Evaluation of gas condensate reservoir behavior using velocity dependent relative permeability during the numerical well test analysis

Arash Azamifard a, Mahnaz Hekmatzadeh b, Bahram Dabir a, * 

a Amirkabir University of Technology, Tehran Polytechnic, 424 Hafez Ave, Tehran, Iran  
b IOR Research Institute, National Iranian Oil Company, Tehran, Iran

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A B S T R A C T

Gas condensate is one of the most different fluids in reservoir simulation due to retrograde condensation in case of pressure reduction. In this kind of fluids, two phenomena named negative inertia and positive coupling, become significant in the high velocity zone around the wellbore. In this study, a modified black oil simulator is developed that take into account the velocity dependent relative permeability. Against the industrial simulator that assumes linear variation of transmissibilities by pressure, modified black oil nonlinear equations are solved directly without linearization. The developed code is validated by ECLIPSE simulator. The behavior of two real gas condensate fluids, a lean and a rich one, are compared with each other. For each fluid, simulations of PVT experiments are carried out to calculate black oil property applying Coats approach for gas condensate fluids. For both fluids, the proposed models for gas condensate velocity dependent relative permeability show different influence of velocity on relative permeability in the same conditions. Moreover, it is observed that higher flow rate of gas production leads to more condensate production during constant rate well testing.

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1. Introduction

At early stages of reservoir production period, the corresponding fluid in gas condensate reservoirs is a single-phase gas. Afterwards, due to reservoir pressure decline below its dew point pressure, condensate formation occurs which is referred to as condensate blockage. Condensate blockage is a central issue in calculation of well productivity that is an important issue in the development of many gas condensate reservoirs [1].

In conventional fluids, as the liquid saturation increases, gas relative permeability decreases. But in gas condensate reservoirs, the relative permeability of both gas and condensate not only depends on saturation but also on capillary pressure [2,3]. The behavior of gas condensate relative permeability divided into two effects; inertia and positive coupling [4]. Inertia effect happens in high fluid velocity and causes an additional pressure drop during movement. This effect is named non-Darcy flow that decrease fluid relative permeability. On the contrary, the positive coupling effect causes an increase in relative permeability in low interfacial tension (or IFT) as velocity increases and/or IFT decreases [5]. In fact, the relative permeability curve moves toward cross relative permeability curves in low IFT values which belongs to miscible two-phase flows. An increase in fluid velocity, which is intensified around the wellbore, makes these two conflicting effects significant, especially positive coupling.

Gas condensates well test involves many phenomena that occur due to this confliction. Hence, it remains an open question that what will happen during gas condensate well test. Jones and Raghavan [6] claim that imbibition is the predominant process regarding the condensate bank build-up. But Whitson and Fevang [3] disprove that idea and introduce relative permeability...
model based on the ratio of relative permeability instead of saturation. They introduced an explicit relation for velocity dependent relative permeability (VDRP) as a function of IFT. Pope et al. [7] used trapping number which involves both capillary and buoyancy effects, to model VDRP. Mott et al. [1] discussed the magnitude of positive coupling effect and inertia effect from low to high gas saturation. It is demonstrated that in high gas saturation, inertia overcomes positive coupling effect and reduces gas permeability. But in low gas saturation, positive coupling becomes significant and increases gas permeability. Broadly speaking, positive coupling has a major effect on relative permeability [1]. Jamiolahmady et al. [8] observed flow patterns in gas condensate reservoirs in micro-models. Afterwards, they proposed a VDRP model as a function of fractional flow and discussed the effect of micro fractures in addition to inertia and positive coupling effect [9].

Two types of well testing in gas condensate are applied, namely analytical and numerical. In analytical well testing, by defining the pseudo pressure concept, it is assumed that only the gas flows through the reservoir. Condensate blockage is represented by skin value or considering damaged zone around the wellbore with lower permeability than the origin permeability of rock. In numerical well testing, simulation of fluid flow in porous media is carried out to reproduce well data of pressure versus time. By adjusting reservoir and fluid parameters in the simulation, a reasonable match is gained in pressure versus time data set.

Table 1
Composition of real fluids.

