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On oscillation of a food-limited population model with impulse and delay ☆

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Abstract

In this paper, we consider a food-limited population model with impulsive effect. Several explicit sufficient conditions are established for oscillation and nonoscillation of solutions of the equations.

Keywords: Oscillation; Nonoscillation; Impulsive delay equation

1. Introduction and preliminaries

The theory of impulsive delay differential equation is emerging as an important area of investigation, since it is a lot richer than the corresponding theory of delay differential equations without impulse effects. Moreover, such equations may describe several real world phenomena in physics, biology, engineering, etc. In recent years, oscillation theory of impulsive delay differential equations attracts attention of many mathematicians and numerous papers have been published on this class of equations (see [2,5,9,10] and references therein). For oscillation theory of nonimpulsive delay differential equations, we refer the reader to the references [3,4,6–8].

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L. Berezansky and E. Braverman [1] investigated oscillation of the nonautonomous food-limited equation with nonconstant delay:

$$\dot{N}(t)=r(t)N(t)\frac{K-N(h(t))}{K+s(t)N(g(t))},\quad t\geqslant 0,\ h(t)\leqslant t,\ g(t)\leqslant t, \tag{1.1}$$

which is a generalization of the equations in [2–4,6,8,11].

The purpose of this paper is to derive sufficient conditions for oscillation and nonoscillation about K of generalized food-limited equation with impulsive effects

$$\begin{cases} \dot{N}(t)=r(t)N(t)\dfrac{K-N(h(t))}{K+\sum_{i=1}^{m}p_i(t)N(g_i(t))}, \\ N\big(t_k^+\big)-N(t_k)=b_k\big(N(t_k)-K\big) \end{cases} \tag{1.2}$$

under the following assumptions:

(A1) $0\leqslant t_1<t_2<\cdots<t_k<\cdots$ are fixed points with $\lim_{k\to\infty}t_k=\infty$, $k=1,2,\ldots;$

(A2) $b_k>-1$, $k=1,2,\ldots,$ K is a positive constant;

(A3) $r(t)$ and $p_i(t)$ are Lebesgue measurable locally essentially bounded functions, $r(t)\geqslant 0$ and $p_i(t)\geqslant 0$, $i=1,2,\ldots,m;$

(A4) $h,g_i:[0,\infty)\to\mathbb{R}$ are Lebesgue measurable functions, $h(t)\leqslant t$, $g_i(t)\leqslant t$, $\lim_{t\to\infty}h(t)=\infty$, $\lim_{t\to\infty}g_i(t)=\infty$, $i=1,2,\ldots,m$.

The results of this are generalizations of those of (1.1) in [1].

We consider the impulsive differential equation

$$\begin{cases} \dot{y}(t)=-r(t)y\big(h(t)\big)\dfrac{1+y(t)}{1+\sum_{i=1}^{m}p_i(t)[1+y(g_i(t))]}, \\ y\big(t_k^+\big)-y(t_k)=b_ky(t_k), \end{cases}\quad t\geqslant T_0\geqslant 0,\ b_k>-1, \tag{1.3}$$

and the initial value problem

$$y(t)=\varphi(t)\geqslant 0,\quad \varphi(T_0)>0,\ t\in[T^-,T_0]. \tag{1.4}$$

Here for any $T_0\geqslant 0$, $T^-=\min_{1\leqslant i\leqslant m}\inf_{t\geqslant T_0}(g_i(t),h(t))$, $\varphi:[T^-,T_0]\to\mathbb{R}_+$ is a Lebesgue measurable function.

Definition 1.1. For any $T_0\geqslant 0$ and $\varphi(t)$, a function $y:[T^-,\infty)\to\mathbb{R}$ is said to be a solution of (1.3) on $[T_0,\infty)$ satisfying the initial value condition (1.4), if the following conditions are satisfied:

(i) $y(t)$ satisfies (1.4);

(ii) $y(t)$ is absolutely continuous in each interval (T_0,t_k), (t_k,t_{k+1}), $t_k>T_0$, $k\geqslant k_0$, $y(t_k^+)$, $y(t_k^-)$ exist and $y(t_k^-)=y(t_k)$, $k>k_0$;

(iii) $y(t)$ satisfies the former equation of (1.3) a.e. in $[T_0,\infty)\backslash\{t_k\}$ and satisfies the latter equation for every $t=t_k$, $k=1,2,\ldots.$

