DIFFERENTIAL GAMES OF CAPITALISM: A SURVEY

Matti Pohjola

Department of Economics, University of Helsinki,
Aleksanterinkatu 7, SF-00100 Helsinki, Finland

Abstract. This paper surveys some recent work done by the author and others on a differential game model of capitalism which was originally developed by Kelvin Lancaster. Economic growth and income distribution are here modelled as a game between workers, who may consume or save, and capitalists, who may consume or invest. The assumptions made and the results obtained are discussed with a view to pointing out possible avenues of future research.

Keywords. Economic growth; differential games; dynamic efficiency.

INTRODUCTION

Nearly sixty years ago Ramsey (1928) pioneered in a new field of economic theory which we nowadays call optimal economic growth. His work was more or less ignored during the rise of Keynesian macroeconomics when unemployment and inflation were regarded as the pressing issues. In many respects Ramsey's analysis was already quite general and it was extended in a number of ways, for example, by allowing for many capital goods, population growth, technical progress and uncertainty in the 1960's [for a survey see, for example, Wan (1971, Ch 10)]. This new theoretical interest in the issues of optimal economic growth arose as a direct consequence of the advances made in growth theory and control theory. Pontryagin's maximum principle was quickly adopted by economists.

In the heydays of the applications of optimal control theory to the modelling of economic growth, distributional issues were much neglected. This must have reflected the bias of the general economics profession to questions of growth as well as the immaturity of the methods which were available to deal with dynamic conflicts.

The first game-theoretic approach to distributional issues was formulated by Phelps and Pollak (1968), who viewed economic growth and distribution as an intergenerational conflict. Assuming that the present generation derives its utility from the consumption pattern of infinitely many nonoverlapping generations but that it can only control its own saving rate, they demonstrated that the Nash equilibrium of this intergenerational game results in under-saving. Their model cannot, however, be expressed in the framework of differential game theory, and, therefore, we shall not review the works based on this approach.

The first study applying differential game theory to the modelling of growth and distribution is due to Lancaster (1973). He took up the issues, studied by the classical economists Malthus, Ricardo and Marx, of capital accumulation and the distribution of income between the social classes and formulated them as a simple two-player noncooperative differential game between workers and capitalists. This model will be described in the next section, after which the existing models generalizing Lancaster's approach are reviewed. In concluding the paper we shall also point out possible avenues of future research.

LANCASTER'S MODEL

Lancaster (1973) considered a one-sector single technique economy whose output $X(t)$ at any given time t may be consumed or added to the existing stock $K(t)$ and in which labour is never a limiting factor and capital lasts forever. It is assumed that the workers can control their share of consumption in total output, $u_1(t) = C_1(t)/X(t)$, within given institutional limits: $c \leq u_1(t) \leq b$ for all t, c and b being constants such that $0 < c < b$ and $1/2 < b < 1$. This is interpreted to mean that this social organization would not survive if the workers' share of consumption in output were less than c or greater than b. The assumption that they can obtain more than half of output is important and, we believe, realistic in the analysis that follows. The capitalists' control variable is $u_2(t) = I(t)/[X(t)-C_1(t)]$, the share of investment in output which is not consumed by the workers. This variable is assumed free to vary over the closed unit interval: $0 \leq u_2(t) \leq 1$ for all t.

Let a denote the output-capital ratio and T be the length of the planning horizon. Assume that workers and capitalists have somehow devised piecewise-continuous (open-loop) strategies $u_1 : [0,T] \to [c,b]$ and $u_2 : [0,T] \to [0,1]$, respectively. Total worker consumption is then given by

$$J_1(u_1,u_2) = \int_0^T C_1(t)dt = \int_0^T aK(t)u_1(t)dt, \quad (1)$$

and total capitalist consumption by

$$J_2(u_1,u_2) = \int_0^T C_2(t)dt = \int_0^T aK(t)\,[1-u_1(t)]\,[1-u_2(t)]\,dt \quad . \quad (2)$$

