Cross-layer Design for Space-time coded MIMO Systems over Rice Fading Channel

Xiangbin Yu\textsuperscript{a,b}, Tingting Zhou\textsuperscript{a}, Xiaoshuai Liu\textsuperscript{a}, Xin Yin\textsuperscript{a}

\textsuperscript{a} College of Information Science and Technology, Nanjing University of Aeronautics and Astronautics, Nanjing, China
\textsuperscript{b} National Mobile Communication Research Laboratory, Southeast University, Nanjing, China

Abstract

A cross-layer design (CLD) scheme for space-time coded MIMO systems over Rice fading channel is presented by combining adaptive modulation and automatic repeat request, and the corresponding system performance is investigated well. The fading gain switching thresholds subject to a target packet error rate (PER) and fixed power constraint are derived. According to these results, and using the generalized Marcum Q-function, the calculation formulae of the average spectrum efficiency (SE) and PER of the system with CLD are derived. As a result, closed-form expressions for average SE and PER are obtained. These expressions include some existing expressions in Rayleigh channel as special cases. With these expressions, the system performance in Rice fading channel is evaluated effectively. Numerical results verify the validity of the theoretical analysis. The results show that the system performance in Rice channel is effectively improved as Rice factor increases, and outperforms that in Rayleigh channel.

1. Introduction

With the fast development of modern communication techniques, the demand for high data rate service is grown increasingly in the limited radio spectrum. For this reason, the future wireless communication system will require spectrally efficient techniques to increase the system capacity. Cross-layer design, as a good work to improve the spectral efficiency (SE) and system throughput while meeting the prescribed quality of service (QoS) requirements, has obtained fast development recently. [1-2]. Especially, a cross-layer design combining adaptive modulation and automatic repeat request (ARQ) is widely accepted as an efficient means to improve the overall performance of transmission in fading channels [3-8]. Multiple antennae approach is another well known SE technique with diversity and/or coding gain [9-10]. Moreover,
multi-antenna system with space-time coding (STC) can provide effective transmit diversity for combating fading effects, has been widely used in wireless communications [10-12]. Therefore, the effective combination of cross-layer design and space-time coding technique has received much attention in the literature [3-7].

A cross-layer design (CLD) combined adaptive modulation and coding in the physical layer (PHY) and automatic repeat request (ARQ) protocol in the data link layer (DLL) over single input and single output (SISO) fading channel is developed in [3]. Based on the idea of CLD in [3], [4] and [5] give the CLD schemes for the multiple input and multiple output (MIMO) systems with orthogonal space-time coding in Rayleigh and Nakagami fading channel, respectively. Based on different space-time code schemes, the performance of MIMO system with CLD is studied over Rayleigh fading channel [6]. The performance of CLD combined with STC is analyzed in optical MIMO system in [7]. Under the feedback constraint, a CLD scheme is presented for multiuser MIMO systems, and the performance is investigated in Rayleigh fading channel [8].

According to the above analysis, the system performance with CLD in Rayleigh fading channel is studied well. In practice, however, the system may experience the Rice fading due to the direct-path propagation. For this reason, in this paper, we will give the CLD scheme for space-time coded MIMO system over Rice fading channel by combining the adaptive modulation (AM) in the physical layer and ARQ in the data link layer. The SE and packet error rate (PER) performances are investigated in Rice channel. Based on the performance analysis, the fading gain switching thresholds for AM are derived under the target PER constraint. Using the obtained switching thresholds and the generalized Marcum Q-function, the closed-form expressions for average PER and SE are obtained by means of the mathematical derivation in detail. With these expressions, the system performance is effectively assessed in theory, and some existing theoretical expressions in Rayleigh fading channel can be included. The numerical results show that the derived theoretical expressions are valid for evaluating the performance over Rice fading channel.

2. System Model

In this section, a wireless communication system with \( N_t \) antennas at the transmitter and \( N_r \) antennas at the receiver are considered, and the system operates over a flat and quasi-static Rice fading channel. Given that \( \mathbf{H} = \{ h_{kn} \} \) is \( N_r \times N_t \) fading channel matrix, where \( h_{kn} \) denotes the complex channel gain from transmit antenna \( n \) to receive antenna \( k \). Considering quasi-static channel, the corresponding channel gains are constant over a frame (\( J \) symbols) and vary from one frame to another. For Rice fading channels, the channel gains \( \{ h_{kn} \} \) are modeled as independent complex Gaussian random variables with respective means \( m_I \) and \( m_Q \) for the real and imaginary parts and variance of 0.5 per real dimension [12-13]. A complex orthogonal space-time code, which is represented by an \( N_t \times J \) transmission matrix \( \mathbf{D} \), is used to encode \( J \) input symbols into an \( N_r \)-dimensional vector sequence of \( T \) time slots. The matrix \( \mathbf{D} \) is a linear combination of \( J \) symbols satisfying the complex orthogonality: \( \mathbf{D}^H \mathbf{D} = \sigma (|d_1|^2 + \ldots + |d_J|^2) \mathbf{I}_{N_0} \), where \( \mathbf{I}_{N_0} \) is the \( N_0 \times N_0 \) identity matrix, \( \{ d_j \}_{j=1,...,J} \) are the \( J \) input symbols, and \( \sigma \) is a constant which depends on the space-time coding transmission matrix [11]. Accordingly, the transmission rate of the space-time code is \( R = J / T \).