<table>
<thead>
<tr>
<th>Component</th>
<th>Composition (fraction)</th>
<th>MW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lean fluid</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Methane</td>
<td>0.8265</td>
<td>16.043</td>
</tr>
<tr>
<td>Ethane</td>
<td>0.0516</td>
<td>30.07</td>
</tr>
<tr>
<td>Propane</td>
<td>0.0185</td>
<td>44.096</td>
</tr>
<tr>
<td>I-Butane</td>
<td>0.0041</td>
<td>58.123</td>
</tr>
<tr>
<td>n-Butane</td>
<td>0.007</td>
<td>58.123</td>
</tr>
<tr>
<td>I-Pentan</td>
<td>0.0032</td>
<td>72.15</td>
</tr>
<tr>
<td>N-pentane</td>
<td>0.0028</td>
<td>72.15</td>
</tr>
<tr>
<td>Hexanes</td>
<td>0.004</td>
<td>86.117</td>
</tr>
<tr>
<td>Carbon_Dioxide</td>
<td>0.0207</td>
<td>44.01</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>0.0349</td>
<td>28.013</td>
</tr>
<tr>
<td>Hydrogen_Sulphide</td>
<td>0.0028</td>
<td>34.082</td>
</tr>
<tr>
<td>pseudo_component_1</td>
<td>0.02037</td>
<td>128.06</td>
</tr>
<tr>
<td>pseudo_component_2</td>
<td>0.00333</td>
<td>216.67</td>
</tr>
<tr>
<td>total</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

| Rich fluid         |                        |     |
| Methane            | 0.6599                 | 16.043 |
| Ethane             | 0.0869                 | 30.07 |
| Propane            | 0.0591                 | 44.096 |
| I-Butane           | 0.0239                 | 58.123 |
| n-Butane           | 0.0278                 | 58.123 |
| I-Pentan           | 0.0157                 | 72.15  |
| N-pentane          | 0.0112                 | 72.15  |
| Hexanes            | 0.0181                 | 86.117 |
| Carbon_Dioxide     | 0.0121                 | 44.01  |
| Nitrogen           | 0.0194                 | 28.013 |
| pseudo_component_1 | 0.05619                | 117.865 |
| pseudo_component_2 | 0.00971                | 267.35 |
| total              | 1                      | 1    |

In this study, simulation of build-up tests is used to investigate the behavior of condensate formation around the wellbore. First of all, a PVT module is written in the Python programming language to calculate modified black oil properties of two real gas condensate fluids with different richness. Using python programming a modified black oil (MBO) simulator is developed which solves nonlinear equations directly without linearization. Linearization reduces accuracy because of assuming linear variation of transmissibilities by pressure variation that is applied in all industrial simulators. To do this, python simulator is developed by the use of Scipy and Numpy libraries. Results show that this modern procedure leads to more accurate material balance during simulation.

This paper organized as follow; first, gas condensate fluid properties are illustrated. Then the methodology of deriving and solving nonlinear equations of fluid flow near the wellbore are described. Thereafter, the model validation is investigated and simulation results are presented for different cases. Finally, the conclusions are presented.

2. Methodology

In this section, first the gas condensate fluids that is used in this study, is introduced. Then Coat approach for calculation of black oil properties is discussed and finally the method for solving equations without linearization is presented.

![Fig. 1. Liquid drop out of two real fluids.](image-url)
Nagarajan et al. [10] proved that synthetic fluids fail to simulate real reservoir gas condensate phase behavior. Then in this study, real lean and rich fluids are used in numerical well test simulation. Compositions of these fluids are presented in Table 1. Liquid drop out of these two fluids is shown in Fig. 1. As shown in Fig. 1, maximum liquid drop out of the two fluids is about 20 and 3 percent for rich and lean one, respectively.

Whitson and Torp [11] proposed a procedure for calculation of black oil properties for gas condensate fluids by flashing it under reservoir temperature in each pressure. Then the resulting gas and liquid flashed again in all stages of separators and stock tank pressures and temperatures.


