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Production of doubly strange hypernuclei via Ξ^- doorways in the ^{16}O(K^-, K^+) reaction at 1.8 GeV/c

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ABSTRACT

We examine theoretically production of doubly strange hypernuclei, $^{16}_{\Xi^-}$C and $^{16}_{\Lambda\Lambda}$C, in double-charge exchange ^{16}O(K^-, K^+) reactions using a distorted-wave impulse approximation. The inclusive K^+ spectrum at the incident momentum $p_{K^-} = 1.8$ GeV/c and scattering angle $\theta_{\rm lab} = 0^\circ$ is estimated in a one-step mechanism, $K^- p \to K^+ \Xi^-$ via Ξ^- doorways caused by a $\Xi^- p$–$\Lambda\Lambda$ coupling. The calculated spectrum in the Ξ^- bound region indicates that the integrated cross sections are on the order of 7–12 nb/sr for significant 1^- excited states with ^{14}C$(0^+, 2^+) \otimes s_\Lambda p_\Lambda$ configurations in $^{16}_{\Lambda\Lambda}$C via the doorway states of the spin-stretched ^{15}N$(1/2^-, 3/2^-) \otimes s_{\Xi^-}$ in $^{16}_{\Xi^-}$C due to a high momentum transfer $q_{\Xi^-} \simeq 400$ MeV/c. The Ξ^- admixture probabilities of these states are on the order of 5–9%. However, populations of the 0^+ ground state with ^{14}C$(0^+) \otimes s^2_\Lambda$ and the 2^+ excited state with ^{14}C$(2^+) \otimes s^2_\Lambda$ are very small. The sensitivity of the spectrum on the ΞN–$\Lambda\Lambda$ coupling strength enables us to extract the nature of ΞN–$\Lambda\Lambda$ dynamics in nuclei, and the nuclear (K^-, K^+) reaction can extend our knowledge of the $S = -2$ world.

1. Introduction

It is important to understand properties of Ξ hypernuclei whose states are regarded as "doorways" to access multi-strangeness systems as well as a two-body ΞN–$\Lambda\Lambda$ system, and it is a significant step to extend study of strange nuclear matter in hadron physics and astrophysics [1]. Because the Ξ hyperon in nuclei has to undergo a strong $\Xi N \to \Lambda\Lambda$ decay, widths of Ξ hypernuclear states give us a clue to a mechanism of Ξ absorption processes in nuclei. A pioneer study of Ξ hypernuclei by Dover and Gal [2] has found that a Ξ–nucleus potential has a well depth of 24 ± 4 MeV in the real part on the analysis of old emulsion data. However, our knowledge of these Ξ–nucleus systems is very limited due to the lack of the experimental data [3]. Indeed, the missing-mass spectra of a double-charge exchange (DCX) reaction (K^-, K^+) on a ^{12}C target have suggested the Ξ well depth of 14–16 MeV [4,5]. Several authors [6] have used the unsettled Ξ–nucleus (optical) potentials such as $V_\Xi = (-24)$–(-14) MeV and $W_\Xi = (-6)$–(-3) MeV in the Woods–Saxon potential to demonstrate the Ξ^- production spectra in the nuclear (K^-, K^+) reactions. There remains a full uncertainty about the nature of doubly strange $(S = -2)$ dynamics caused by the ΞN and ΞN–$\Lambda\Lambda$ interaction in nuclei at the present stage. More experimental information is earnestly desired.

The (K^-, K^+) reaction is one of the most promising ways of studying doubly strange systems such as Ξ^- hypernuclei for the forthcoming J-PARC experiments [3]. One expects that these experiments will confirm the existence of Ξ hypernuclei and establish properties of the Ξ–nucleus potential, e.g., binding energies and widths. This reaction can also populate a $\Lambda\Lambda$ hypernucleus through a conventional DCX two-step mechanism as $K^- p \to \pi^0 \Lambda$ followed by $\pi^0 p \to K^+ \Lambda$ [7–9], as shown in Fig. 1(a). Such an inclusive K^- spectrum in the $\Lambda\Lambda$ bound region is rather clean with much less background experimentally. Early theoretical predictions for two-step ^{16}O(K^-, K^+) reactions at the incident momentum $p_{K^-} = 1.1$ GeV/c and scattering angle $\theta_{\rm lab} = 0^\circ$ [7,8] have indicated small cross sections for the $\Lambda\Lambda$ states, for example, ~ 0.1 nb/sr for the $0^+(s^2_\Lambda)$ ground state and ~ 2 nb/sr for the $2^+(s^2_\Lambda)$ excited state in $^{16}_{\Lambda\Lambda}$C when we took 0.61 mb/sr and 0.32 mb/sr as the laboratory cross sections at 0° for $K^- p \to \pi^0 \Lambda$ and $\pi^0 p \to K^+ \Lambda$, respectively.

It should be noticed that another exotic production of $\Lambda\Lambda$ hypernuclei in the (K^-, K^+) reactions is a one-step mechanism, $K^- p \to K^+ \Xi^-$ via Ξ^- doorways caused by a $\Xi^- p \to \Lambda\Lambda$ tran-

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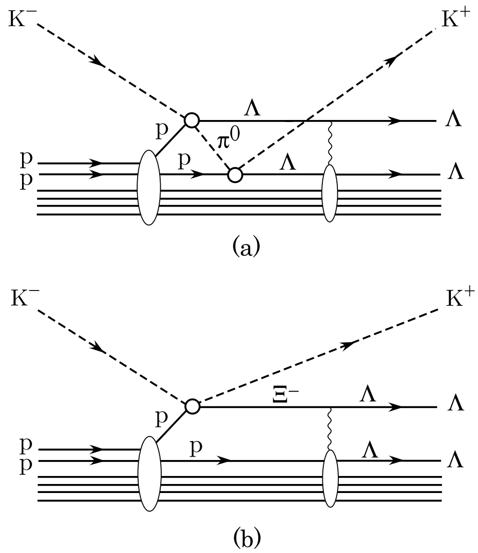