For any $t\geqslant 0$, consider the nonlinear delay differential equation

$$\dot{x}(t)=-r(t)\Big(\prod_{h(t)\leqslant t_k<t}(1+b_k)^{-1}\Big)x\big(h(t)\big)\cdot\frac{1+(\prod_{T_0\leqslant t_k<t}(1+b_k))x(t)}{1+\sum_{i=1}^{m}p_i(t)[1+(\prod_{T_0\leqslant t_k<g_i(t)}(1+b_k))x(g_i(t))]}. \tag{1.5}$$

Definition 1.2. A solution of (1.3) or (1.5) is said to be nonoscillatory if it is either eventually positive or eventually negative. Otherwise, it is called oscillatory. A solution $N(t)$ of (1.2) is said to be nonoscillatory about K if $N(t) - K$ is either eventually positive or eventually negative. Otherwise, it is called oscillatory about K.

It is easy to see that the change of variable $y(t) = \frac{N(t)}{K} - 1$ reduces (1.2) to (1.3). Thus we have the following lemma.

Lemma 1.3. *Assume that* (A1)–(A4) *hold, then the solution $N(t)$ of* (1.2) *oscillates about K if and only if the solution $y(t)$ of* (1.3) *oscillates about zero.*

Lemma 1.4. *Assume that* (A1)–(A4) *hold. For any $T_0 \geqslant 0$, $y(t)$ is a solution of* (1.3) *on* $[T_0, \infty)$ *if and only if*

$$x(t) = \left(\prod_{T_0 \leqslant t_k < t} (1 + b_k) \right)^{-1} y(t) \tag{1.6}$$

is a solution of nonimpulsive delay differential equation (1.5).

The proof of Lemma 1.4 is similar to that in [9, Theorem 1] and will be omitted.

Remark 1.5. By Lemmas 1.3 and 1.4 we see that the solution $N(t)$ of (1.2) is oscillatory about K if and only if the solution $y(t)$ of (1.3) is oscillatory.

2. Main results

In this paper, consider only such solutions of (1.3) for which the following condition holds:

$$1 + y(t) > 0 \quad \text{for } t \geqslant T_0, \tag{2.1}$$

and hence, in view of (1.6),

$$1 + \left(\prod_{T_0 \leqslant t_k < t} (1 + b_k) \right) x(t) > 0 \quad \text{for } t \geqslant T_0. \tag{2.2}$$

Since $y(t) = \frac{N(t)}{K} - 1$ and (2.1), (2.2), it follows that

$$N(t) = K\left(1 + \prod_{T_0 \leqslant t_k < t} (1 + b_k) x(t) \right) > 0, \quad t \geqslant T_0.$$

Thus for the initial condition $N(t) = \varphi(t) : [T^-, T_0] \to \mathbb{R}_+$, $\varphi(T_0) > 0$, the solution of (1.2) is positive on $[T_0, \infty)$.

We begin with the following lemma.

Lemma 2.1. *Assume that* (A1)–(A4) *hold,*

$$\int_0^\infty \frac{r(t)}{1 + \sum_{i=1}^m p_i(t)}\, dt = \infty \tag{2.3}$$

and

$$\prod_{T_0\leqslant t_k<t}(1+b_k) \quad \textit{is bounded and} \quad \liminf_{t\to\infty}\prod_{T_0\leqslant t_k<t}(1+b_k)>0. \tag{2.4}$$

If $y(t)$ is a nonoscillatory solution of (1.3)*, then* $\lim_{t\to\infty} y(t)=0$.

Proof. Suppose first $y(t)>0$ for $t\geqslant T_1\geqslant 0$. From (1.6) and (A2), $x(t)>0$ for $t\geqslant T_1$. Then there exists $T_2\geqslant T_1$ such that

$$h(t)\geqslant T_2, \qquad g_i(t)\geqslant T_2, \quad i=1,2,\dots,m, \tag{2.5}$$

for $t\geqslant T_2$. Denote

$$u(t)=-\frac{\dot x(t)}{x(t)}, \quad t\geqslant T_2. \tag{2.6}$$

Then $u(t)\geqslant 0$, $t\geqslant T_2$,

$$x(t)=x(T_2)\exp\left\{-\int_{T_2}^{t}u(s)\,ds\right\}, \quad t\geqslant T_2. \tag{2.7}$$