Simulation of a CVD experiment with Coats approach allows us to calculate the black oil properties of gas and condensing liquid in the near wellbore condition. The coats’s approach needs simulation of separator test in addition to CVD experiments in each pressure step of CVD test. Fig. 2 presents how to combine CVD and separator tests by flash calculation. Fig. 3 illustrate the procedure of PVT experiment simulation in order to gain black oil properties. According to this figure, after solving sets of linear equations mentioned in Appendix 1, solution gas in the oil and solution oil in the gas are calculated by dividing volumes of producing gas and liquid as below, respectively:

\[
R_g = \frac{\text{Gas} 3 + \text{Gas} 4}{\text{Liq} 2}
\]

\[
R_s = \frac{\text{Liq} 1}{\text{Gas} 1 + \text{Gas} 2}
\]

Volumes in the above relations must be in the standard condition. Figs. 4–6 show calculated solution gas, solution oil and gas formation volume factor for two lean and rich real fluids, respectively. The observed trend of properties versus pressure is approved with results of Coats [1].

### 2.2. Solving equations

Modified black oil equations must be solved in order to perform a numerical well test analysis in gas condensate reservoirs. Differential and transmissibility form of these equations are as below:

Oil component:

\[
T_{0.5} \left[ P_{0.1} - P_0 \right] + T_{0.5} \left[ P_{0.1} - P_0 \right] - q_g' + (R_cT_g\frac{1}{2}) \left[ P_{0.1} - P_0 \right] + (P_{cog,1} - P_{cog})
\]

\[
(R_cT_g\frac{1}{2}) \left[ P_{0.1} - P_0 \right] + (P_{cog,1} - P_{cog}) - R_cq_g'
\]

\[
= C_{poo} \left( P_0 - p_{0g} \right) + C_{Sgo} \left( S_g - S_g' \right)
\]
Gas component:

\[ T_{g+1/2} \left[ (P_{o_{i-1}} - P_{o_i}) + (P_{cog_{i-1}} - P_{cog_{i}}) \right] + T_{g-1/2} \left[ (P_{o_{i-1}} - P_{o_i}) + (P_{cog_{i-1}} - P_{cog_{i}}) \right] \]

\[ + (P_{cog_{i-1}} - P_{cog_{i}}) - q'_{g} + (R_{s}T_{o_{i+1}}) \left[ (P_{o_{i+1}} - P_{o_i}) \right] \]

\[ + (R_{s}T_{o_{i}}) \left[ (P_{o_{i+1}} - P_{o_i}) - R_{s}q_{g} \right] \]

\[ = C_{pog} \left( P_{o_i} - P_{o_{i-1}} \right) + C_{sgg} \left( S_{g} - S_{g}^i \right) \]

More details of deriving above equations are explained in Appendix 2. Implicit approach for pressure and explicit approach for saturation (IMPES) is impracticable for solving mentioned equations because of the constant saturation assumption during time step in IMPES. Hence, we have to solve this set of nonlinear equations by simultaneous solution (SS) approach. By the way, assumption of explicit transmissibilities or pressure independent transmissibilities is not valid because of severe variation of gas formation volume factor in the denominator of gas.
transmissibility. This variation is a consequence of pressure variation. A python simulator is developed by the use of Scipy and Numpy libraries to solve nonlinear equations directly without linearization. Specification of simulated models are presented in Table 2.

![Fig. 7. Validation of prototype oil-water simulator by material balance calculation.](image7)

![Fig. 8. Validation of gas condensate simulator by material balance calculation.](image8)

![Fig. 9. Simulation of specific reservoir with different time steps to validate gas condensate simulator.](image9)

Table 2
Specification of simulated models.

<table>
<thead>
<tr>
<th>Specification of simulation model</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Porosity</td>
<td>0.2 [dimensionless]</td>
</tr>
<tr>
<td>Permeability (all directions)</td>
<td>75 mD</td>
</tr>
<tr>
<td>Grid length in x direction</td>
<td>10 ft</td>
</tr>
<tr>
<td>Grid length in y direction</td>
<td>10 ft</td>
</tr>
<tr>
<td>Grid length in z direction</td>
<td>4 ft</td>
</tr>
<tr>
<td>Rock compressibility</td>
<td>0.0 psi⁻¹</td>
</tr>
<tr>
<td>Initial solution oil in gas</td>
<td>0.0006 stb/Mscf</td>
</tr>
<tr>
<td>Initial solution gas in oil</td>
<td>1.6 Mscf/stb</td>
</tr>
<tr>
<td>Gas production rate in producing well</td>
<td>5000 Mscf/day</td>
</tr>
</tbody>
</table>

Table 3
Validation of modified Black oil property calculation. Two columns of obtained gas formation volume factor are from two different approaches.