Fig. 1. Diagrams for DCX nuclear (K^-, K^+) reactions: (a) a two-step mechanism, $K^- p \to \pi^0 \Lambda$ followed by $\pi^0 p \to K^+ \Lambda$, and (b) a one-step mechanism, $K^- p \to K^+ \Xi^-$ via Ξ^- doorways caused by the $\Xi^- p$–$\Lambda\Lambda$ coupling.

sition, as shown in Fig. 1(b). The ΞN–$\Lambda\Lambda$ coupling induces the Ξ^- admixture and the $\Lambda\Lambda$ energy shift $\Delta B_{\Lambda\Lambda} \equiv B_{\Lambda\Lambda}({}^{A}_{\Lambda\Lambda}\mathrm{Z}) - 2B_{\Lambda}({}^{A-1}_{\Lambda}\mathrm{Z})$ in the $\Lambda\Lambda$-nuclear states [10–14], and its coupling strength is also related to widths of Ξ-hypernuclear states [15, 16]. For a viewpoint of $S = -2$ studies, it is very important to extract quantitative information concerning the ΞN–$\Lambda\Lambda$ coupling from spectroscopy of the Ξ and $\Lambda\Lambda$ hypernuclei [17,18].

In this Letter, we study theoretically production of a doubly strange hypernucleus in the DCX (K^-, K^+) reaction on an ^{16}O target at $p_{K^-} = 1.8$ GeV/c and $\theta_{\mathrm{lab}} = 0°$ within a distorted-wave impulse approximation (DWIA). Thus we focus on the $\Lambda\Lambda$–Ξ spectrum for ${}^{16}_{\Lambda\Lambda}\mathrm{C}$ and ${}^{16}_{\Xi^-}\mathrm{C}$ in the Ξ^- bound region considering the one-step mechanism, $K^- p \to K^+ \Xi^-$ via Ξ^- doorways caused by the ΞN–$\Lambda\Lambda$ coupling in the nuclear (K^-, K^+) reaction, rather than the two-step mechanism as $K^- p \to \pi^0 \Lambda$ followed by $\pi^0 p \to K^+ \Lambda$ [7,8]. These different mechanisms are well separated kinematically. The forward cross section for the $K^- p \to K^+ \Xi^-$ elementary process is at its maximum at $p_{K^-} = 1.8$–1.9 GeV/c, whereas the $K^- p \to \pi^0 \Lambda$ reaction at $p_{K^-} = 1.1$ GeV/c leads to the maximal cross section for the $\pi^0 p \to K^+ \Lambda$ process. The present study is the first attempt to evaluate a production spectrum of the $\Lambda\Lambda$–Ξ hypernucleus via the ΞN–$\Lambda\Lambda$ coupling from the inclusive (K^-, K^+) reaction, and to extract the Ξ^- admixture probability in the $\Lambda\Lambda$ hypernucleus from the spectrum. We also discuss a contribution of the two-step processes in the (K^-, K^+) reactions within the eikonal approximation.

2. Calculations

Let us consider the DCX (K^-, K^+) reaction on the ^{16}O target at 1.8 GeV/c within a DWIA and examine the production cross sections and wave functions of the doubly strange hypernucleus. To fully describe the one-step process via Ξ^- doorways, as shown in Fig. 1(b), we perform nuclear $\Lambda\Lambda$–Ξ coupled-channel calculations [13,14], which are assumed to effectively represent the coupling nature in omitting other $\Lambda\Sigma$ and $\Sigma\Sigma$ channels for simplicity. Here we employ a multichannel coupled wave function of the $\Lambda\Lambda$–Ξ nuclear state for a total spin J_B within a weak coupling basis. It is written as

$$\begin{aligned}
&\left|\Psi_{J_B}\left({}_{\Lambda\Lambda-\Xi}^{16}\mathrm{C}\right)\right\rangle \\
&\quad = \sum_{J J'' j_1 j_2} \left[\left[\Phi_J\left({}^{14}\mathrm{C}\right), \varphi_{j_1}^{(\Lambda)}(\boldsymbol{r}_{\Lambda_1})\right]_{J''}, \varphi_{j_2}^{(\Lambda)}(\boldsymbol{r}_{\Lambda_2})\right]_{J_B} \\
&\qquad + \sum_{J J' j_p j_3} \left[\Phi_{J'}\left({}^{15}\mathrm{N}\right), \varphi_{j_3}^{(\Xi^-)}(\boldsymbol{r}_{\Xi})\right]_{J_B}
\end{aligned} \tag{1}$$

with $\Phi_{J'}({}^{15}\mathrm{N}) = \mathcal{A}[\Phi_J({}^{14}\mathrm{C}), \varphi_{j_p}^{(p)}(\boldsymbol{r}_p)]_{J'}$, where $\boldsymbol{r}_{\Lambda_1}$ ($\boldsymbol{r}_p$) denotes the relative coordinate between the ^{14}C core-nucleus and the Λ (proton), and $\boldsymbol{r}_{\Lambda_2}$ ($\boldsymbol{r}_{\Xi}$) denotes the relative coordinate between the center of mass of the ^{14}C–Λ (^{15}N) subsystem and the Λ (Ξ^-). Thus $\varphi_{j_{1,2}}^{(\Lambda)}$, $\varphi_{j_3}^{(\Xi^-)}$ and $\varphi_{j_p}^{(p)}$ describe the relative wave functions of shell model states (that occupy $j_{1,2}$, j_3 and j_p orbits) for the Λ, Ξ^- and proton, respectively; $\Phi_J(^{14}\mathrm{C})$ is a wave function of the ^{14}C core-nucleus state, and $\mathcal{A}$ is the antisymmetrized operator for nucleons. The energy difference between $^{15}\mathrm{N} + \Xi^-$ and $^{14}\mathrm{C} + \Lambda + \Lambda$ channels is $\Delta M = M(^{15}\mathrm{N}) + m_{\Xi^-} - M(^{14}\mathrm{C}) - 2m_\Lambda = 18.4$ MeV, where $M(^{15}\mathrm{N})$, $M(^{14}\mathrm{C})$, m_{Ξ^-} and m_Λ are masses of the ^{15}N nucleus, the ^{14}C nucleus, the Ξ^- and Λ hyperons, respectively. We take the ^{15}N core-nucleus states with $J^\pi = 1/2^-$(g.s.) and $3/2^-$(6.32 MeV), and the ^{14}C core-nucleus states with $J^\pi = 0^+$(g.s.) and 2^+(7.01 MeV) that are given in $(0p_{1/2}^{-1}0p_{1/2}^{-1})_{0^+}$, $(0p_{3/2}^{-1}0p_{1/2}^{-1})_{2^+}$ and $(0p_{3/2}^{-1}0p_{3/2}^{-1})_{0^+,2^+}$ configurations on ^{16}O(g.s.) [7,8]. Because we assume only natural-parity $\pi = (-1)^{J_B}$ states via Ξ^- doorways that are selectively formed by non-spin-flip processes in the forward $K^- p \to K^+ \Xi^-$ reaction, we consider a spin $S = 0$, $\Lambda\Lambda$ pair in the hypernucleus. If the $\Lambda\Lambda$ component is dominant in a bound state, we can identify it as a state of the $\Lambda\Lambda$ hypernucleus ${}^{16}_{\Lambda\Lambda}\mathrm{C}$, in which the Ξ^- admixture probability can be estimated by

