Adapting Interactive Methods In The Teaching Of Linear Algebra –
Results From Pilot Studies

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Abstract
This study presents the results of five experiments which restructure the lab exams of a traditional Linear Algebra course by implementing interactive methods of education and software technologies. The student participants are encouraged to work cooperatively in class and to use a discussion forum for their homework. Each topic of the curriculum consists of practical problems adapted for the interactive methods of education used in class. The study uses software products as means of helping students to solve Linear Algebra problems from their homework and submitting the resulting solutions virtually. In the pedagogic specialties the results from the experiments show that the use of such organizational models leads to a higher average score in final evaluations; however it only affects the more active students in the experimental groups. The students with less drive and motivation tend to repeat previous performances regardless of the available interactive methods, software products and discussion forum tools. Similar results were repeated with the specialty “Informatics”; however with less dispersion.

Keywords: interactive methods; web 2.0; mathematics education; linear algebra; mathematical competences

1. Introduction

The curriculum of the Linear Algebra course in the Faculty of Mathematics and Natural Sciences in South-West University “Neofit Rilski” is structured as combination of lectures (3 hours per week) and lab exams (2 hours per week) in one semester. There are two homework assignments. The practice shows that the time is insufficient for
implementation of more complex interactive methods of education and the lab exams are leaded more like seminars where the assistant-professor solves problems in front of the students each followed by short discussion.

The study made an effort to reorganize the lab exams and the homework assignments in a way that the students will prepare better prior the lab exams which should free time for implementation of interactive methods of education in class. Similar to the researches in Gibbs & Simpson (2004) and Jaworski, Treffert & Bartsch (2011), the homework was divided in smaller parts which were assigned after each lab exam in a Web 2.0 educational environment via discussion forum. The students were encouraged to discuss their solutions, help each other in topics or private messages and to use software tools like MatrixCalc.org and Microsoft Mathematics 4.0 (Purwanti & Pustari, 2013) to help themselves in solving the problems from their homework assignments.

2. Methods

2.1. Interactive methods in the teaching of Linear Algebra


- Using a 4x4 matrix every student is given a self-dependent task to solve its determinant with the Laplace’s formula but each starting with different row or column. When the students are ready they compare their results and discuss which row or column leads to less calculations then others. The “discussion by circles” method showed to be practical here;
- When multiplying two 5x5 matrices, groups of 3-4 students are formed and each group is given a task to find the numbers in one of the rows (or columns) of the resulting matrix. When all groups are ready, the assistant-professor combines the results and presents the final solution;
- When finding an inverse of an invertible 4x4 matrix by finding the adjugate matrix, each student in the class is given a task to compute one of the cofactors. When every student is ready, the assistant-professor combines the results and presents the final solution;
- Due to the hardness of the topics from the second part of the linear algebra course, the assistant-professor should be taking a more central role in the interactive education. More efforts are put into the discussions instead of solving. Like in Cowen (1997), the assistant-professor is often asking misleading questions like “how many basses this linear space have?” which aim at focusing the attention of the students in the important details of the topic. The tutor should praise every effort for joining the discussion even when the answer is incorrect;
- When solving problems using the Gram-Schmidt process different students are asked one by one to get in front of the group to find the next vector from the orthogonal basis on the whiteboard. When the vectors are found, the desk is divided on three or four parts and different students are asked to normalize the vectors simultaneously;
- On the eigenvalues and eigenvectors topic and on the quadratic forms topic the assistant-professor is trying to provoke larger discussions by asking quick questions. The target is not only to teach the new material but also to make a retrospection of everything learned so far and to clarify eventual gapes in the knowledge.

Furthermore at the end of each topic the students are also presented with practical-oriented problems in which the linear algebra apparatus is used to solve different real life issues using mathematical modelling. The students are not forced to solve them on their own but the assistant-professor is presenting the solutions instead. It is essential that such problems and solutions must be discussed and explained very well either in class or (if there is not enough time) later in the discussion forum. In some topics the problems in the homework are assigned like analogical practical problems to the presented in class. Authors like Schoenfeld, (1992), and Gantchev, (2007) point that the practical problems are interesting for the students and lead to a higher cognitive dissonance. They also increase the interdisciplinary relationships which is important for universities Gyudzhenov, (2007).
2.2. The discussion forum and software provisioning

After some research from different recent articles like Hijon-Neira & Velazquez-Iturbide, (2009) and Liao& She , (2009), Simple Machines was chosen as the base system for electronic education. The interface of the forum software is standard and suitable for general computer user. It comes with built-in statistics which are not as good as the statistics in LCMS systems like Moodle but they are still enough to collect all required data for the experiment. The presentation layer was upgraded to use MathJax and CodeCogs equation editor. An adaptation with Moodle is planned for future.

