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Static Analysis of Electrostatically Actuated Micro Cantilever Beam

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Abstract

Electrostatically actuated micro devices experience a fundamental limit on their stable travel range due to a phenomenon called as the pull-in Instability. Accurate determination of pull-in parameters (pull-in displacement and pull-in voltage) is vital in the design of electrostatic micro actuators. A systematic method of analysis of prismatic type of electrostatic beam is discussed in this paper. Using Galerkin method static analysis is carried out. Behaviour of interaction of nonlinear electrostatic force with linear restoring force of the micro cantilever beam is studied. Static analysis using COMSOL multiphysics finite element package is done to validate the results. The results are also compared with existing literature.

Keywords: Electrostatic Actuator, Pull-in instability, MEMS

Nomenclature

$u(x)$	Downward deflection of the micro beam towards the fixed electrode.
A	Electrostatic area of overlap
I	Moment of inertia of the beam
g_0	Initial gap between two electrode
ϵ	Permittivity of the free space
E	Young's modulus of the micro-beam material
x	Length co-ordinate varies from 0 to L
V	Applied actuation voltage

1. Introduction

The Micro-Electro-Mechanical Systems (MEMS) is the integration of mechanical elements, actuators, sensors and electronics on a common silicon substrate through the utilization of micro-fabrication technology. The Micro-Electro-Mechanical systems (MEMS) have experienced a lot of progress recently. Their light weight, low cost, small dimensions, low-energy consumption and durability attracted them widely. The successful and commercialized MEMS actuators include digital micro mirror device, automotive crash sensors, ink jet printer nozzles, catheter tip pressure sensors, etc. [1]. Newer applications of MEMS technology abound in the areas that include, but are not limited to, the telecommunication, biomedical, semiconductor, and aerospace industries. The present day national and international initiatives in the area of MEMS reveal the ever growing scientific and industrial interest in this field. There are two basic components of MEMS, sensors and actuators. A sensor gathers the information from the environment by utilizing mechanical, thermal, biological, chemical, optical, and magnetic properties. The actuator contains the mechanical members, which are acted upon by various mechanisms like electromagnetic, thermo actuation, use of shape memory alloys, piezo actuation, magneto static actuation and electrostatic actuation. Out of all these, electrostatic actuation is widely used. The popularity of electrostatic actuation is due to ease of fabrication, low power consumption and higher energy density.

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Various analytical and numerical studies have been carried out on the pull-in parameters of the MEMS devices [1-5]. The biggest disadvantage of using electrostatically driven actuators is the well-known pull-in instability in which one of the movable capacitor plates strikes its fixed counterpart after travelling a certain distance. This causes the snapping of two electrodes, which can cause a short circuit and it will make the device non-functional [3]. The root cause of the pull-in instability lies in the interaction of nonlinear electrostatic force and the linear mechanical restoring force. The pull-in displacements as percentages of the original gaps in many structural models of these actuators are 33.33% for parallel plates, 45% for cantilevers, 44.04% for torsional actuators and 35.8% for fixed-fixed beams [5]. Looking to the results quoted in literature, energy techniques has been widely used for the analysis of micro actuator. The study of literature indicates that more than 50% of the initial gap is unutilised for the travel range. The motivation behind this article is to study the total displacement in micro cantilever beam using Galerkin approach and to compare pull-in parameters with existing literature and commercial finite element package (COMSOL).

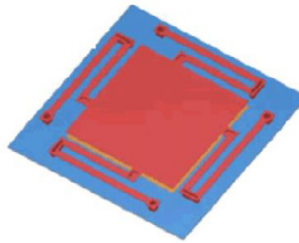


Figure 1 Electrostatic actuator used in MEMS [6]

2. Static Analysis of Electrostatically Actuated Micro cantilever beam

The micro-beam requires mechanical energy to obtain a vibrating or translating motion based on the application of MEMS. An electric charge is created around the electric field due to potential difference between two conductors (Refer Figure 2). When the electrostatic force of attraction is in action, the stiffness of the movable electrode offers the required restoring force to maintain the system in equilibrium. The electrostatic force being a surface force, varies as per the inverse square law, i.e. the force of attraction between the two electrodes is inversely proportional to the square of the distance between them. The mechanical restoring force offered by movable plate is directly proportional to its displacement. The interaction of this linear and non-linear pair of force leads to pull-in instability [7].