Let $c=x(T_2)$, we have

$$\begin{aligned}
u(t)&=\frac{r(t)}{x(t)}\left(\prod_{h(t)\leqslant t_k<t}(1+b_k)^{-1}\right)x\big(h(t)\big)\\
&\quad\cdot\frac{1+(\prod_{T_0\leqslant t_k<t}(1+b_k))x(t)}{1+\sum_{i=1}^m p_i(t)(1+(\prod_{T_0\leqslant t_k<g_i(t)}(1+b_k))x(g_i(t)))}\\
&\geqslant\frac{r(t)}{x(t)}\left(\prod_{h(t)\leqslant t_k<t}(1+b_k)^{-1}\right)x(t)\frac{1}{1+\sum_{i=1}^m p_i(t)(1+\prod_{T_0\leqslant t_k<g_i(t)}(1+b_k)c)}\\
&=\frac{r(t)}{1+\sum_{i=1}^m p_i(t)}\left(\prod_{h(t)\leqslant t_k<t}(1+b_k)^{-1}\right)\\
&\quad\cdot\frac{1+\sum_{i=1}^m p_i(t)}{1+\sum_{i=1}^m p_i(t)(1+\prod_{T_0\leqslant t_k<g_i(t)}(1+b_k)c)}\\
&\geqslant\frac{r(t)}{1+\sum_{i=1}^m p_i(t)}\\
&\quad\cdot\frac{1}{(\prod_{h(t)\leqslant t_k<t}(1+b_k))(1+\sum_{i=1}^m p_i(t)(1+\prod_{T_0\leqslant t_k<g_i(t)}(1+b_k)c))}.
\end{aligned}$$

Then, by (2.3) and (2.4), $\int_{T_2}^{\infty}u(t)\,dt=\infty$.

Now suppose $-1<y(t)<0$. Hence in view of (2.1), $-1<\prod_{T_0\leqslant t_k<t}(1+b_k)x(t)<0$, $t>T_1$. Then there exists $T_2>T_1$ such that (2.5) holds for $t>T_2$. Suppose $u(t)$ is denoted by (2.6) and $c=x(T_2)$. Then from (1.5) and (2.2) $u(t)\geqslant 0$, $-1<c<0$, we obtain

$$\begin{aligned}
u(t) &= \frac{r(t)}{x(t)}\left(\prod_{h(t)\leqslant t_k<t}(1+b_k)^{-1}\right)x\big(h(t)\big) \\
&\quad\cdot \frac{1+(\prod_{T_0\leqslant t_k<t}(1+b_k))x(t)}{1+\sum_{i=1}^m p_i(t)(1+(\prod_{T_0\leqslant t_k<g_i(t)}(1+b_k))x(g_i(t)))} \\
&\geqslant \frac{r(t)}{x(h(t))}\left(\prod_{h(t)\leqslant t_k<t}(1+b_k)^{-1}\right)x\big(h(t)\big)\frac{1+(\prod_{T_0\leqslant t_k<t}(1+b_k))c}{1+\sum_{i=1}^m p_i(t)} \\
&= \left(\prod_{h(t)\leqslant t_k<t}(1+b_k)^{-1}\right)\left(1+\left(\prod_{T_0\leqslant t_k<t}(1+b_k)\right)c\right)\cdot\frac{r(t)}{1+\sum_{i=1}^m p_i(t)}.
\end{aligned}$$

Then by (2.2)–(2.4), again $\int_{T_2}^{\infty} u(t)\,dt = \infty$.

Equation (2.7) implies $\lim_{t\to\infty} x(t) = 0$. Use (1.6), then $\lim_{t\to\infty} y(t) = 0$. □

Theorem 2.2. *Assume that* (A1)–(A2)*,* (2.3) *and* (2.4) *hold and for some* $\epsilon > 0$*, all solutions of the linear equation*

$$\dot{x}(t) + (1-\epsilon)\prod_{h(t)\leqslant t_k<t}(1+b_k)^{-1}\frac{r(t)}{1+\sum_{i=1}^m p_i(t)}x\big(h(t)\big) = 0 \tag{2.8}$$

are oscillatory. Then all solutions of (1.3) *are oscillatory.*

Proof. Suppose $y(t)$ is an eventually positive solution of (1.3). Then $x(t)$ is an eventually positive solution of (1.5). Lemma 2.1 implies that there exists $T_1 \geqslant 0$ such that

$$0 < \left(\prod_{T_0\leqslant t_k<t}(1+b_k)\right)x(t) < \epsilon \quad \text{for } t \geqslant T_1.$$