The forum is designed to be shared by students from different specialties. It is divided in two parts – public and private. The public category consists of general discussions board and resources section where the assistant-professor provides additional resources for review and download. The resources board includes:
- Notes from the lab exams – detailed solutions of all problems that are solved in class;
- Multimedia section with different video and interactive tutorials for teaching students in technology;
- Software section where students can find links and downloads of software products.

The private category of the forum is visible only by the students from each specific experimental group. In a personal topic in the board of the group the students are receiving their homework assignments and they are submitting solutions. Different students from the one group can share materials and help each other. One student can ask questions or provide different solutions before reaching the correct answer. When specific problem is solved correctly, the assistant-professor may contact that particular student personally by a private channel and asks additional questions over the submitted solution. When these questions are answered correctly, the solution is accepted and points are added to the students file. For the control groups, where the homework assignments are presented on paper, the situation was similar – when a student was submitting his solutions, the assistant-professor was asking one question to verify if the student understands what he pass as a solution to him.

The software products and educational materials are preferred to be in Bulgarian language. That involved the first author of the study to contact the developers and to make translations of software products like Code Cogs equation editor and the tools on the MatrixCalc.org website which were later integrated with the forum.

Students were not forced to use the software technology for insertion of mathematical formulas directly in the forum. They were allowed to write their texts in Word or any other software of their own choice and to provide their solutions as attachments. Some students even preferred to write their solution on paper and then photograph it – this was also accepted as valid approach. The study summarized that the usage of Microsoft Word was mostly preferred.

3. The experiments and their results

3.1. Experiment 1 – a failed effort to implement the model

The first experiment was done in academic year 2011/2012 with control group of students from speciality “Pedagogy of Teaching Mathematics and Informatics” (POMI) and an experimental group of speciality “Pedagogy of Teaching Physics and Mathematics” (POFM). Both specialties are not very popular among candidate-students.

This experiment was called “pilot” as it was used to mainly test and eventually smooth-out unplanned organizational and software issues. Such issues appeared as the specialty of the experimental group was formed by only 12 students compared to 22 signed in the control group. That way the smaller group may have advantage as the tutor can pay more attention to personal questions in class. The bigger problem was that the students from the experimental group did not involve with the required enthusiasm in submitting their homework assignments in the discussion forum. That was a major issue for the lab exams plan because the students were coming unprepared and the education was forced to be leded more like the traditional way. Most of the planned interactive methods were not implemented at all. Both groups suffered from a non-attendance of some students. These issues resulted in insignificantly higher results of the experimental group compared to the control one.
3.2. Experiment 2 – a successful implementation of the model

The failure of the first experiment forced some corrections in the experimental model. The main focus was to change the attitude of the tutor to the homework assignments. The discussion forum was no longer presented as a requirement but it more like a source of materials and software which will help the students to solve their homework more easily. That change in the attitude from negative rule towards positive opportunity shown to be a big success. The experimental group was formed from 25 students combined in one group from the two specialties POMI and POFM in academic year 2012/2013. The control group was chosen to be the control group from experiment 1. The average score from the high school diplomas are almost identical and the Mann-Whitney U-test showed no difference in the entry level of the two groups: U=213 with Ucrit=182. The results from the experiment were quite encouraging - the increase in the average final points was over 70% for the experimental group (14.25 points in the control and 25.36 points in the experimental group)! The greatly increased average score of the experimental group was however with a huge dispersion. Comparison of the groups with the Mann-Whitney U-test showed no significant difference in the median of the final results: U=204 with Ucrit=182. That confirmed the observation of the tutor that only a relatively small group of the students in the experimental group were affected well by the interactive education in the lab exams. It was no surprise that they were also the students who attended actively in the discussion forum and received excellent scores on the two tests.