$$P_{\Xi^-} = \sum_{j_p j_3} \left\langle \varphi_{j_p}^{(p)} \varphi_{j_3}^{(\Xi^-)} \middle| \varphi_{j_p}^{(p)} \varphi_{j_3}^{(\Xi^-)} \right\rangle, \tag{2}$$

under the normalization of

$$\sum_{j_1 j_2} \left\langle \varphi_{j_1}^{(\Lambda)} \varphi_{j_2}^{(\Lambda)} \middle| \varphi_{j_1}^{(\Lambda)} \varphi_{j_2}^{(\Lambda)} \right\rangle + \sum_{j_p j_3} \left\langle \varphi_{j_p}^{(p)} \varphi_{j_3}^{(\Xi^-)} \middle| \varphi_{j_p}^{(p)} \varphi_{j_3}^{(\Xi^-)} \right\rangle = 1.$$

After we set up the ${}^{15}_{\Lambda}\mathrm{C}$ and ^{15}N configurations in our model space with Eq. (1), we calculate the wave functions of $\varphi_{j_2}^{(\Lambda)}(\boldsymbol{r}_{\Lambda_2})$ and $\varphi_{j_3}^{(\Xi^-)}(\boldsymbol{r}_{\Xi})$ taking into account their channel coupling. Thus, the complete Green's function $\boldsymbol{G}(\omega)$ [19] describes all information concerning $({}^{15}_{\Lambda}\mathrm{C} \otimes \Lambda) + ({}^{15}\mathrm{N} \otimes \Xi^-)$ coupled-channel dynamics, as a function of the energy transfer ω. It is numerically obtained as a solution of the N-channels radial coupled equations with a hyperon–nucleus potential $\boldsymbol{U}$ [20,21], which is written in an abbreviated notation as

$$\boldsymbol{G}(\omega) = \boldsymbol{G}^{(0)}(\omega) + \boldsymbol{G}^{(0)}(\omega)\boldsymbol{U}\boldsymbol{G}(\omega) \tag{3}$$

with

$$\boldsymbol{G}(\omega) = \begin{pmatrix} G_\Lambda(\omega) & G_X(\omega) \\ G_X(\omega) & G_\Xi(\omega) \end{pmatrix}, \qquad \boldsymbol{U} = \begin{pmatrix} U_\Lambda & U_X \\ U_X & U_\Xi \end{pmatrix}, \tag{4}$$

where $\boldsymbol{G}^{(0)}(\omega)$ is a free Green's function. In our calculations, for example, we deal with $N = 28$ for the $J_B^\pi = 1^-$ state. The nuclear optical potentials U_Y ($Y = \Xi$ or Λ) can be written as

$$U_Y(r) = V_Y f(r, R, a) + iW_Y f(r, R', a') + iW_Y^{(D)} g(r, R', a'), \tag{5}$$

where f is the Woods–Saxon (WS) form, $f(r, R, a) = [1 + \exp((r - R)/a)]^{-1}$, and g is the derivative of the WS form, $g(r, R', a') = -4a'(d/dr) f(r, R', a')$. The spin–orbit potentials are neglected. In ^{15}N–Ξ^- channels, we assume the strength parameter of $V_\Xi =$

-24 or -14 MeV with $a = 0.6$ fm and $R = 1.10(A-1)^{1/3} = 2.71$ fm in $U_\Xi(r)$ [2,5,6], taking into account the Coulomb potential with the nuclear finite size $R_C = 1.25A^{1/3} = 3.15$ fm [22]. The spreading imaginary potential in Eq. (5), $\mathrm{Im}\,U_Y$, expresses complicated excited states via $\Xi^- N \to \Lambda\Lambda$ conversion processes in ${}^{16}_{\Xi^-}$C or ${}^{16}_{\Lambda\Lambda}$C above the ${}^{15}_{\Lambda\Lambda}\mathrm{C}+n$ threshold at 8.2 MeV, as a function of the excitation energy E_{ex} measured from an energy of the ${}^{16}_{\Lambda\Lambda}$C ground state, as often used in nuclear optical models. Since we have no criterion for a choice of W_Ξ or $W_\Xi^{(D)}$ in the limited experimental data, we adjust appropriately the strength parameter of W_Ξ in the WS-type to give widths of Ξ^- quasibound states in recent calculations [5,6,23]. In ^{14}C–$\Lambda\Lambda$ channels, we should use a ${}^{15}_{\Lambda}$C–Λ potential, which can be constructed in folded potential models [24]:

$$U_\Lambda(r) = \int \rho_{J''}(\boldsymbol{r}_\Lambda)\big[U_{C\Lambda}\big(|\boldsymbol{r}+\lambda_\Lambda \boldsymbol{r}_\Lambda|\big) + V_{\Lambda\Lambda}\big(|\boldsymbol{r}-\nu_\Lambda \boldsymbol{r}_\Lambda|\big)\big]\,d\boldsymbol{r}_\Lambda, \tag{6}$$