The state variable, i.e. the capital stock, is determined by

$$\dot{K}(t) = I(t) = aK(t)[1-u_1(t)]u_2(t) \; ;$$

$$K(0) = K_0 > 0 \; . \qquad (3)$$

We are now interested in the situation where each player chooses his strategy, u_1 or u_2, so as to maximize the respective criterion function (1) or (2), subject to the state equation (3) and the assumption made about the rationality of the other player. Lancaster (1973) derived the open-loop Nash equilibrium of this differential game and showed the solution to consist of two phases: in the first phase workers consume minimally ($u_1(t) = c$) and capitalists invest maximally ($u_2(t) = 1$), whilst in the second both classes consume at the maximum rate ($u_1(t) = b$, $u_2(t) = 0$), which means that accumulation has ceased. The switch takes place at $\bar{t} = T - 1/a\,(1-b)$, where T is assumed large enough to make $\bar{t}$ positive. It is the instant at which the value of investment to the capitalists falls below that of consumption. As Lancaster points out, the workers value investment higher than consumption at $\bar{t}$ since they can receive more than half of future output ($b > 1/2$). However, it is not optimal for them to save when the capitalists have stopped investing. Workers cannot force capitalists to accumulate against the latter's best interest.

Lancaster's open-loop equilibrium also qualifies as the feedback equilibrium because the differential game defined in equations (1) - (3) is state separable, i.e. neither the Hamiltonian-maximizing conditions nor the costate equations of the players depend on the state variable.

By comparing the noncooperative equilibrium with the cooperative solution which was obtained under the assumption that saving and investment decisions are derived so as to maximize total (worker plus capitalist) consumption, Lancaster demonstrated that both social classes could obtain more consumption under cooperation. This result follows from the fact that the optimal switch point is $t^* = T - 1/a$ and, thus, $t^* > \bar{t}$. His important conclusion was that the Keynesian separation of saving and investment decisions results in dynamic inefficiency. Later Hoel (1978) showed that the game solution does not usually belong to the set of Pareto-optimal solutions and that it results in lower capital accumulation than all the cooperative solutions.

Before turning to consider these extensions of the Lancaster model let us observe that his model has both a (neo-)Marxian and Keynesian flavour. It is distinctly Marxian in the sense that he studies a labour-surplus economy and assumes that the workers can control income distribution in any given period by being able to set the share of their consumption in total output. Thus, distribution is prior to accumulation. The capitalists are assumed to control accumulation by being able to choose the share of investment in the surplus, i.e. in output which is not consumed by the workers. This disjunction of saving and investment decisions brings in some Keynesian flavour.

EXTENSIONS OF THE LANCASTER MODEL

Table 1 summarizes the dynamic game models based on the Lancaster approach. It also contains the contributions of Hamada (1967), Marglin (1976), Stanley(1978) and Hammer (1981), in which the framework is somewhat different from Lancaster's. In these studies the government chooses the time path of income redistribution from workers to capitalists whose fixed saving rate is higher than that of the former class so as to maximize a weighted average of the worker and capitalist welfare. We have included these applications of optimal control theory in the Pareto-optimal solutions of "capitalism games".

The original Lancaster model has been technically extended in a number of ways. Hoel (1978) studied the implications of diminishing returns to capital, which make the state equation (3) nonlinear in the capital stock. Infinite horizon, nonlinear utility functions and the full-employment constraint have been introduced in Pohjola (1985).

The explanation for the dynamic inefficiency displayed in these studies is a dynamic externality. One group's decision to save and invest for the future is affected by the fact that the accumulated amount may be consumed by some other group. Rationally acting players take into account this public good nature of provision for the future and, consequently, save and invest less than they would do under cooperation. Externalities also arise under other specifications of growth and distribution similar to those arising from the worker-capitalist conflict. An example is an economy where the working class is not a single decision-making unit but has organized itself in a number of competing unions [Pohjola (1984a)]. I have argued in Pohjola (1985) that we can explain in terms of this externality many current economic problems, such as high real wages, slow growth and unemployment, from a new viewpoint.