Utilizing the complex orthogonality of space-time coding, the instantaneous SNR per symbol after space-time block decoding is expressed as [11]

\[
\gamma = \frac{E_s}{N_0} \frac{1}{N_t} \sum_{k=0}^{N_r-1} \sum_{n=0}^{N_t-1} |h_{kn}|^2 = \frac{\bar{\gamma}}{N_t} \| \mathbf{H} \|_F^2
\]

where \( \bar{\gamma} = E_s / N_0 \) is the average SNR per receive antenna and \( \| \mathbf{H} \|_F^2 \) is the square of Frobenius norm of \( \{ h_{kn} \} \). For Rice fading channel, \( \| \mathbf{H} \|_F^2 \) is noncentral chi-square distributed with \( 2N_rN_t \) degrees of freedom. According to [13] and using the transformation of random variable, the probability density function (pdf) of \( \gamma \) with Rice factor \( K = (m_I^2 + m_Q^2) / (2\sigma^2) \) can be obtained as
\[ f(\gamma) = \left( \frac{N_i R}{\sqrt{\gamma}} \right)^{(N_i N_R + 1)/2} \left( \frac{\gamma}{(N_i N_R K)} \right)^{(N_i N_R - 1)/2} \]  
\[ \cdot e^{-N_i N_R K - N_i R \gamma / \sqrt{\gamma}} I_{N_i N_R - 1} \left( 2\sqrt{RKN_i N_i^2 \gamma / \sqrt{\gamma}} \right) \]  \tag{2}

where \( I_v(x) \) is the \( v \)-th order modified Bessel function of the first kind and \( \sigma^2=0.5 \) [13-14]. With (2), the pdf of \( \gamma \) for Rayleigh fading can be obtained by setting \( K=0 \).

I. CROSS-LAYER DESIGN FOR SPACE-TIME CODED SYSTEMS

In this section, we will give a cross-layer design scheme for MIMO system with space-time coding, and square MQAM is considered for modulation in the system due to its inherent SE and ease of implementation. For discrete-rate MQAM, the constellation size \( M_l \) is defined as \( \{M_0=0, M_1=2, M_l=2^{2l-2}, l=2,\ldots,q\} \), where \( M_0 \) means no data transmission. The instantaneous SNR range is divided into \( q \) fading regions with switching thresholds \( \{\gamma_0, \gamma_1, \ldots, \gamma_q, \gamma_{q+1}; \gamma_0=0, \gamma_{q+1}=+\infty\} \). The MQAM of constellation size \( M_l \) is used for modulation when \( \gamma \) falls in the \( l \)-th region \([\gamma_l, \gamma_{l+1})\). Consequently, the data rate is \( b_l=\log_2 M_l \) bits/symbol with \( b_0=0 \).

According to [3], the PER of MQAM with two dimensional Gray code over additive white Gaussian noise (AWGN) channel for the received SNR \( \gamma \) and constellation size \( M_l \) is approximately given by

\[ \text{PER}_l(\gamma) = \begin{cases} 1, & \gamma < \gamma_{pl} \\ a_l \exp(-g_l \gamma), & \gamma \geq \gamma_{pl} \end{cases} \]  \tag{3}

where \( \{a_l, g_l, \gamma_{pl}\} \) are constellation and packet-size dependent constants, and they can be obtained by fitting (3) to the exact PER. Specifically, their values can be found in Table I in [3].

In our paper, adaptive modulation in the physical layer and ARQ protocol in the data link layer are employed for CLD. We first define the target packet loss rate (PLR) for the data link layer as \( P_{\text{loss}} \). Since truncated ARQ is used in the data link layer, the packets in error may be retransmitted up to \( \max_r \). Hence, the target PER is \( P_o = P_{\text{loss}}^{1/(N_r \max_r + 1)} \) in the physical layer, which is generally limited as \( P_o<1 \). The switching thresholds \( \{\gamma_l\} \) can be set to be the required SNR to achieve the target PER, \( P_o \), over an AWGN channel. By inverting the \( P_o \) in (3), we can obtain the switching threshold values [3] as follows:

\[ \gamma_l = \begin{cases} 0, & l=0 \\ \frac{B_l}{g_l}, & l=1,\ldots,q \\ +\infty, & l=q+1 \end{cases} \]  \tag{4}

where \( B_l=-\ln(P_o/a_l) \) is a factor dependent on the choosing of the modulation mode \( l \).