We suppose (2.5) holds for $t \geqslant T_2$, we have

$$\begin{aligned}
&\frac{(1+\sum_{i=1}^m p_i(t))(1+(\prod_{T_0\leqslant t_k<t}(1+b_k))x(t))}{1+\sum_{i=1}^m p_i(t)[1+(\prod_{T_0\leqslant t_k<t}(1+b_k))x(g_i(t))]} \\
&\quad\geqslant \frac{1+\sum_{i=1}^m p_i(t)}{1+\sum_{i=1}^m p_i(t)(1+\epsilon)} \geqslant \frac{1+\sum_{i=1}^m p_i(t)}{(1+\epsilon)(1+\sum_{i=1}^m p_i(t))} = \frac{1}{1+\epsilon} \geqslant 1-\epsilon.
\end{aligned}\tag{2.9}$$

Equation (1.5) implies

$$\dot{x}(t) + (1-\epsilon)\prod_{h(t)\leqslant t_k<t}(1+b_k)^{-1}\frac{r(t)}{1+\sum_{i=1}^m p_i(t)}x\big(h(t)\big) \leqslant 0, \quad t \geqslant T_2. \tag{2.10}$$

By a known result (see [7, p. 67]), (2.8) has a positive solution, which is a contradiction.

Now we suppose $-\epsilon < (\prod_{T_0\leqslant t_k<t}(1+b_k))x(t) < 0$ for $t \geqslant T_1$ and (2.3) holds for $t \geqslant T_2 \geqslant T_1$. Then for $t \geqslant T_2$, we also get

$$\begin{aligned}
&\frac{(1+\sum_{i=1}^m p_i(t))(1+(\prod_{T_0\leqslant t_k<t}(1+b_k))x(t))}{1+\sum_{i=1}^m p_i(t)(1+(\prod_{T_0\leqslant t_k<t}(1+b_k))x(g_i(t)))} \\
&\quad\geqslant \frac{(1+\sum_{i=1}^m p_i(t))(1-\epsilon)}{1+\sum_{i=1}^m p_i(t)} = 1-\epsilon.
\end{aligned}\tag{2.11}$$

So we easily know that (2.8) has a nonoscillatory solution and we again obtain a contradiction which completes the proof. □

From Theorem 2.2 and [7, Theorem 3.4.1], we have result

Corollary 2.3. *Assume that* (A1)–(A4), (2.3) *and* (2.4) *hold and if*

$$\liminf_{t\to\infty}\int_{h(t)}^{t}\prod_{h(s)\leqslant t_k<s}(1+b_k)^{-1}\frac{r(s)}{1+\sum_{i=1}^{m}p_i(s)}\,ds>\frac{1}{e},\tag{2.12}$$

then all solutions of (1.3) *are oscillatory.*

Theorem 2.4. *Assume that* (A1)–(A4) *hold and*

$$\prod_{T_0\leqslant t_k<t}(1+b_k)\quad \textit{is convergent.}\tag{2.13}$$

Moreover, for some $\epsilon>0$ *there exists a nonoscillatory solution of the linear delay differential equation*

$$\dot{x}(t)+(1+\epsilon)\prod_{h(t)\leqslant t_k<t}(1+b_k)^{-1}\frac{r(t)}{1+\sum_{i=1}^{m}p_i(t)}x\bigl(h(t)\bigr)=0,\tag{2.14}$$

then there exists a nonoscillatory solution of (1.3).

Proof. Suppose that $x(t)>0$ for $t>T_0$ is a solution of (2.14). Then by (1.5) and [7, Corollary 3.1.2], there exist $T_0\geqslant 0$ and $w_0(t)\geqslant 0$, $t\geqslant T_0$; $w_0(t)=0$, $T_0^-\leqslant t\leqslant T_0$ such that

$$w_0(t)\geqslant(1+\epsilon)\frac{r(t)}{1+\sum_{i=1}^{m}p_i(t)}\Biggl(\prod_{h(t)\leqslant t_k<t}(1+b_k)^{-1}\Biggr)\exp\Biggl\{\int_{h(t)}^{t}w_0(s)\,ds\Biggr\}.\tag{2.15}$$