where $\rho_{J''}(\boldsymbol{r}_\Lambda) = \sum_{j_1 m_1}\langle JMj_1m_1|J''M''\rangle^2|\varphi^{(\Lambda)}_{j_1}(\boldsymbol{r}_\Lambda)|^2$ and $\lambda_\Lambda = 1-\nu_\Lambda = m_\Lambda/(M({}^{14}\mathrm{C})+m_\Lambda)$. $U_{C\Lambda}$ and $V_{\Lambda\Lambda}$ denote an optical potential for ^{14}C–Λ as given in Eq. (5) and a $\Lambda\Lambda$ residual interaction, respectively. Here we neglected $V_{\Lambda\Lambda}$ for simplicity. The real part of $U_{C\Lambda}$ leads to $B_\Lambda = 12.2$ MeV for the $(0s_\Lambda)$ state and $B_\Lambda = 1.6$ MeV for the $(0p_\Lambda)$ state in ${}^{15}_{\Lambda}$C [25], and its imaginary part exhibits a flux loss of the wave functions through the core excitations of ^{14}C*. We assume $W_\Lambda \simeq \frac{1}{4}W_N$ and $W_\Lambda^{(D)} \simeq \frac{1}{4}W_N^{(D)}$ where parameters of W_N and $W_N^{(D)}$ for nucleon were obtained in Ref. [26] because the well depth of the imaginary potential for Λ is by a factor of 4 weaker than that for nucleon in g-matrix calculations [27].

The $\Lambda\Lambda$–Ξ coupling potential U_X in off-diagonal parts of $\boldsymbol{U}$ is the most interesting object in this calculation [10–16]. It can be obtained by a two-body ΞN–$\Lambda\Lambda$ potential $v_{\Xi N,\Lambda\Lambda}(\boldsymbol{r}',\boldsymbol{r})$ with the ${}^1S_0$, isospin $T=0$ state. Here we use a zero-range interaction $v_{\Xi N,\Lambda\Lambda}(\boldsymbol{r}',\boldsymbol{r}) = v^0_{\Xi N,\Lambda\Lambda}\delta_{S,0}\delta(\boldsymbol{r}'-\boldsymbol{r})$ in a real potential for simplicity, where $v^0_{\Xi N,\Lambda\Lambda}$ is the strength parameter that should be connected with volume integral $\int v_{\Xi N,\Lambda\Lambda}(\boldsymbol{r})\,d\boldsymbol{r} = v^0_{\Xi N,\Lambda\Lambda}$ [13,14,16]. Thus the matrix elements can be easily estimated by use of Racah algebra [29]:

$$\begin{aligned} U_X(r) &= \Big\langle\big[\Phi_{J'}\big({}^{15}\mathrm{N}\big)\otimes\mathcal{Y}^{(\Xi^-)}_{j'\ell's'}(\hat{\boldsymbol{r}})\big]_{J_B}\Big|\sum_i v_{\Xi N,\Lambda\Lambda}\big(\nu_i\boldsymbol{r}'_i,\boldsymbol{r}\big) \\ &\quad\times\Big|\big[\big[\Phi_J\big({}^{14}\mathrm{C}\big),\varphi^{(\Lambda)}_{j_1}\big]_{J''}\otimes\mathcal{Y}^{(\Lambda)}_{j\ell s}(\hat{\boldsymbol{r}})\big]_{J_B}\Big\rangle \\ &= \sum_{LSK}\sqrt{1/2}\,v^0_{\Xi N,\Lambda\Lambda}\delta_{S,0}C^{J_B}_{LSK}\big(J'J''\big)\mathcal{F}^{J'J''}_{LSK}(r), \end{aligned} \tag{7}$$

where $\mathcal{Y}_{j\ell s} = [Y_\ell \otimes X_{\frac{1}{2}}]_j$ is a spin–orbit function and $C^{J_B}_{LSK}(J'J'')$ is a purely geometrical factor [29]; $\mathcal{F}^{J'J''}_{LSK}(r)$ is the nuclear form factor including a recoupling coefficient of $U(Jj_1J''K;J'j_p)$ [16], a parentage coefficient for proton removal from ${}^{15}\mathrm{N}(1/2^-,3/2^-)$ [30] and the center-of-mass correction of a factor $\sqrt{A/(A-1)}$ [31]. The factor $\sqrt{1/2}$ comes from the procedure handling a transition between $p\Xi^-$ and $\Lambda\Lambda$ states in the nucleus.

The inclusive K^+ double-differential laboratory cross section of the $\Lambda\Lambda$–Ξ production in the nuclear (K^-,K^+) reaction can be written within the DWIA [32,33] using the Green's function method [19]. In the one-step mechanism, $K^-p\to K^+\Xi^-$ via Ξ^- doorways, it is given [21] as

$$\left(\frac{d^2\sigma}{d\Omega_K dE_K}\right)_{\mathrm{lab}} = \beta\frac{1}{[J_A]}\sum_{M_z}\sum_{\alpha'\alpha}\left(-\frac{1}{\pi}\right) \times \mathrm{Im}\left[\int d\boldsymbol{r}'\,d\boldsymbol{r}\,F^{\alpha'\dagger}_\Xi(\boldsymbol{r}')G^{\alpha'\alpha}_\Xi(\omega,\boldsymbol{r}',\boldsymbol{r})F^\alpha_\Xi(\boldsymbol{r})\right] \tag{8}$$

for the target with a spin J_A and its z-component M_z, where $[J_A] = 2J_A+1$, and a kinematical factor β [34] that expresses the translation from the two-body K^-–p laboratory system to the K^-–^{16}O laboratory system [2]. The production amplitude F^α_Ξ is

$$F^\alpha_\Xi(\boldsymbol{r}) = \bar{f}_{K^-p\to K^+\Xi^-}\chi^{(-)*}_{\boldsymbol{p}_{K^+}}\left(\frac{M_C}{M_B}\boldsymbol{r}\right)\chi^{(+)}_{\boldsymbol{p}_{K^-}}\left(\frac{M_C}{M_A}\boldsymbol{r}\right) \times\langle\alpha|\hat{\psi}_p(\boldsymbol{r})|\Psi_{J_AM_z}\rangle, \tag{9}$$