With the obtained switching thresholds, we will evaluate the performance of the MIMO system with space-time coding and cross-layer design by calculating the average PER and SE in the following section. When the switching thresholds are chosen according to (4), the system will operate with a PER below target \( P_o \), as will be confirmed in the following numerical results in section V.

3. Performance Analysis

We now derive closed-form expressions for the average PER and SE of the system, which will be used to assess the performance of CLD with space-time coding.

Based on the switching thresholds described in (4), using (2) and the generalized Marcum Q-function \( Q_m(k,v) \) [13-14], we can calculate the probability of the effective SNR \( \gamma \) falls in the \( l \)-th region \([\gamma_l, \gamma_{l+1})\), denoted as \( P_l \),

\[ P_l = \int_{\gamma_l}^{\gamma_{l+1}} f(\gamma) d\gamma = \int_{\gamma_l}^{+\infty} f(\gamma) d\gamma - \int_{\gamma_{l+1}}^{+\infty} f(\gamma) d\gamma \]  \tag{5}
For discrete-rate adaptive scheme, the average SE in the physical layer is defined as the ensemble average of effective transmission rate. So the average SE of the adaptive MQAM scheme with space-time coding can be given by

$$\eta_{\text{phy}} = \sum_{i=1}^{q} Rb_i \int_{\gamma_i}^{\gamma_i} f(\gamma) d\gamma = \sum_{i=1}^{q} Rb_i P_i$$

$$= R \sum_{i=1}^{q} b_i [Q_{N,N_r} \left( \sqrt{2N_t N_r K}, \sqrt{2RN_t \gamma_i} / \bar{\gamma} \right) - Q_{N,N_r} \left( \sqrt{2N_t N_r K}, \sqrt{2RN_t \gamma_{i+1}} / \bar{\gamma} \right) ] \tag{6}$$

The average SE of the system in the PHY for Rayleigh fading case can be obtained by setting $K=0$. That is, by using $K=0$, (6) can be changed to

$$\eta_{\text{phy}} = R \sum_{i=1}^{q} b_i [\Gamma(N,N_r, RN_t \gamma_i / \bar{\gamma}) - \Gamma(N,N_r, RN_t \gamma_{i+1} / \bar{\gamma})] / \Gamma(N,N_r) \tag{7}$$

where $\Gamma(\cdot, \cdot)$ is incomplete Gamma function. This equation is the average SE of the system in the physical layer for Rayleigh fading case.

We define ensemble average PER in the physical layer for multi-antenna system combined with CLD and space-time coding as

$$\overline{\text{PER}}_l = \int_{\gamma_i}^{\gamma_i} \text{PER}_{l}(\gamma) f(\gamma) d\gamma = a_l \left( \frac{N R}{\bar{\gamma} c_l} \right)^s \exp(-\frac{SKg_l}{c_l})$$

$$\times Q_2 \left( \frac{2RN(SK)}{\bar{\gamma} c_l}, \sqrt{2c_l \gamma_i} \right) - Q_2 \left( \frac{2RN(SK)}{\bar{\gamma} c_l}, \sqrt{2c_l \gamma_{i+1}} \right) \tag{8}$$

where $c_l = N_r / \bar{\gamma} + g_l$, $S=N_tN_r$.

Substituting (9) and (5) into (8), the average PER can be evaluated as follows:

$$\overline{\text{PER}}_l = \sum_{i=1}^{q} b_i a_l \left( \frac{SKg_l}{c_l} \right)^s \exp(-\frac{SKg_l}{c_l}) \times Q_2 \left( \frac{2RN(SK)}{\bar{\gamma} c_l}, \sqrt{2c_l \gamma_i} \right) - Q_2 \left( \frac{2RN(SK)}{\bar{\gamma} c_l}, \sqrt{2c_l \gamma_{i+1}} \right) \tag{9}$$

where $\bar{\gamma} = \bar{\gamma} / (N,R)$. Eq.(10) is a closed-form expression of the PER for MIMO system with space-time coding and cross-layer design. Using $K=0$, (10) can be reduced to the average PER under Rayleigh fading case, i.e.,

$$\overline{\text{PER}}_l = \sum_{i=1}^{q} b_i a_l \left[ \Gamma(S,c_l \gamma_i) - \Gamma(S,c_l \gamma_{i+1}) \right] / (\bar{\gamma} c_l)^s \tag{10}$$

$$\sum_{i=1}^{q} b_i a_l \left[ \Gamma(S, \gamma_i / \bar{\gamma}) - \Gamma(S, \gamma_{i+1} / \bar{\gamma}) \right] \tag{11}$$