Since $\prod_{T_0\leqslant t_k<t}(1+b_k)$ is convergent constant, there exists a positive constant c such that $0<c(\prod_{T_0\leqslant t_k<t}(1+b_k))<\epsilon$, and consider two sequences:

$$\begin{aligned}
w_n(t)&=r(t)\Biggl(\prod_{h(t)\leqslant t_k<t}(1+b_k)^{-1}\Biggr)\exp\Biggl\{\int_{h(t)}^{t}w_{n-1}(s)\,ds\Biggr\}\\
&\quad\cdot\frac{1+c(\prod_{T_0\leqslant t_k<t}(1+b_k))\exp\{-\int_{T_0}^{t}v_{n-1}(s)\,ds\}}{1+\sum_{i=1}^{m}p_i(t)(1+c(\prod_{T_0\leqslant t_k<g_i(t)}(1+b_k))\exp\{-\int_{T_0}^{g_i(t)}w_{n-1}(s)\,ds\})},\\
&\quad n=1,2,\ldots;\\
v_n(t)&=r(t)\Biggl(\prod_{h(t)\leqslant t_k<t}(1+b_k)^{-1}\Biggr)\exp\Biggl\{\int_{h(t)}^{t}v_{n-1}(s)\,ds\Biggr\}\\
&\quad\cdot\frac{1+c(\prod_{T_0\leqslant t_k<t}(1+b_k))\exp\{-\int_{T_0}^{t}w_{n-1}(s)\,ds\}}{1+\sum_{i=1}^{m}p_i(t)(1+c(\prod_{T_0\leqslant t_k<g_i(t)}(1+b_k))\exp\{-\int_{T_0}^{g_i(t)}v_{n-1}(s)\,ds\})},\\
&\quad n=1,2,\ldots,
\end{aligned}\tag{2.16}$$

where w_0 was defined above and $v_0 \equiv 0$. We have

$$\begin{aligned}
w_1(t) &= \frac{r(t)}{1+\sum_{i=1}^m p_i(t)} \Biggl(\prod_{h(t)\leqslant t_k<t}(1+b_k)^{-1}\Biggr)\exp\Biggl\{\int_{h(t)}^{t} w_0(s)\,ds\Biggr\} \\
&\quad \cdot \frac{(1+\sum_{i=1}^m p_i(t))(1+c(\prod_{T_0\leqslant t_k<t}(1+b_k)))}{1+\sum_{i=1}^m p_i(t)(1+c(\prod_{T_0\leqslant t_k<g_i(t)}(1+b_k))\exp\{-\int_{T_0}^{g_i(t)} w_0(s)\,ds\})} \\
&\leqslant \frac{r(t)}{1+\sum_{i=1}^m p_i(t)} \Biggl(\prod_{h(t)\leqslant t_k<t}(1+b_k)^{-1}\Biggr) \\
&\quad \cdot \exp\Biggl\{\int_{h(t)}^{t} w_0(s)\,ds\Biggr\} \frac{(1+\sum_{i=1}^m p_i(t))(1+\epsilon)}{1+\sum_{i=1}^m p_i(t)} \\
&\leqslant w_0(t).
\end{aligned} \tag{2.17}$$

It is evident that $v_1(t) \geqslant v_0(t)$, $w_0(t) \geqslant v_0(t)$.

Hence by induction,

$$\begin{aligned}
&0 \leqslant w_n(t) \leqslant w_{n-1}(t) \leqslant \cdots \leqslant w_0(t), \\
&v_n(t) \geqslant v_{n-1}(t) \geqslant \cdots \geqslant v_0(t) = 0, \quad n = 1, 2, \ldots,
\end{aligned} \tag{2.18}$$

and $w_n(t) \geqslant v_n(t)$.

There exist pointwise limits of nonincreasing nonnegative sequence $w_n(t)$ and of nondecreasing sequence $v_n(t)$. If we denote $w(t) = \lim_{n\to\infty} w_n(t)$, $v(t) = \lim_{n\to\infty} v_n(t)$, then by the Lebesgue Convergence Theorem, we conclude that

$$\begin{aligned}
w(t) &= r(t)\Biggl(\prod_{h(t)\leqslant t_k<t}(1+b_k)\Biggr)\exp\Biggl\{\int_{h(t)}^{t} w(s)\,ds\Biggr\} \\
&\quad \cdot \frac{1+c(\prod_{T_0\leqslant t_k<t}(1+b_k))\exp\{-\int_{T_0}^{t} v(s)\,ds\}}{1+\sum_{i=1}^m p_i(t)(1+c(\prod_{T_0\leqslant t_k<g_i(t)}(1+b_k))\exp\{-\int_{T_0}^{g_i(t)} w(s)\,ds\})}, \\
v(t) &= r(t)\Biggl(\prod_{h(t)\leqslant t_k<t}(1+b_k)\Biggr)\exp\Biggl\{\int_{h(t)}^{t} v(s)\,ds\Biggr\} \\
&\quad \cdot \frac{1+c(\prod_{T_0\leqslant t_k<t}(1+b_k))\exp\{-\int_{T_0}^{t} w(s)\,ds\}}{1+\sum_{i=1}^m p_i(t)(1+c(\prod_{T_0\leqslant t_k<g_i(t)}(1+b_k))\exp\{-\int_{T_0}^{g_i(t)} v(s)\,ds\})},
\end{aligned} \tag{2.19}$$