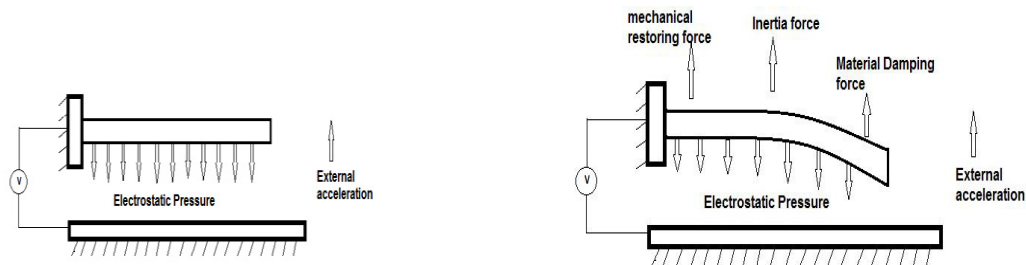


Figure 2 Principle of electrostatic actuation

For analyzing the static behavior of micro cantilever beam, Galerkin's approach is used to arrive at the precise values of the static pull-in parameters. It also explains the dependence of static pull-in parameters on various non-dimensional quantities, which would enhance the overall understanding of the static pull-in behaviour of prismatic and non-prismatic micro-beam.

The governing differential equation of a electrostatically actuated micro-beam is nonlinear in nature and is expressed by,

$$EI \frac{d^4 u}{dx^4} = \frac{\epsilon AV^2}{2(g_0 - u)^2} \quad (1)$$

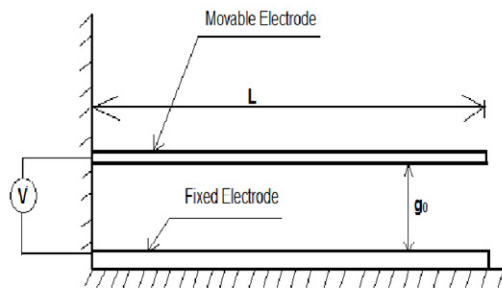


Figure 3: Schematic of an Electrostatically Actuated Micro cantilever beam

An electrostatically actuated micro-cantilever beam is schematically shown in Figure 3. When the voltage V is applied across the two electrodes, electrostatic force is generated. Electrostatic force acting on a beam is proportional to the deflection of the micro-beam. Since the deflection varies along the length, the intensity of the electrostatic force also varies proportionately. In case of the micro-cantilever beam, the maximum electrostatic force is experienced by its tip, since it undergoes the maximum deflection. The deflection function is given as under,

$$u(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 \tag{2}$$

Satisfying following boundary conditions

$$u|_{x=0} = 0, u'|_{x=0} = 0 \tag{3}$$

$$EIu''|_{x=L} = 0, EIu'''|_{x=L} = 0 \tag{4}$$

Equation for deflection can be written as

$$u(x) = a_4(6x^2L^2 - 4x^3L + x^4) \tag{5}$$

Normalising the Eq.(5)

$$\hat{u}(\hat{x}) = a(6\hat{x}^2 - 4\hat{x}^3 + \hat{x}^4) \tag{6}$$

$$\hat{u}(\hat{x}) = aF \tag{7}$$

where,

$$F = (6\hat{x}^2 - 4\hat{x}^3 + \hat{x}^4) \tag{8}$$

Governing differential Eq. (1) can be rewritten in the normalised form as under

$$a \frac{d^4 F}{dx^4} = \frac{\hat{V}^2}{(1 - aF)^2} \tag{9}$$

Non dimensional parameters are defined as

$$\hat{x} = \frac{x}{L}, \hat{u} = \frac{u}{g_0}, \hat{V} = \sqrt{\frac{\epsilon b V^2 L^4}{2EIg_0^3}} \tag{10}$$

The domain residual can be written as

$$R_d(x) = 24a - \frac{\hat{V}^2}{(1 - aF)^2} \tag{11}$$

By using Taylor series expansion, the above Eq. (11) can be rewritten as

$$R_d(x) = 24a - \hat{V}^2 (1 + 2aF + 3a^2F^2 + 4a^3F^3 + 5a^4F^4 + 6a^5F^5 + 7a^6F^6 + \dots) \tag{12}$$