<table>
<thead>
<tr>
<th>Relative error</th>
<th>Gas formation volume factor by Coats et al. approach, bbl/scf</th>
<th>Gas formation volume factor by definition, bbl/scf</th>
<th>Pressure, psia</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.082267</td>
<td>0.003461</td>
<td>0.003176</td>
<td>984</td>
</tr>
<tr>
<td>0.053117</td>
<td>0.002759</td>
<td>0.002613</td>
<td>1184</td>
</tr>
<tr>
<td>0.033269</td>
<td>0.002293</td>
<td>0.002216</td>
<td>1384</td>
</tr>
<tr>
<td>0.019651</td>
<td>0.001962</td>
<td>0.001923</td>
<td>1584</td>
</tr>
<tr>
<td>0.01042</td>
<td>0.001717</td>
<td>0.001699</td>
<td>1784</td>
</tr>
<tr>
<td>0.004359</td>
<td>0.00153</td>
<td>0.001523</td>
<td>1984</td>
</tr>
<tr>
<td>0.000608</td>
<td>0.001383</td>
<td>0.001382</td>
<td>2184</td>
</tr>
<tr>
<td>0.147</td>
<td>0.001265</td>
<td>0.001267</td>
<td>2384</td>
</tr>
<tr>
<td>0.236</td>
<td>0.001169</td>
<td>0.001172</td>
<td>2584</td>
</tr>
<tr>
<td>0.239</td>
<td>0.001089</td>
<td>0.001092</td>
<td>2784</td>
</tr>
<tr>
<td>0.179</td>
<td>0.001023</td>
<td>0.001025</td>
<td>2984</td>
</tr>
<tr>
<td>0.7</td>
<td>0.000966</td>
<td>0.000967</td>
<td>3184</td>
</tr>
<tr>
<td>0.000796</td>
<td>0.000918</td>
<td>0.000917</td>
<td>3384</td>
</tr>
<tr>
<td>0.002676</td>
<td>0.000876</td>
<td>0.000874</td>
<td>3584</td>
</tr>
<tr>
<td>0.004939</td>
<td>0.00084</td>
<td>0.000836</td>
<td>3784</td>
</tr>
<tr>
<td>0.007615</td>
<td>0.000809</td>
<td>0.000803</td>
<td>3984</td>
</tr>
<tr>
<td>0.010747</td>
<td>0.000782</td>
<td>0.000774</td>
<td>4184</td>
</tr>
</tbody>
</table>

2.3. Validation

First of all, PVT module validation is discussed. Afterwards python simulator is validated by ECLIPSE. PVT module uses Coats approach. Calculated properties by Coats approach is validated by its definition. For example, validation of the gas formation volume factor, $B_g$ (bbl/scf), with its definition can be done as follow with using compressibility factor $Z$, temperature $T$, and pressure $p$: 
Table 3 compares calculated values by definition and Coats approach for the lean gas condensate fluid. The comparison shows great proximity between two calculating ways for this property. If one of the properties is validated, the others are surely validated because they are calculated at the same time from the same flash calculation. Thus, all properties are validated along with gas formation volume factor.

Simulator validation is accomplished by ECLIPSE simulator with the same input data. At the first stage, prototype of the gas condensate simulator is developed in python for oil-water simulation. Results of oil-water simulation for a specific synthetic reservoir are shown in Fig. 7. Exact match between ECLIPSE and python simulators reveals efficiency of the approach using Scipy and Numpy for solving non-linear equations. Material balance approves that solving without linearization reduces material balance deviation during simulation. After validation of prototype simulator, gas condensate simulator is developed with the same style of programming. Fig. 8 shows validation of this simulator with ECLIPSE with real condensate fluid. Comparing Figs. 7 and 8, reveals that the proposed approach of programming leads to more accurate response when dealing with a more nonlinear case. In both mentioned figures, an exact match for pressure versus time data is observed between Eclipse simulator and developed code.