we fix $b \geqslant T_0$ and define operator $T : L_\infty[T_0, b] \to L_\infty[T_0, b]$ by the following equality:

$$\begin{aligned}
(Tu)(t) &= r(t)\Biggl(\prod_{h(t)\leqslant t_k<t}(1+b_k)\Biggr)\exp\Biggl\{\int_{h(t)}^{t} u(s)\,ds\Biggr\} \\
&\quad \cdot \frac{1+c(\prod_{T_0\leqslant t_k<t}(1+b_k))\exp\{-\int_{T_0}^{t} u(s)\,ds\}}{1+\sum_{i=1}^m p_i(t)(1+c(\prod_{T_0\leqslant t_k<g_i(t)}(1+b_k))\exp\{-\int_{T_0}^{g_i(t)} u(s)\,ds\})},
\end{aligned} \tag{2.20}$$

where $L_\infty[T_0, b]$ is the space of all essentially bonded on $[T_0, b]$ functions with the usual norm.

For every function u from the interval $v \leqslant u \leqslant w$, we have $v \leqslant Tu \leqslant w$. The result of [2, Lemma 3] implies that operator T is a compact operator on the space $L_\infty[T_0, b]$. Then by Schauder's fixed-point theorem there exists a nonnegative solution of equation $u = Tu$.

Denote

$$x(t) = \begin{cases} c\exp\{-\int_{T_0}^t u(s)\,ds\}, & t \geqslant T_0, \\ c, & T^- \leqslant t < T_0. \end{cases} \tag{2.21}$$

Then $x(t)$ is a nonoscillatory solution of (1.5). Thus by Lemma 1.1,

$$y(t) = \left(\prod_{T_0 \leqslant t_k < t} (1 + b_k)^{-1}\right) x(t)$$

is a nonoscillatory solution of (1.3) which completes the proof of Theorem 2.4. □

By Theorem 2.4 and [8, Theorem 3.3.1] we have the following result.

Corollary 2.5. *Assume that* (A1)–(A4) *and* (2.13) *hold and if there exists a constant* $\varepsilon > 0$ *such that*

$$(1 + \varepsilon) \int_{h(t)}^{t} \prod_{h(s) \leqslant t_k < s} (1 + b_k)^{-1} \frac{r(s)}{1 + \sum_{i=1}^m p_i(s)}\,ds \leqslant \frac{1}{e}, \tag{2.22}$$

then (1.3) *has a nonoscillatory solution.*

Now we consider the impulsive delay logistic equation (1.2). From Lemma 1.3 and Corollaries 2.3 and 2.5, we have the following results.

Corollary 2.6. *Assume that* (A1)–(A4), (2.3), (2.4) *and* (2.12) *hold, then all solutions of* (1.2) *are oscillatory about* K.

Corollary 2.7. *Assume that* (A1)–(A4), (2.13) *and* (2.22) *hold, then* (1.2) *has a nonoscillatory solution about* K.

Remark 2.8. Similarly, we can study oscillation and nonoscillation about K for the following models:

$$\begin{cases} \dot{N}(t) = r(t)N(t)\dfrac{K - N(h(t))|N(h(t))|^{l-1}}{K + \sum_{i=1}^m p_i(t)N(g_i(t))|N(g_i(t))|^{l-1}}, \\ N(t_k^+) - N(t_k) = b_k(N(t_k) - K), \end{cases} \tag{2.23}$$

$$\begin{cases} \dot{N}(t) = \displaystyle\sum_{j=1}^n r_j(t)N(t)\frac{K - N(h_j(t))}{K + \sum_{i=1}^m p_{ij}(t)N(g_{ij}(t))}, \\ N(t_k^+) - N(t_k) = b_k(N(t_k) - K), \end{cases} \tag{2.24}$$

and some relevant models.

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