Ways of reducing the welfare loss which arises from the lack of cooperation between the social classes have been examined in a few studies. In Pohjola (1983b) it is demonstrated that a partial transfer of control over the investment decision from capitalists to workers improves both player's welfare. Such a transfer of economic power is an essential feature of the worker investment funds established recently in Sweden. Buhl and Machaczek (1985) have extended this approach by considering the implications of the worker ownership of capital. Their conclusion is that inefficiency is sharply reduced because such an ownership makes workers less dependent on capitalists' behaviour.

In Pohjola (1983a) we have compared the Nash solution of the Lancaster game with the open-loop Stackelberg solution. It turned out that both players prefer the Stackelberg formulation but that neither workers nor capitalists want to act as the leader. This means that the game is in a stalemate. The result can be interpreted along the following lines. As was explained in section 2, the workers value investment higher than consumption at $\bar{t}$, the instant at which the capitalists stop invest-

Table 1. Capitalism Games

Information structure	Noncooperative solutions: Nash	Noncooperative solutions: Stackelberg	Cooperative solutions: Pareto	Cooperative solutions: Nash's threat bargaining
Open loop	Lancaster (1973) Hoel (1978) Pohjola (1983b, 1984a, 1985) Buhl & Machaczek (1985)	Pohjola (1983a)	Hamada(1967) Lancaster (1973) Marglin (1976) Hoel (1978) Stanley (1978) Hammer (1981) Pohjola (1985)	Pohjola(1984b)
Feedback	Lancaster (1973) Pohjola (1983b, 1984a, 1985) Başar, Haurie & Ricci (1985)	Başar, Haurie & Ricci (1985)		
Closed loop	Haurie & Pohjola (1985)			

ing. This means that the workers would save for a longer period if they could control investment. As the leader in the Stackelberg game they obtain such a control by offering their savings over a given interval beyond $\bar{t}$ as compensation to capitalists for an extension of investment activity. The workers have to follow this strategy since they have no control over the investment decision. Under the capitalists' leadership they obtain similar results without such a sacrifice in consumption.

Our conclusion is, however, sensitive to the assumption about the information structure, as Başar, Haurie and Ricci (1985) have pointed out. They showed that both players want the capitalists to act as the leader in the feedback Stackelberg game. Under the workers' leadership the Stackelberg solution is equivalent to the feedback Nash equilibrium. In economic terms this conclusion follows from the neo-Marxian assumption concerning income distribution and it demonstrates the importance to the workers of a period of commitment. This assumption places the players in an asymmetric position since the capitalists' decision $u_2(t)$ in period t is independent of the workers' choice $u_1(t)$. The investment share $u_2(t)$ does, however, depend on $u_1(s), s > t$. This means that as the leader of the game workers can influence capitalists' current choice of $u_2(t)$ by committing themselves to low wage shares only in the future, i.e. only by applying an open-loop control. This explains why the open-loop Stackelberg solution with the workers as the leader may dominate the corresponding Nash solution and also why the feedback Stackelberg solution with the workers as the leader is equivalent to the feedback Nash solution. The latter result follows from the fact that in the feedback Stackelberg game the leader can commit himself to the current period choice only. The situation is different under the capitalists' leadership since $u_1(t)$ depends on $u_2(t)$ in a direct way as well. Consequently, the leader can influence the follower's choice even under the feedback information structure. It is interesting to observe that in the practice of incomes policy it is the workers who apply open-loop strategies.

The Nash bargaining solution and the optimal threats announced by the social classes to affect the negotiated solution to their own advantage are examined in Pohjola (1984b). It is demonstrated that the workers' threat takes the form of refusing to accept low wages while capitalists threaten to refrain from investing. The threats as well as the possible gains from cooperation determine the players' relative importance, or bargaining power, and it is shown that capitalists are in general in a stronger position than workers.