Another validation is applied with different time steps for the specified simulation. Fig. 9 shows that, independent of time step, simulator shows same pressure versus time data for the specified reservoir simulation.

3. Results and discussions

The results of the developed simulator are presented for three different cases. In the first case various velocity dependent relative permeability models are compared with each other. The second one discussed a case that condensate saturation become greater than the critical condensate saturation. The last one illustrates the influence of capillary number on relative permeability.

3.1. Case 1: saturation behavior of production cell

In the first case, simulation is started at a specified pressure below the dew point pressure. The purpose of this case is to compare various VDRP models. In this case, only one well is located in reservoir to investigate the behavior of gas condensate during draw down followed by a build up well testing. Fig. 10
shows the saturation of the block in which the well is located. For the lean fluid, Jamialahmadi et al. [4] VDRP model behaves identical to the velocity independent relative permeability case, while Whitson et al. (2003) model shows a slight increase in condensate saturation early in the production time, but follows by a subsequent reduction. This is due to positive coupling and subsequently increasing in relative permeability in high velocity around the wellbore. Reduction in late time is due to lack of condensate in reservoir outer boundary and condensate return away from the well.

Fig. 11 illustrates different relative permeability model results for the same condition of production and build up periods for rich fluid. This figure shows that Whitson et al. (2003) model considers velocity into relative permeability more effectively, resulting in more reduction in condensate saturation. Stronger ability to condense in rich fluid is the reason of intensifying in velocity effect on relative permeability. In other words, positive coupling effect is more pronounced in higher condensate saturation.

3.2. Case 2: critical condensate saturation

In this case, the effect of passing from the critical condensate saturation is studied during the simulation. Jamialahmadi et al. [4] velocity dependent relative permeability model is conducted for this purpose. Critical saturation

![Fig. 13. Total underground condensate in different production rates a) Rich fluid b) Lean fluid.](image)

![Fig. 14. Total production of condensate in different production rates, a) Rich fluid b) Lean fluid.](image)

![Fig. 15. Ratio of produced to underground condensate in different production rates, a) Rich fluid b) Lean fluid.](image)
is equal to 0.01. Fig. 12 compares the behavior of saturation increase during production and build up periods. Lean fluid shows the inflection point after 0.01 but rich fluid crosses the critical saturation without any inflection point. This is because of the strong ability of rich one to condense which prevents the reduction in the ascending rate of the condensate saturation.

3.3. Case 3: condensate build up and pressure drop compensation

In the final case the ratio of the underground condensate to the produced condensate volume is investigated for different production rates. Fig. 13 shows total underground condensate in different production rates for both fluids. This parameter in rich fluid is independent to production rate at low rates. But for the lean fluid, it is always dependent on production rate. This dependency for lean fluid is due to inability to produce sufficient condensate in all production rates, but rich fluid can compensate condensate production completely resulting in constant value of underground condensate. Total production of condensate is shown in Fig. 14 with the same trend with different values for two fluids under constant gas production during well testing. It is also observed that higher flow rate of gas production leads to more condensate production when no other limitation such as bottom-hole pressure limitation is violated during well testing. Fig. 15 compares the ratio of produced to underground condensate for two fluids. Again, the same trend with different values is observed.

4. Conclusion

In this study, nonlinear equations of gas condensate are programmed without linearization. This makes the simulation more accurate in terms of material balance. Three supplementary cases are studied to investigate the behavior of the gas condensate in various aspects that shows bellow results:

- Different VDRP models result in different responses to the same problem. This is due to the experimental nature of these models.
- Rich fluid causes an increase in condensate saturation regardless of passing from critical saturation. The critical saturation effect is highly pronounced in lean gas condensate.
- The produced condensate and ratio of the produced condensate to underground condensate with the same production rates has same trend for two lean and rich gas condensate fluids.