where $\bar{f}_{K^-p\to K^+\Xi^-}$ is a Fermi-averaged amplitude for the $K^-p\to K^+\Xi^-$ reaction in nuclear medium [2], and $\chi^{(-)}_{\boldsymbol{p}_{K^+}}$ and $\chi^{(+)}_{\boldsymbol{p}_{K^-}}$ are the distorted waves for outgoing K^+ and incoming K^- mesons, respectively; the factors of M_C/M_B and M_C/M_A take into account the recoil effects, where M_A, M_B and M_C are masses of the target, the final state and the core-nucleus, respectively. $\langle\alpha|\hat{\psi}_p|\Psi_{J_AM_z}\rangle$ is a hole-state wave function for a struck proton in the target, where α denotes the complete set of eigenstates for the system. It should be recognized that the $\Lambda\Lambda$–Ξ coupled-channel Green's function with the spreading potential provides an advantage of estimating contributions from sources both as $\Lambda\Lambda$ components in Ξ^-–nucleus eigenstates [16] and as $\Xi^-p\to\Lambda\Lambda$ quasi-scattering processes in the nucleus [15].

Because the momentum transfer is very high in the nuclear (K^-,K^+) reaction at 1.8 GeV/c, i.e., $q_{\Xi^-}\simeq 360$–430 MeV/c, the distorted waves for outgoing K^+ and incoming K^- in Eq. (9) are calculated with the help of the eikonal approximation [32,35]. As the distortion parameters, we use total cross sections of $\sigma_{K^-N} = 28.9$ mb for K^-N scattering and $\sigma_{K^+N} = 19.4$ mb for K^+N scattering [6], and $\alpha_{K^-N} = \alpha_{K^+N} = 0$. We take 35 μb/sr as the laboratory cross section of $d\sigma/d\Omega = \bar{\alpha}|\bar{f}_{K^-p\to K^+\Xi^-}|^2$ including the kinematical factor $\bar{\alpha}$ [9,5]. For the target nucleus ^{16}O with $J^\pi_A = 0^+$, we assume the wave functions for the proton hole-states in the relative coordinate, which are calculated with central (WS-type) and spin–orbit potentials [22], by fitting to the charge rms radius of 2.72 fm [36]. For the energies (widths) for proton-hole states, we input 12.1 (0.0), 18.4 (2.5) and 36 (10) MeV for $0p^{-1}_{1/2}$, $0p^{-1}_{3/2}$ and $0s^{-1}_{1/2}$ states, respectively.

Three parameters, V_Ξ, W_Ξ and $v^0_{\Xi N,\Lambda\Lambda}$, are very important for calculating the inclusive spectra with the one-step mechanism. These parameters are strongly connected each other for the shape of the spectrum and its magnitude, as well as for the Ξ^- binding energies and widths of the Ξ^- states. Several authors [10,16,14] investigated the effects of the ΞN–$\Lambda\Lambda$ coupling in light nuclei evaluating the volume integrals for k_F-dependent ΞN–$\Lambda\Lambda$ effective interactions based on Nijmegen potentials [28], in which these values are strongly model dependent; for example, 250.9, 370.2, 501.5, 582.1 and 873.9 MeV fm^3 for NHC-D, NSC97e, NSC04a, NHC-F and NSC04d potentials ($k_F = 1.0$ fm^{-1}), respectively [14,28]. The $\Xi^-p\to\Lambda\Lambda$ conversion cross section of $(v\sigma)_{\Xi^-p\to\Lambda\Lambda}\simeq 7.9$ mb also yields to be about 544 MeV fm^3 [16]. To see the dependence of the spectrum on the ΞN–$\Lambda\Lambda$ coupling strength, here, we choose typical values of $v^0_{\Xi N,\Lambda\Lambda} = 250$ and 500 MeV, which approximate the volume integrals of NHC-D and NSC04a, respectively. We take the spreading potential of $\mathrm{Im}\,U_\Xi$ to be $W_\Xi\simeq -3$ MeV at the ${}^{15}\mathrm{N}+\Xi^-$ threshold [5,6,14,18]. It should be noticed that this spreading potential expresses nuclear core breakup processes

