \equiv \bar{D}(q) = \bar{D}^{\mathrm{INP}}(q) + \bar{D}^{\mathrm{PT}}(q)$ (this has been rigorously proven in Ref. [13]). In principle, two different cases should be considered due to the distribution nature of the simplest NP IR singularity $(q^2)^{-2}$, which saturates its INP part in Eq. (4.2).

(I) If there is an explicit integration over the gluon momentum, then from Eq. (4.2) it follows

$$\bar{D}^{\mathrm{INP}}(q) \equiv \bar{D}^{\mathrm{INP}}_{\mu\nu}(q,\bar{\Delta}) = iT_{\mu\nu}(q)\bar{\Delta}^2\pi^2\delta^4(q), \qquad (4.3)$$

i.e., in this case we have to replace the NP IR singularity $(q^2)^{-2}$ in Eq. (4.2) by its δ-type regularization (2.10) at $k = 0$, which scales as $1/\epsilon$. We also always should take into account the relation (3.3) for the IR renormalization of the mass gap (which scales as ϵ) in order to express all relations in terms of the IR renormalized quantities. The δ-type regularization is valid even for the multi-loop skeleton diagrams, where the number of independent loops is equal to the number of the gluon propagators. In the multi-loop skeleton diagrams, where these numbers do not coincide (for example, in the diagrams containing three or four-gluon proper vertices), the general regularization (2.10) should be used (i.e., derivatives of the δ-functions), and not the product of the δ-functions at the same point, which has no mathematical meaning in the DT sense [14].

(II) If there is no explicit integration over the gluon momentum, then the full gluon propagator is reduced to $D(q) \equiv \bar{D}_{\mu\nu}(q) = \bar{D}^{\mathrm{PT}}_{\mu\nu}(q)$ in the $\epsilon \to 0^+$ limit, since in this case the function $(q^2)^{-2}$ in Eq. (4.2) cannot be treated as the distribution. Only the relation (3.3) again comes out into the play. So the INP part of the full gluon propagator (4.2) in this case disappears as ϵ as $\epsilon \to 0^+$, namely

$$\bar{D}^{\mathrm{INP}}_{\mu\nu}(q, \bar{\Delta}^2) \sim \epsilon, \quad \epsilon \to 0^+. \tag{4.4}$$

This means that any amplitude (more precisely its INP part) for any number of soft-gluon emissions (no integration over their momenta) will vanish in the IR limit in our picture. In other words, there are no transverse gluons in the IR, i.e., at large distances (small momenta) there is no possibility to observe gluons experimentally as free particles. So this behavior can be treated as the gluon confinement criterion (see also Ref. [12]), and it supports the consistency of the exact solution (4.2) for the INP part of the full gluon propagator. Evidently, this behavior does not explicitly depend on the gauge choice in the full gluon propagator, i.e., it is a manifestly gauge-invariant as it should be, in principle. Concluding, it is worth underlining that the gluon confinement criterion (4.3) is valid in the general case as well, i.e., before explicitly showing that the QCD coupling constant squared and the gauge fixing parameter are IR finite [13].

Separating between the NP IR singularities and the PT IR ones on dynamical ground and introducing on this basis the mass gap, we naturally come to the inevitable existence of the INP phase in QCD. In terms of the gluon propagator it can be defined as follows [13]: (i). It is always transverse, i.e., depends only on physical degrees of freedom of gauge bosons, (ii). Before the IR renormalization, the presence of the NP IR singularities $(q^2)^{-2-k}$, $k = 0, 1, 2, 3, \ldots$ is only possible, (iii). After the IR renormalization, the INP part of the full gluon propagator is fully saturated by the simplest NP IR singularity and all other NP IR singularities will be additionally suppressed in the $\epsilon \to 0^+$ limit, (iv). There is an inevitable dependence on the mass gap Δ^2, so that when it formally goes to zero, then the INP phase vanishes, while the PT phase survives.

5. Conclusions

Emphasizing the highly nontrivial structure of the true QCD ground state in the deep IR region, one can conclude.

(1) The self-interaction of massless gluons is only responsible for the ZMME effect in the true QCD vacuum, which in its turn, is to be taken into account by the deep IR structure of the full gluon propagator.

(2) The full gluon propagator thus is inevitably more singular in the IR than its free counterpart (the smooth in the IR gluon propagator is a gauge artifact, since it is possible in the Landau gauge only).

(3) This requires the existence of a mass gap, which is responsible for the NP dynamics in the true QCD vacuum. It appears on dynamical ground due to the self-interaction of massless gluons only.

(4) We define the NP and the PT IR singularities as more severe than and as much severe as $1/q^2$, respectively, which is the power-type, exact IR singularity of the free gluon propagator.