APPENDIX 1

In this appendix MBO property calculation proposed by Coats [12] is reviewed. During expansion from \( P_1 \) to lower pressure \( P_2 \), mass material balance is indefeasible:

\[
\text{Gas : } V_2 (b_g R_g S_g + b_o R_o S_o) = V_1 (b_g R_g S_g + b_o R_o S_o) \quad (A-1)
\]

\[
\text{Liquid : } V_2 (b_g R_g S_g + b_o S_o) = V_1 (b_g R_g S_g + b_o S_o) \quad (A-2)
\]

\[
\rho_{o2} = b_o 2 (\rho_{oast} + c_1 R_o) \quad (A-3)
\]

Where

\[
c_1 = \frac{14.7 M_g}{10.73520 \times 5.6144} \quad (A-4)
\]

\( M_g \): Molecular weight of final stage gas, and,
\( \rho_{oast} \): Final step oil density, lbm/cu ft.

If relation A-3 is rearranged, below relation is obtained:

\[
(b_o R_o) = \frac{\rho_{oast}}{c_1} b_o + \frac{\rho_{oast}}{c_1} = \alpha b_o + \beta \quad (A-5)
\]

If last relation is substituted into material balance relations, A-1 & A-2, below relations are obtained for lower pressure of \( P_2 \) (subscript 2 is deleted for simplicity):

\[
S_g b_g + S_o \alpha b_o = \frac{V_1}{V_2} (S_g b_g + b_o R_o S_o) - S_o \beta \quad (A-6)
\]

\[
S_g \alpha b_g + S_o b_o = \frac{V_1}{V_2} (b_o R_o S_o + b_o S_o) \quad (A-7)
\]

Above relations are solved simultaneously to calculate \( b_o \) and \( b_o \). Afterwards, \( R_o \) is obtained by relation A-5.

APPENDIX 2

In this appendix derivation of modified black oil formulation is reviewed. For each phase in case of no other dissolved phase present, continuity equation is as below:

\[
-\frac{d}{dx} (\rho_x u_x) = -\frac{d}{dx} (\phi_x S_x) \quad (B-1)
\]

In MBO simulation we have both oil dissolved in gas and gas dissolved in oil. Therefore, density of each phase is divided into two parts, solution part and free part. For example, oil density is as below:

\[
\rho_o = \rho_{os} + \rho_{og} R_o \quad \frac{B_o}{B_o} + \rho_{os} R_o \quad \frac{B_o}{B_o} = \rho_{ol} + \rho_{og} \quad (B-2)
\]

In above relation, “L” and “G” in subscript are related to liquid and gas phase, respectively. Hence continuity equation for gas is as below:

\[
-\frac{d}{dx} (\rho_g u_g + \rho_{og} U_o) = -\frac{d}{dx} (\phi (\rho_g S_g + \rho_{og} S_o)) \quad (B-3)
\]

And for oil:

\[
-\frac{d}{dx} (\rho_o u_o + \rho_{og} U_g) = -\frac{d}{dx} (\phi (\rho_o S_o + \rho_{og} S_g)) \quad (B-4)
\]

Darcy law for each phase is as below:

\[
u_o = -\frac{K K_o}{\mu_o} \frac{d P_b}{d x} \quad (B-5)
\]

\[
u_g = -\frac{K K_g}{\mu_g} \frac{d P_g}{d x} \quad (B-6)
\]

Implementing Darcy law into continuity equation results in:

\[
\frac{d}{d x} \left( \frac{K K_g}{\mu_g} \frac{d P_g}{d x} \right) + \frac{R K K_{og} \phi}{\mu_o b_o} \frac{d P_o}{d x} - q_g - R_s q_o = \frac{d}{d x} \left( \frac{\phi S_g}{B_g} + R_o \frac{S_o}{B_o} \right) \quad (B-7)
\]
Oil:

\[
\frac{\partial}{\partial x} \left( \frac{KK_{G0} \partial P_0}{\mu_0 B_0} + R_{K_{G0}} \frac{\partial P_0}{\partial x} \right) - q'_0 - R_s q'_G = \frac{\partial}{\partial t} \left( \frac{\phi s_o B_0}{B_g} + R_s \frac{\phi s_G}{B_g} \right) \tag{B-8}
\]

LHS of above relations should be rearranged as transmissibility terms. For example, first section of LHS of gas relation is rearranged as below:

\[
\frac{\partial}{\partial x} \left( \frac{KK_{G0} \partial P_0}{\mu_0 B_0} + R_{K_{G0}} \frac{\partial P_0}{\partial x} \right) = T_{s_G} \left( p_{G_0} - p_G \right) + T_{s_0} \left( p_{G_0} - p_G \right) \tag{B-9}
\]