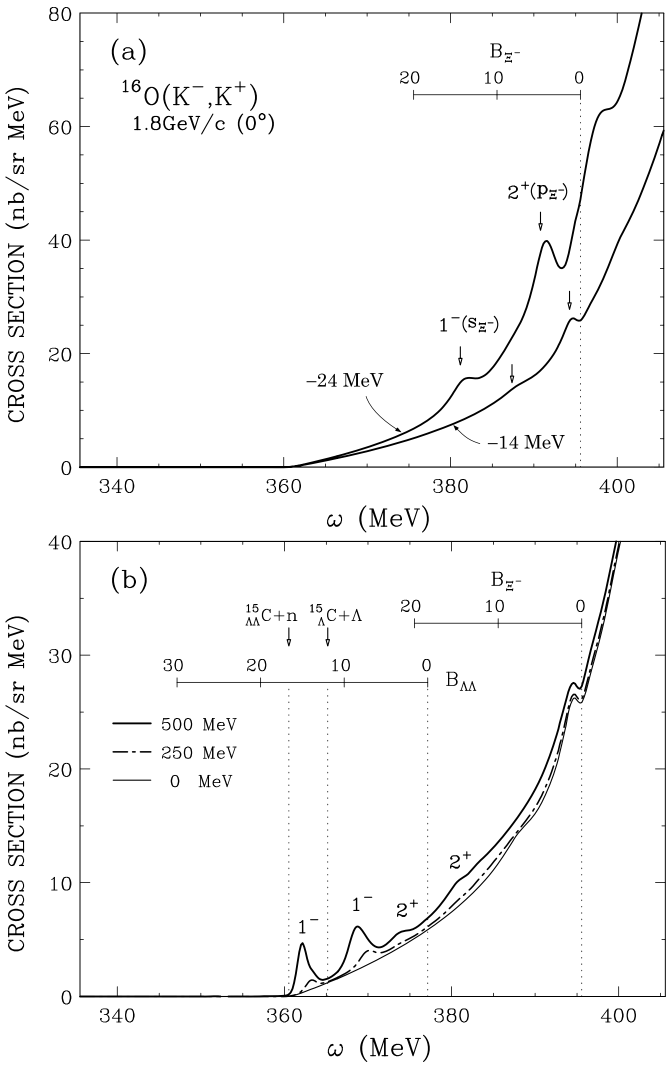

Fig. 2. Calculated inclusive $\Lambda\Lambda$–Ξ spectra by the one-step mechanism in the ^{16}O(K^-, K^+) reaction at 1.8 GeV/c (0°), with a detector resolution of 1.5 MeV FWHM; (a) $V_\Xi = -24$ or -14 MeV without the $\Lambda\Lambda$–Ξ coupling potential. The Ξ conversion decay occurs above the $^{15}_{\Lambda\Lambda}$C$+n$ threshold at $\omega = 360.4$ MeV; (b) $V_\Xi = -14$ MeV with the $\Lambda\Lambda$–Ξ coupling potential obtained by $v^0_{\Xi N,\Lambda\Lambda} = 0$, 250 and 500 MeV.

caused by the $\Xi^- p \to \Lambda\Lambda$ conversion in the ^{15}N nucleus, and its effect cannot be involved in U_X.

3. Results and discussion

Now let us discuss the inclusive spectrum in the ^{16}O(K^-, K^+) reaction at 1.8 GeV/c (0°) in order to examine the dependence of the spectrum on the parameters of V_Ξ and $v^0_{\Xi N,\Lambda\Lambda}$. We consider contributions of the $\Lambda\Lambda$–Ξ nuclear bound and resonance states to the $\Xi^- p \to \Lambda\Lambda$ conversion processes in the Ξ^- bound region.

In Fig. 2(a), we show the calculated spectra in the Ξ^- bound region without the $\Lambda\Lambda$–Ξ coupling potential when we use $V_\Xi = -24$ MeV or -14 MeV with the Coulomb potential. The calculated spectra are in agreement with the spectra obtained by previous works [6]. In the case of $V_\Xi = -24$ MeV, we find that a broad peak of the $[^{15}\text{N}(1/2^-) \otimes s_{\Xi^-}]_{1^-}$ quasibound state in $^{16}_{\Xi^-}$C is located at $B_{\Xi^-} = 13.4$ MeV with a sizable width of $\Gamma = 3.5$ MeV, and a clear peak of the $[^{15}\text{N}(1/2^-) \otimes p_{\Xi^-}]_{2^+}$ quasibound state at $B_{\Xi^-} = 3.7$ MeV with $\Gamma = 3.1$ MeV. Integrated cross sections indicate $d\sigma(0°)/d\Omega \simeq 28$ nb/sr for the 1^- state and 77 nb/sr for the 2^+ state in $^{16}_{\Xi^-}$C. In the case of $V_\Xi = -14$ MeV, which is favored in recent calculations [6,13,14,18], we have the $[^{15}\text{N}(1/2^-) \otimes s_{\Xi^-}]_{1^-}$ state at $B_{\Xi^-} = 6.8$ MeV with $\Gamma = 3.8$ MeV and the $[^{15}\text{N}(1/2^-) \otimes p_{\Xi^-}]_{2^+}$ at $B_{\Xi^-} = 0.5$ MeV with $\Gamma = 1.1$ MeV. The integrated cross sections indicate $d\sigma(0°)/d\Omega \simeq 6$ nb/sr for the 1^- state and 9 nb/sr for the 2^+ state. Note that the $\Xi^- p \to \Lambda\Lambda$ conversion processes that can be described by the absorption potential $\text{Im}\, U_\Xi$, must appear above the $^{15}_{\Lambda\Lambda}$C$+n$ decay threshold at $\omega = 360.4$ MeV (which corresponds to $B_{\Lambda\Lambda} = 16.7$ MeV). We confirm that no clear signal of the Ξ^- bound state is measured if V_Ξ is sallow such as $-V_\Xi \leqslant 14$ MeV and/or W_Ξ is sizably absorptive ($-W_\Xi \geqslant 3$ MeV at the $^{15}\text{N} + \Xi^-$ threshold) in U_Ξ. Nevertheless, the production of these Ξ^- states as well as Ξ^- states coupled to a ^{15}N($3/2^-$) nucleus is essential in this model because these states act as doorways when we consider the $\Lambda\Lambda$ states formed in the one-step mechanism. We also expect to extract properties of the Ξ–nucleus potential such as V_Ξ and W_Ξ from the Ξ^- continuum spectra in the (K^-, K^+) reactions on nuclear targets, as already discussed for studies of the Σ^-–nucleus potential in nuclear (π^-, K^+) reactions [37,38].

On the other hand, the $\Lambda\Lambda$–Ξ coupling plays an important role in making a production of the $\Lambda\Lambda$ states via Ξ^- doorways below the $^{15}\text{N} + \Xi^-$ threshold. The positions of their peaks must be slightly shifted downward by the energy shifts $\Delta B_{\Lambda\Lambda}$ due to the coupling potential in Eq. (7). When $v^0_{\Xi N,\Lambda\Lambda} = 500$ MeV (250 MeV), we obtain $\Delta B_{\Lambda\Lambda} = 1.17$ MeV (0.15 MeV) and the Ξ^- admixture probability $P_{\Xi^-} = 5.24\%$ (0.87%) in the $[^{14}\text{C}(0^+) \otimes s_\Lambda p_\Lambda]_{1^-}$ excited state and $\Delta B_{\Lambda\Lambda} = 0.38$ MeV (0.09 MeV) and $P_{\Xi^-} = 0.58\%$ (0.14%) in the $[^{14}\text{C}(0^+) \otimes s^2_\Lambda]_{0^+}$ ground state. The value of P_{Ξ^-} in the 1^- state is by a factor of 6–9 as large as that in the 0^+ state. These values are strongly connected with the magnitude of the peak for the $\Lambda\Lambda$ state in the spectrum.