With (10), the average PLR in the data link layer with the maximum number of retransmissions, $N_r^{\text{max}}$, is

$$\overline{\text{PLR}} = \overline{\text{PER}}_l N_r^{\text{max}} = p_r^{N_r^{\text{max}}+1} \tag{12}$$

where $p_r = \overline{\text{PER}}$. Thus, the average number of transmissions per packet can be calculated as:

$$\overline{N_r} = 1 + p_r + p_r^2 + \ldots + p_r^{N_r^{\text{max}}} = (1 - p_r^{N_r^{\text{max}}}) / (1 - p_r) \tag{13}$$
Using (6) and (13), the overall average SE of multi-antenna system with STC and CLD can be obtained as:

$$\eta = \frac{\eta_{\text{phy}}}{\bar{N}}$$

(14)

When $N_{r}^{\text{max}}$ is set to be zero, $\bar{N}=1$, and (14) is reduced to $\eta_{\text{phy}}$, which corresponds to the average SE for AM in the physical layer only.

4. Numerical Results and Analysis

In this section, we use the derived performance formulae to evaluate the average PER and spectrum efficiency of the multi-antenna system with cross-layer design and space-time coding over Rice fading channel. The system is referred to as CLD-STC. The channel is assumed to be quasi-static flat fading and Gray code is employed to map the data bits to MQAM constellations. The set of MQAM constellations is $\{M_{l}\}_{l=0,1,\ldots,s} = \{0, 2, 4, 16, 64\}$. The target PLR in the data link layer, $P_{\text{loss}} = 10^{-3}$, the maximum number of ARQ retransmissions, $N_{r}^{\text{max}}$ is set to be 2, and thus the target PER, $P_{o}=0.1$. Different space-time codes, such as $G_{2}$, $G_{3}$, and $H_{3}$, are adopted for evaluation and comparison. In the following figures, $xTyR$ denotes a multiple antenna system with $x$ transmit antennas and $y$ receive antennas.

In Fig.1, we plot the average spectrum efficiency of the MIMO with cross-layer design and space-time coding for different transmit antennas and one receive antenna. Rice factor $K=1$ and $3$ are considered. For comparison, Raleigh fading channel ($K=0$) is also considered. The spectrum efficiency is calculated by using (14) with the switching thresholds defined by (4). As shown in Fig.1, CLD is able to increase spectrum efficiency with SNR. The 2T1R system with $G_{2}$ code provides larger spectrum efficiency than that of 3T1R system with $H_{3}$ code because the $G_{2}$ is a full rate code while $H_{3}$ code has code rate less than one. Moreover, the spectrum efficiency of CLD with $G_{2}$ code is also higher than that with one transmit antenna because more antennas are used. From the figures, it is found that the spectrum efficiency of CLD with space-time coding in Rice fading channel is higher than that in Rayleigh fading channel due to the existence of direct path. The bigger the value of Rice factor $K$ is, the larger the spectrum efficiency is. Namely, the spectrum efficiency of system with $K=3$ is higher than that with $K=1$ because of the stronger path gain, and due to the same reason, the spectrum efficiency of system with $K=1$ is higher than that with $K=0$, which accords with the existing knowledge. The above results show that the derived spectrum efficiency expression is effective and reasonable.
Figure 1. Average SE of CLD-STC with multiple transmit antennas and one receive antenna for different Rice factor

In Fig.2, we plot the average PER of the MIMO with cross-layer design and space-time coding for two transmit antennas and one receive antenna, where $K=0$, 1 and 3 are considered, and $G_2$ code is used for space-time coding. The average PER is calculated by (10) with the switching thresholds defined by (4). As shown in Fig.2, the values of average PER is below the target PER (i.e. $P_o$). It means that the cross-layer design is successful in increasing spectrum efficiency while the target PER is maintained. Moreover, with the increase of Rice factor $K$, the PER performance of the system is effectively improved as expected. Besides, it is observed that the average PER curves have ripples phenomenon due to the available discrete-rate adaptation. The above results show that the derived PER expression is also valid and reasonable.
5. Conclusions

We have investigated the performance of space-time coded MIMO system with cross-layer design over Rice fading channel. Subject to the target PER and fixed power constraint, we achieve the switching thresholds of the fading gain by using approximate PER expression. Based on the achieved switching thresholds, using the probability density function of instantaneous SNR and the generalized Marcum $Q$-function, the closed-form expressions of the average PER and SE are derived. Using these expressions, the system performance is effectively assessed in theory. Moreover, these expressions include some existing expressions under Rayleigh channel as special cases. Numerical results verify the validity of the derived performance formulae, and it is shown that the system performance is obviously better than that in Rayleigh channel due to the existence of direct-path.

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