Where:

\[
T_{s_G} = \frac{2 \lambda_{s_G}}{\Delta x} \frac{(\Delta x/\lambda_{s_G})}{\Delta x} \tag{B-10}
\]

\[
T_{s_0} = \frac{2 \lambda_{s_0}}{\Delta x} \frac{(\Delta x/\lambda_{s_0})}{\Delta x} \tag{B-11}
\]

\[
\lambda_s = \frac{K_{s_G}}{\mu_{s_G} B_g} \tag{B-12}
\]

\( \lambda_s \) Should be calculated in the boundary of two adjacent simulation blocks which is shown by subscript \( i \) or \( j \). In this study, upstream values are used. RHS of relation of gas and oil also should be rearranged in terms of coefficients for oil pressure and gas saturation. First, oil relation RHS is discussed:

\[
\frac{\partial}{\partial t} \left( \frac{\phi s_o B_0}{B_g} + R_s \frac{\phi s_G}{B_g} \right) = \frac{\partial}{\partial t} \left( \frac{\phi s_o B_0}{B_g} \right) + \frac{\partial}{\partial t} \left( R_s \frac{\phi s_G}{B_g} \right) \tag{B-13}
\]

\[
R_s \frac{\phi s_G}{B_g} = R_s \frac{\partial}{\partial t} \left( \frac{\phi s_G}{B_g} \right) + \frac{\partial}{\partial t} \left( R_s \frac{\phi s_G}{B_g} \right) \tag{B-14}
\]

\[
\frac{\partial}{\partial t} \left( \frac{\phi s_G}{B_g} \right) = \frac{\partial}{\partial t} \left( \frac{\phi s_G}{B_g} \right) = \frac{\partial}{\partial t} \left( R_s \frac{\phi s_G}{B_g} \right) \tag{B-15}
\]

Therefore, in form of coefficient, relation below is obtained:

\[
\frac{\partial}{\partial t \left( \frac{\phi s_o B_0}{B_g} + R_s \frac{\phi s_G}{B_g} \right) = C_{poo} \left( p_o - p_o^e \right) + C_{sgo} \left( s_G - s_G^e \right) \tag{B-16}
\]

Where:

\[
C_{sgo} = \frac{\phi}{\Delta t} \left[ \frac{1}{B_o} + (R_s S_G) \left( \frac{c_r}{B_g} + \frac{d(1/B_g)}{dp_g} \right) \right] + \frac{R_s}{B_g} \tag{B-17}
\]

And

\[
C_{poo} = \frac{\phi}{\Delta t} \left[ \frac{(1-S_G) \left( \frac{c_r}{B_g} + \frac{d(1/B_g)}{dp_g} \right) + (R_s S_G) \left( \frac{c_r}{B_g} + \frac{d(1/B_g)}{dp_g} \right)}{\left( \frac{dR_s}{dp_g} \right)} \right] \tag{B-18}
\]

All in all, oil continuity equation is as below:

\[
t_o \left[ p_{o_1} - p_o \right] + t_o \left[ p_{o_1} - p_o \right] - q'_o
\]

\[
= (R_s T_g) \left[ p_{o_1} - p_o \right] + (P_{cos} - P_{pog}) \right) - R_s q'_G \tag{B-19}
\]

Next step is to rearrange gas continuity equation. First, RHS is discussed:

\[
\frac{\partial}{\partial t \left( \frac{\phi s_G}{B_g} \right) = \frac{\partial}{\partial t} \left( \frac{\phi s_G}{B_g} \right) + \frac{\partial}{\partial t} \left( R_s \frac{\phi s_G}{B_g} \right) \tag{B-20}
\]

\[
\frac{\partial}{\partial t} \left( R_s \frac{\phi s_G}{B_g} \right) = R_s \frac{\partial}{\partial t} \left( \frac{\phi s_G}{B_g} \right) + \frac{\partial}{\partial t} \left( R_s \frac{\phi s_G}{B_g} \right) \tag{B-21}
\]