In Fig. 2(b), we show the calculated spectra with the $\Lambda\Lambda$–Ξ coupling potential when $V_\Xi = -14$ MeV. We recognize that the shape of these spectra is quite sensitive to the value of $v^0_{\Xi N,\Lambda\Lambda}$, and it is obvious that no ΞN–$\Lambda\Lambda$ coupling cannot describe the spectrum of the $\Lambda\Lambda$ states below the $^{14}\text{C} + \Lambda + \Lambda$ threshold. The calculated spectrum for $v^0_{\Xi N,\Lambda\Lambda} = 500$ MeV has a fine structure of the $\Lambda\Lambda$ excited states in $^{16}_{\Lambda\Lambda}$C. We find that significant peaks of the 1^- excited states with $^{14}\text{C}(0^+) \otimes s_\Lambda p_\Lambda$ at $\omega = 362.1$ MeV ($B_{\Lambda\Lambda} = 15.1$ MeV) and $^{14}\text{C}(2^+) \otimes s_\Lambda p_\Lambda$ at $\omega = 368.5$ MeV ($B_{\Lambda\Lambda} = 8.7$ MeV), and small peaks of the 2^+ excited states with $^{14}\text{C}(0^+) \otimes p^2_\Lambda$ at $\omega = 373.8$ MeV ($B_{\Lambda\Lambda} = 3.4$ MeV) and $^{14}\text{C}(2^+) \otimes p^2_\Lambda$ at $\omega = 380.4$ MeV ($B_{\Lambda\Lambda} = -3.2$ MeV). This result arises from the fact that the high momentum transfer $q_{\Xi^-} \simeq 400$ MeV/c leads to a preferential population of the spin-stretched Ξ^- doorways states followed by the $[^{15}\text{N}(1/2^-, 3/2^-) \otimes s_{\Xi^-}]_{1^-} \to [^{14}\text{C}(0^+, 2^+) \otimes s_\Lambda p_\Lambda]_{1^-}$ and $[^{15}\text{N}(1/2^-, 3/2^-) \otimes p_{\Xi^-}]_{2^+} \to [^{14}\text{C}(0^+, 2^+) \otimes p^2_\Lambda]_{2^+}$ transitions, to which a sum of their continuum states may contribute predominately in the (K^-, K^+) reactions. Fig. 3 also displays partial-wave decomposition of the calculated inclusive spectrum for $^{16}_{\Lambda\Lambda}$C in the $\Lambda\Lambda$ bound region when $V_\Xi = -14$ MeV and $v^0_{\Xi N,\Lambda\Lambda} = 500$ MeV. The integrated cross sections at $\theta_{\text{lab}} = 0°$ for the 1^- excited states with $^{14}\text{C}(0^+) \otimes s_\Lambda p_\Lambda$ and $^{14}\text{C}(2^+) \otimes s_\Lambda p_\Lambda$ are respectively

$$\frac{d\sigma}{d\Omega_L}\left[{}^{16}_{\Lambda\Lambda}\text{C}(1^-)\right] \simeq 7 \text{ nb/sr and } 12 \text{ nb/sr}, \tag{10}$$

where the Ξ^- admixture probabilities of these states amount to $P_{\Xi^-} = 5.2\%$ and 8.8%, respectively. It should be noticed that the cross sections are on the same order of magnitude as those for the 1^- and 2^+ quasibound states that are located at $B_{\Xi^-} = 6.8$ MeV and 0.5 MeV, respectively, in the $^{16}_{\Xi^-}$C hypernucleus. Therefore,

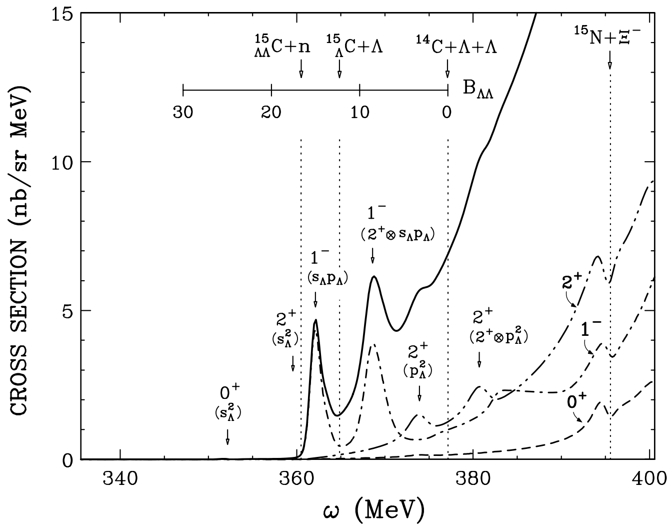

Fig. 3. Partial-wave decomposition of the calculated inclusive spectrum by the one-step mechanism near the $_{14}\mathrm{C} + \Lambda + \Lambda$ threshold in the $^{16}\mathrm{O}(K^-, K^+)$ reaction at 1.8 GeV/c (0°). $V_\Xi = -14$ MeV and $v^0_{\Xi N,\Lambda\Lambda} = 500$ MeV were used. The labels $0^+(s^2_\Lambda)$, $1^-(s_\Lambda p_\Lambda)$ and $2^+(p^2_\Lambda)$ denote the J^π $\Lambda\Lambda$ nuclear states of $(0s_\Lambda)^2$, $(0s_\Lambda)(0p_\Lambda)$ and $(0p_\Lambda)^2$ coupled with $^{14}\mathrm{C}(0^+)$, respectively. The labels $2^+(s^2_\Lambda)$, $1^-(2^+ \otimes s_\Lambda p_\Lambda)$ and $2^+(2^+ \otimes p^2_\Lambda)$ denote the states of $(0s_\Lambda)^2$, $(0s_\Lambda)(0p_\Lambda)$ and $(0p_\Lambda)^2$ coupled with $^{14}\mathrm{C}(2^+)$, respectively.

such $\Lambda\Lambda$ excited states below the $^{14}\mathrm{C} + \Lambda + \Lambda$ threshold will be measured experimentally at the J-PARC facilities [3].