3.3. Experiment 3 – the model in specialty “Informatics” 2012/2013

The experimental group was formed of 20 students while the control group was with 19. Mann-Whitney U-test over the high school grades gave no significant difference: U=178 with Ucrit=112. The subject in that experiment was “Linear Algebra and Analytic Geometry”. While it is possible to implement the model in Analytic Geometry topics, the authors preferred to focus on the Linear Algebra part of for the experiment and the topics from Analytic Geometry were covered as much as possible like the traditional model (less interactivity in class and traditional homework assignments). During the semester it was no surprise to see that the better motivated students from the experimental group tended to prefer the Linear Algebra topics. This clearly shows that the interactive methods are making the subject more attractive because like many authors note the Linear Algebra subject is usually much harder for students due its high abstraction while Analytic Geometry is “more visual” and easier to understand. The students with less drive and motivation on the other side were apathetic to both subjects’ topics. The results from this experiment repeated the results from Experiment 2; however with less dispersion in the results (this is due to the higher average score of all students in Informatics compared to the pedagogic specialties). The percentage growth in the average final score of the students was 34% (22.61 points in the control compared to 30.48 points in the experimental group). There was a high dispersion in the average score in the experimental group and only a subset of students increased their score. The students with low grades were not affected by the model. This is confirmed by the Mann-Whitney U-test which showed no significant difference - U=149 with Ucrit=112.

3.4. Experiment 4 – the level of voluntarily involvement in the discussion forum

The experiment was made with one group from specialty “Computer Systems and Technologies”. The lab exams were leaded traditionally and the usage of the discussion forum was not a requirement – the students were having the opportunity to provide their solutions on paper if they prefer so. The purpose of the experiment was to see how many of the students will join the discussion forum voluntarily. The students in this experimental group were 30 and their profile is with much higher grades in mathematics from high school compared to the other experiments. The results from the experiment show that more than half of the students in the group joined the discussion forum with activity but only three of them were submitting homework assignments. The others were mainly using the resources section and preferred to provide their homework solutions on paper. A non-formal survey across some students confirmed the conclusions – the software tools for inputting mathematical text on computer is what is repulsing the students from using the system. When looking at the students final score two of the three “forum-active” students were those who shown best scores in the group and the third one was above average. This still cannot be taken as confirmation
that the discussion forum helped the students to raise their results. It is also very likely that these three students used the forums because they want to emphasise to the tutor their leadership in the group.

### 3.5. Experiment 5 – the effect of the forum over the traditional lab exams

This experiment aimed to measure how much the discussion forum itself influence the students. The experimental group was “Communication Technics and Technologies” (25 students) and the control group was “Electronics” (16 students) – both in Technical College in South-West University “Neofit Rilski”. The high school grades were compared with Mann-Whitney U-test to have no significant difference: U=159 with Ucrit=126. The lab exams between the two groups are leaded as equally as possible. The difference was that the students from the experimental group are required to provide the solutions of their homework in the discussion forum. All of the students in the experimental group registered and used the forum; however many of them never submitted their homework assignments. The results from the experiment were quite surprising - the students from the experimental group had much higher scores on the tests (7.52 points average for the experimental group and only 3.75 for the control one); however their average final score was lower (23.96 for the experimental group compared to 31.56 on the control group). The Mann-Whitney U-test showed a significant difference in the final scores of the groups: U=113.5 with Ucrit=126. The reason for this abnormality (higher scores on tests with lower final grades) is clearly the much less points taken from homework solutions. The control group students submitted twice more homework assignment solutions on paper than the experimental group did in the forum! The requirement for submission of the homework solutions in the electronic system is more a drawback than a positive.

### 4. Conclusions

The proposed model should have better results if used with students which have better mathematical skills and higher motivation. Despite the increase in the average scores it was not able to gain the expected interest in students who come from high school with lower grades in mathematics.

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### References

Cowen C. (1997). On the centrality of linear algebra in the curriculum. The Deborah and Franklin Teper Haimo award for distinguished college or university teaching at the Joint Meetings, San Diego, California.


Gibbs G. Simpson C. (2004). Conditions under which assessment supports students learning. Learning and teaching in higher education Issue 1