In terms of coefficients:

\[
C_{pog} = \frac{\phi}{\Delta t} \left( \frac{S_G \left( \frac{c_r}{B_g} + \frac{d(1/B_g)}{dp_g} \right) + R_s \left( 1-S_G \right) \left( \frac{c_r}{B_g} + \frac{d(1/B_g)}{dp_g} \right)}{\left( \frac{dR_s}{dp_g} \right)} \right) \tag{B-22}
\]
\[ C_{Sgg} = \frac{\phi}{\Delta T} \left[ S_g \left( \frac{C_P}{B_g} + d \left( \frac{1}{B_g} \right) \frac{dP_{cog}}{dS_g} \frac{R_s}{B_o} + \frac{1}{B_g} \right) \right] \quad (B-23) \]

Finally, gas equation can be rearranged as below:

\[
\begin{align*}
T_{\frac{1}{2}g} \left[ (P_{o,i} - P_{o}) + (P_{cog,i} - P_{cog,i}) \right] + T_{\frac{1}{2}g} \left[ (P_{o,i} - P_{o}) \\
+ (P_{cog,i} - P_{cog,i}) \right] - q_g + (R_s T_{0,i}) \frac{1}{2} \left[ (P_{o,i} - P_{o}) \right] \\
+ (R_s T_{0,i}) \frac{1}{2} \left[ (P_{o,i} - P_{o}) \right] - R_o q_o \\
= C_{pog} \left( P_o - P_o^g \right) + C_{Sgg} \left( S_g - S_g^o \right)
\end{align*}
\]

(B-24)

In this appendix, for simplification, one dimension simulation equations are explained. Two dimensional equations have a little difference of two extra transmissibility terms in y direction in addition to these x direction transmissibilities. RHS of equations are the same for one dimension and two dimension simulations. Thus final form for two dimensional simulation for gas and oil is as below:

Gas:

\[
\begin{align*}
T_{\frac{1}{2}g} \left[ (P_{o,i} - P_{o}) + (P_{cog,i} - P_{cog,i}) \right] + T_{\frac{1}{2}g} \left[ (P_{o,i} - P_{o}) \\
+ (P_{cog,i} - P_{cog,i}) \right] - q_g + (R_s T_{0,i}) \frac{1}{2} \left[ (P_{o,i} - P_{o}) \right] \\
+ (R_s T_{0,i}) \frac{1}{2} \left[ (P_{o,i} - P_{o}) \right] - R_o q_o \\
= C_{pog} \left( P_o - P_o^g \right) + C_{Sgg} \left( S_g - S_g^o \right)
\end{align*}
\]

(B-25)

Oil:

\[
\begin{align*}
T_{\frac{1}{2}o} \left[ (P_{o,i} - P_{o}) \right] + T_{\frac{1}{2}o} \left[ (P_{cog,i} - P_{cog,i}) \right] + T_{\frac{1}{2}o} \left[ (P_{o,i} - P_{o}) \right] \\
+ (P_{cog,i} - P_{cog,i}) \left[ (R_s T_{0,i}) \frac{1}{2} \left[ (P_{o,i} - P_{o}) \right] \right] \\
+ (P_{cog,i} - P_{cog,i}) \left[ (R_s T_{0,i}) \frac{1}{2} \left[ (P_{o,i} - P_{o}) \right] \right] \\
+ (P_{cog,i} - P_{cog,i}) \left[ (R_s T_{0,i}) \frac{1}{2} \left[ (P_{o,i} - P_{o}) \right] \right] \\
- q_o - R_o q_o \\
= C_{pog} \left( P_o - P_o^o \right) + C_{Sgg} \left( S_g - S_g^o \right)
\end{align*}
\]

(B-26)

References


Nomenclature

R_o: solution gas in oil
r_c: solution oil in gas
T: transmissibility
C_{pog}: coefficient of oil pressure in oil equation
C_{sog}: coefficient of gas saturation in oil equation
C_{pog}: coefficient of pressure in gas equation
C_{sog}: coefficient of gas saturation in gas equation
B_g: gas formation volume factor
Z: gas compressibility factor
T: temperature
P: pressure

Subscript

o: oil
g: gas

cog: capillary between oil and gas
i + 1/2 between block “i” and “i+1”
i − 1/2 between block “i” and “i-1”

c: value at previous time step