On the other hand, it is extremely difficult to populate the 0^+ ground state with $^{14}\mathrm{C}(0^+) \otimes s^2_\Lambda$ at $\omega \simeq 352.3$ MeV ($B_{\Lambda\Lambda} \simeq 24.9$ MeV) and also the 2^+ excited state with $^{14}\mathrm{C}(2^+) \otimes s^2_\Lambda$ at $\omega \simeq 359.6$ MeV ($B_{\Lambda\Lambda} \simeq 17.5$ MeV) in the one-step mechanism via Ξ^- doorways in the (K^-, K^+) reactions. The high momentum transfer of $q_\Xi \simeq 400$ MeV/c necessarily leads to the non-observability with $\Delta L = 0$. Thus the integrated cross section of the 0^+ state is found to be about 0.02 nb/sr, of which the q dependence is approximately governed by a factor of $\exp(-\frac{1}{2}(\tilde{b}q_\Xi)^2)$ where a size parameter $\tilde{b} = 1.84$ fm. There is no production in the 2^+ state with $^{14}\mathrm{C}(2^+) \otimes s^2_\Lambda$ under the angular-momentum conservation in the $^{16}\mathrm{O}(K^-, K^+)$ reactions by the one-step mechanism. The contribution of these states to the $\Lambda\Lambda$ spectrum in the one-step mechanism is completely different from that in the two-step mechanism as obtained in Refs. [7,8].

In the (K^-, K^+) reaction, $\Lambda\Lambda$ hypernuclear states can be also populated by the two-step mechanism, $K^- p \to \pi^0 \Lambda$ followed by $\pi^0 p \to K^+ \Lambda$ [7–9], as shown in Fig. 1(a). Following the procedure by Dover [7,9], a crude estimate can be obtained for the contribution of this two-step processes in the eikonal approximation using a harmonic oscillator model. The cross section at 0° for quasielastic $\Lambda\Lambda$ production at $p_{K^-} = 1.8$ GeV/c in the two-step mechanism, which is summed over all final state, is given [9] as

$$\sum_f \left(\frac{d\sigma_f^{(2)}}{d\Omega_L}\right)_{0^\circ} \approx \frac{2\pi\xi}{p_\pi^2} \left\langle \frac{1}{r^2} \right\rangle \left(\alpha \frac{d\sigma}{d\Omega_L}\right)_{0^\circ}^{K^- p \to \pi^0 \Lambda} \times \left(\alpha \frac{d\sigma}{d\Omega_L}\right)_{0^\circ}^{\pi^0 p \to K^+ \Lambda} N_{\mathrm{eff}}^{pp}, \tag{11}$$

where $\xi = 0.022$–0.019 mb^{-1} is a constant nature of the angular distributions of the two elementary processes, $p_\pi \simeq 1.68$ GeV/c is the intermediate pion momentum, and $\langle 1/r^2 \rangle \simeq 0.028$ mb^{-1} is the mean inverse-square radial separation of the proton pair. $N^{pp}_{\mathrm{eff}} \simeq 1$ is the effective number of proton pairs including the nuclear distortion effects [7]. The elementary laboratory cross section $(\alpha d\sigma/d\Omega_L)_{0^\circ}$ is estimated to be 1.57–1.26 mb/sr for $K^- p \to \pi^0 \Lambda$ and 0.070–0.067 mb/sr for $\pi^0 p \to K^+ \Lambda$ depending on the nuclear medium corrections. This yields

$$\sum_f \left(\frac{d\sigma_f^{(2)}}{d\Omega_L}\right)_{0^\circ} \simeq 0.06\text{–}0.04\ \mu\mathrm{b/sr}, \tag{12}$$

which is half smaller than ~ 0.14 μb/sr at 1.1 GeV/c. Considering a high momentum transfer $q \simeq 400$ MeV/c in the (K^-, K^+) reactions by comparison with the (π^+, K^+) reaction [39], we expect that the production probability for the $\Lambda\Lambda$ bound states does not exceed 1% in the quasielastic $\Lambda\Lambda$ production, so that an estimate of the $\Lambda\Lambda$ hypernucleus in the two-step mechanism may be on the order of 0.1–1 nb/sr. This cross section is smaller than the cross section for the $\Lambda\Lambda$ 1^- states we mentioned above in the one-step mechanism. Consequently, we believe that the one-step mechanism acts in a dominant process in the (K^-, K^+) reaction at 1.8 GeV/c (0°) when $v^0_{\Xi N,\Lambda\Lambda} = 400$–$600$ MeV. This implies that the (K^-, K^+) spectrum provides valuable information concerning ΞN–$\Lambda\Lambda$ dynamics in the $S = -2$ systems such as $\Lambda\Lambda$ and Ξ hypernuclei, which are often discussed in a full coupling scheme [40].

4. Summary and conclusion

We have examined theoretically production of doubly strange hypernuclei in the DCX $^{16}\mathrm{O}(K^-, K^+)$ reaction at 1.8 GeV/c within DWIA calculations using coupled-channel Green's functions. We have shown that the Ξ^- admixture in the $\Lambda\Lambda$ hypernuclei plays an essential role in producing the $\Lambda\Lambda$ states in the (K^-, K^+) reaction.

In conclusion, the calculated spectrum for the $^{16}_{\Xi^-}\mathrm{C}$ and $^{16}_{\Lambda\Lambda}\mathrm{C}$ hypernuclei in the one-step mechanism $K^- p \to K^+ \Xi^-$ via Ξ^- doorways predicts promising peaks of the $\Lambda\Lambda$ bound and excited states in the $^{16}\mathrm{O}(K^-, K^+)$ reactions at 1.8 GeV/c (0°). It has been shown that the integrated cross sections for the significant 1^- excited states in $^{16}_{\Lambda\Lambda}\mathrm{C}$ are on the order of 7–12 nb/sr depending on the ΞN–$\Lambda\Lambda$ coupling strength and also the attraction in the Ξ–nucleus potential. The Ξ^- admixture probabilities are on the order of 5–9%. The sensitivity to the potential parameters indicates that the nuclear (K^-, K^+) reactions have a high ability for the theoretical analysis of precise wave functions in the $\Lambda\Lambda$ and Ξ hypernuclei. New information on $\Lambda\Lambda$–Ξ dynamics in nuclei from the (K^-, K^+) data at J-PARC facilities [3] will bring the $S = -2$ world development in nuclear physics.

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