The weighted residue form of the Eq. (9) can be written as follows

$$\int_0^L R_d(x)w(x)dx = 0 \tag{13}$$

Here,

$$w(x) = F(\hat{x}) = (6\hat{x}^2 - 4\hat{x}^3 + \hat{x}^4) \tag{14}$$

After inserting the domain residue and weighted function into Eq. (13)

$$\int_0^1 F(24a - \hat{V}^2 (1 + 2aF + 3a^2F^2 + 4a^3F^3 + 5a^4F^4 + 6a^5F^5 + 7a^6F^6))dx = 0 \tag{15}$$

The above equation will be resulted in

$$1199.2\hat{V}^2a^6 + 636.25\hat{V}^2a^5 + 127.25\hat{V}^2a^4 + 40.83\hat{V}a^3 + 12.835\hat{V}^2a^2 + (3.815\hat{V}^2 - 24)a + \hat{V}^2 = 0 \tag{16}$$

3. Results and discussion

The pull-in voltage and pull-in displacement are to be obtained from the governing differential equation subjected to suitable boundary conditions. Using Galerkin’s approach and Taylor series, the governing differential equation is converted into algebraic form (Eq. 16). Since, there is one equation and two unknown (pull-in voltage and pull-in displacement), the solution can not be obtained directly. Initially a voltage ranges from 1 to 100 with the step size of 1 is taken and roots are checked. The voltage corresponding to which roots are changing nature (real to complex), is the approximate value of the pull in voltage. This value then can be improved by reducing step size in the range near the voltage value obtained in first iteration. After three to four iterations very precise value of the pull in voltage can be obtained. Based on this normalized pull in voltage the constant ‘a’ can be found. Now using Eq. (6), pull-in displacement is found. The normalized pull-in parameters and their comparison with existing literature can be seen from Table 1.

Table 1 Static pull-in parameters of prismatic beam

Micro-beam type	u _{ps}			V _{ps}		
	Present Approach	Joglekar [8]	% diff.	Present Approach	Joglekar [8]	% diff.
Cantilever Beam	0.4479	0.447	0.2	1.2999	1.3020	0.16

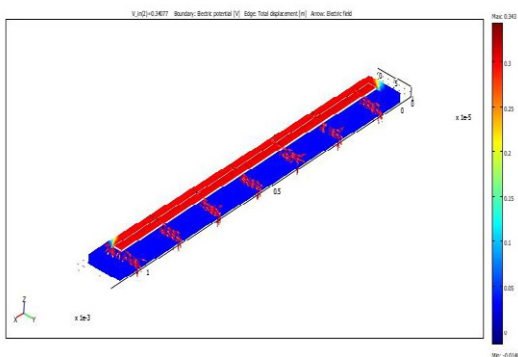


Figure 2 FEA model using COMSOL

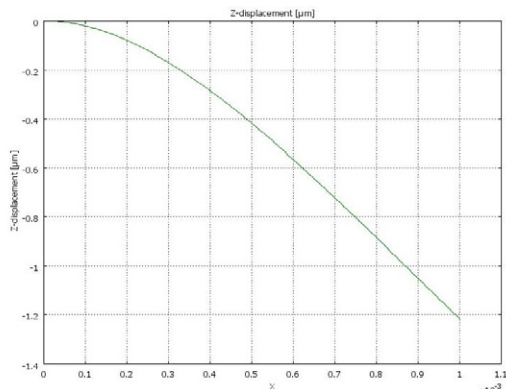


Figure 3 The deflection curve of beam under application of

electrostatic force

To validate the results through Finite Element Analysis, COMSOL software is used in multi physics environment. Modelling of cantilever beam is done in multi-physics environment. Figure 2 shows finite element model of movable electrode and fixed electrode. The deflection curve of beam under application of electrostatic force is shown Figure 3.

Table 2 Comparison of pull-in parameters with COMSOL

Pull-in Parameters	Analytical	Joglekar [8]	COMSOL	%diff.
u_{ps}	0.4479	0.447	0.4418	1.15%
V_{ps}	1.2999	1.3020	1.3245	1.72%

Here from Table 2 the value of pull-in parameter found by using COMSOL is much closed to the analytical one with the difference as 1.15% in pull-in displacement and 1.72% in pull-in voltage.

4. Conclusion

In this work, the results of static analysis of electrostatically exited micro-cantilever beam have been presented. Pull-in analysis of the continuous mechanical systems is carried out. Authors have devised a numerical scheme based on the Galerkin method and voltage iteration scheme is used to solve the polynomial equation. Results are validated with the article published in the literature and also compared the results with COMSOL software.

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