The basic weakness of the axiomatic bargaining approach, which was originally developed for games in the normal form, is its essentially static nature. It is necessary to assume that the agreement reached in the beginning of the game is binding for both players over the whole time horizon on which the game is played. This assumption is most restrictive since it practically eliminates the possibility for the players to adapt their controls dynamically. Economic growth and distributions are, after all, the results of the inherent conflict and complementarity of labour and capital in a fully dynamic context with the possibility of making as well as breaking agreements which may be either formal or informal. It is this aspect of the real life which is modelled by Haurie and Pohjola (1985) by applying closed-loop strategies with memory to an infinite-horizon version of the Lancaster model developed in Pohjola (1985). They demonstrate that efficient equilibria can be constructed by assuming that the players choose a compound strategy whereby they play an agreed upon cooperative policy if neither of them has cheated in the past whereas they apply the feedback

Nash controls for the rest of the game otherwise. The original Lancaster result as well as its extension are consequently shown to be fragile with respect to the assumption concerning the information structure of the game. Thus neither the Marxian conflict between wages and profits nor the Keynesian disjunction between saving and investment decisions, as perceived in this game-theoretic framework, necessarily explain the observed inefficiencies in the development of capitalist economies. They suggest that we have to turn to games of imperfect information to explain these phenomena.

CONCLUSIONS

The games of distribution and growth surveyed here have not yet reached the maturity of optimal growth theory models. Much technical work is needed before we will be able to characterize the solutions to these problems without having to resort to special utility and production functions. Other possible avenues of future research might also include attempts to generalize the distribution theory applied in the Lancaster type models. An obvious alternative to the Marxian approach is the Keynesian theory in which effective demand plays a crucial role. Attempts by growth theorists to explain the distribution of wealth between the social classes would also benefit from a game-theoretic approach.

ACKNOWLEDGEMENT

Financial support from the Yrjö Jahnsson Foundation is gratefully acknowledged.

REFERENCES

Başar, T., A.Haurie and G.Ricci (1985). On the dominance of capitalists' leadership in a "Feedback-Stackelberg" solution of a differential game model of capitalism. Journal of Economic Dynamics and Control, 9, 121-125.

Buhl, Hans Ulrich and Werner Machaczek (1985). Workers' investment and return in a differential game model. Mimeo.

Hamada, K. (1967). On the optimal transfer and income distribution in a growing economy. Review of Economic Studies, 34, 295-299.

Hammer, Jeffrey S. (1981). Optimal growth and income redistribution. Metroeconomica, 33, 145-157.

Haurie, A. and M.Pohjola (1985). Efficient equilibria in a differential game of capitalism. Discussion and Working Paper No. 234. Department of Economics, University of Helsinki.

Hoel, Michael (1978). Distribution and growth as a differential game between workers and capitalists. International Economic Review, 19, 335-350.

Lancaster, Kelvin (1973). The dynamic inefficiency of capitalism. Journal of Political Economy, 81, 1092-1109.

Marglin, Stephen A. (1976). Value and Price in a Labour-surplus Economy. Oxford University Press, London.

Phelps, E.S. and R.A.Pollak (1968). On second-best national saving and game equilibrium growth. Review of Economic Studies, 35, 185-199.

Pohjola, Matti (1983a). Nash and Stackelberg solutions in a differential game model of capitalism. Journal of Economic Dynamics and Control, 6, 173-186.

Pohjola, Matti (1983b). Workers' investment funds and dynamic inefficiency of capitalism. Journal of Public Economics, 20, 271-279.

Pohjola, Matti (1984a). Union rivalry and economic growth: a differential game approach. Scandinavian Journal of Economics, 86, 365-370.

Pohjola, Matti (1984b). Threats and bargaining in capitalism: a differential game view. Journal of Economic Dynamics and Control, 8, 291-302.

Pohjola, Matti (1985). Growth, distribution and employment modelled as a differential game. In G.Feichtinger (ed.), Optimal Control Theory and Economic Analysis, vol.2. North-Holland, Amsterdam, pp. 581-591.

Ramsey, Frank (1928). A mathematical model of saving. Economic Journal, 38, 543-559.

Stanley, Owen (1978). Distributional goals and optimal growth. Review of Economic Studies, 45, 389-390.

Wan, Henry Y., Jr. (1971). Economic Growth. Harcourt Brace Jovanovich, New York.