(5) We decompose algebraically (i.e., exactly) the full gluon propagator as a sum of its INP and PT parts. We additionally distinguish between them dynamically by the different character of the IR singularities in each part.

(6) We have exactly established the deep IR structure of the full gluon propagator, represented by its

INP part, as an infinite sum over all possible NP IR singularities.

(7) The next step is to regularize them correctly, i.e., to use the Laurent expansion (2.10) that is dimensionally regularized with respect to the IR regularization parameter ϵ.

(8) The IR renormalization program is based on an important observation that the NP IR singularities $(q^2)^{-2-k}$, being distributions, always scale as $1/\epsilon$, not depending on the power of the singularity k, i.e., $(q^2)^{-2-k} \sim 1/\epsilon$. It is easy to understand that otherwise none of the IR renormalization program in the INP phase of QCD and QCD as a whole would be possible.

(9) The IR renormalization of the initial mass gap (Eq. (3.3)) is only needed in order to fix uniquely and exactly the IR structure of the full gluon propagator in QCD. It is saturated by the simplest NP IR singularity, the famous $(q^2)^{-2}$, which leads to the linearly rising potential between heavy quarks "seen" by lattice simulations [3]. The final mass gap gains contributions from all orders of PT in the QCD coupling constant squared, which remains IR finite.

(10) On this basis, we have formulated the ZMME model of the true QCD ground state. Due to the distribution nature of the NP IR singularities, two different types of the IR renormalization of the INP part of the full gluon propagator are required.

(11) This makes it possible to establish the gluon confinement criterion in a manifestly gauge-invariant way.

(12) In the same way, we define exactly the INP phase in QCD at the fundamental gluon level. The corresponding decomposition of the full gluon propagator is only needed in order to firmly control the IR region in QCD within our approach (only it contains explicitly the mass gap).

(13) All this makes it possible to determine unambiguously the interaction between quarks concerning its pure gluon (i.e., nonlinear) contribution.

These somehow astonishingly results have been achieved at the expense of the PT part of the full gluon propagator. It remains of arbitrary covariant gauge and its functional dependence cannot be determined. However, let us note in advance that our theory (which we call INP QCD) will be additionally defined by the subtraction of all types of the PT contributions in order to completely decouple it from QCD as a whole (the first step in these subtractions has been already done in Eq. (2.1). See also the extended paper in Ref. [13]).

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References

[1] W. Marciano, H. Pagels, Phys. Rep. C 36 (1978) 137.
[2] L. von Smekal, A. Hauck, R. Alkofer, Ann. Phys. 267 (1998) 1; K.-I. Kondo, hep-th/0303251.
[3] K.D. Born, et al., Phys. Lett. B 329 (1994) 325; V. Miller, et al., Phys. Lett. B 335 (1994) 71.
[4] V.N. Gribov, Eur. Phys. J. C 10 (1999) 71.
[5] D.J. Gross, F. Wilczek, Phys. Rev. D 8 (1973) 3633.
[6] J. Kogut, L. Susskind, Phys. Rev. D 9 (1974) 3501.
[7] H. Fritzsch, M. Gell-Mann, H. Leutwyler, Phys. Lett. B 47 (1973) 365.
[8] S. Weinberg, Phys. Rev. Lett. 31 (1973) 494.
[9] H. Georgi, S. Glashow, Phys. Rev. Lett. 32 (1974) 438.
[10] L. Susskind, J. Kogut, Phys. Rep. 23C (1976) 348.
[11] J.L. Gervais, A. Neveu, Phys. Rep. 23C (1976) 240.
[12] H. Pagels, Phys. Rev. D 15 (1977) 2991.
[13] V. Gogohia, hep-ph/0311061; V. Gogohia, hep-ph/0304163.
[14] I.M. Gel'fand, G.E. Shilov, Generalized Functions, vol. I, Academic Press, New York, 1968.
[15] G. 't Hooft, M. Veltman, Nucl. Phys. B 44 (1972) 189.
[16] G. Leibbrandt, Noncovariant Gauges, World Scientific, Singapore, 1994; A. Bassetto, G. Nardelli, R. Soldati, Yang–Mills Theories in Algebraic Non-Covariant Gauges, World Scientific, Singapore, 1991.
[17] V. Rubakov, Classical Theory of Gauge Fields, Princeton Univ. Press, Princeton, NJ, 2002.
[18] V. Gogohia, Gy. Kluge, Phys. Rev. D 62 (2000) 076008.
[19] V. Gogohia, Gy. Kluge, H. Toki, T. Sakai, Phys. Lett. B 453 (1999) 281.