Production of doubly strange hypernuclei via $\Xi^-$ doorways in the $^{16}\text{O}(K^-, K^+)$ reaction at 1.8 GeV/c

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We examine theoretically production of doubly strange hypernuclei, $^{16}\text{C}$ and $^{16}\text{C}_{\Lambda\Lambda}$, in double-charge exchange $^{16}\text{O}(K^-, K^+)$ reactions using a distorted-wave impulse approximation. The inclusive $K^+$ spectrum at the incident momentum $p_{K^+} = 1.8$ GeV/c and scattering angle $\theta_{lab} = 0^\circ$ is estimated in a one-step mechanism, $K^- p \rightarrow K^+ \Xi^- \rightarrow \Xi^- \rightarrow \Xi^- \Lambda\Lambda$ via $\Xi^-$ doorways caused by a $\Xi^-$–$\Lambda\Lambda$ coupling. The calculated spectrum in the $\Xi^-$ bound region indicates that the integrated cross sections are on the order of $7$–$12$ nb/sr for significant $1^- \Xi^- \Lambda\Lambda$ excited states with $14C(0^+;2^+) \otimes s_2 P_A$ configurations in $^{16}\text{C}_{\Lambda\Lambda}$ via the doorway states of the spin-stretched $^{15}\text{N}(1/2^-; 3/2^-) \otimes s_2 -$ in $^{16}\text{C}$ due to a high momentum transfer $q_\Xi^- \approx 400$ MeV/c. The $\Xi^-$ admixture probabilities of these states are on the order of $8$–$9\%$. However, populations of the $0^+$ ground state with $14C(0^+;2^+) \otimes s_2^2$ and the $2^+$ excited state with $14C(2^+;2^+) \otimes s_2^2$ are very small. The sensitivity of the spectrum on the $\Xi N$–$\Lambda\Lambda$ coupling strength enables us to extract the nature of $\Xi N$–$\Lambda\Lambda$ dynamics in nuclei, and the nuclear $(K^-, K^+)$ reaction can extend our knowledge of the $S = -2$ world.

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1. Introduction

It is important to understand properties of $\Xi$ hypernuclei whose states are regarded as “doorways” to access multi-strangeness systems as well as a two-body $\Xi N$–$\Lambda\Lambda$ system, and it is a significant step to extend study of strange nuclear matter in hadron physics and astrophysics [1]. Because the $\Xi$ hyperon in nuclei has to undergo a strong $\Xi N$–$\Lambda\Lambda$ decay, widths of $\Xi$ hypernuclear states give us a clue to a mechanism of $\Xi$ absorption processes in nuclei. A pioneer study of $\Xi$ hypernuclei by Dover and Gal [2] has found that a $\Xi$–nucleus potential has a well depth of $24 \pm 4$ MeV in the real part on the analysis of old emulsion data. However, our knowledge of these $\Xi$–nucleus systems is very limited due to the lack of the experimental data [3]. Indeed, the missing-mass spectra of a double-charge exchange (DCX) reaction $(K^-, K^+)$ on a $^{12}\text{C}$ target have suggested the $\Xi^-$ well depth of $14$–$16$ MeV [4,5]. Several authors [6] have used the unsettled $\Xi$–nucleus (optical) potentials such as $V_\Xi = -(24)$–$(-14)$ MeV and $W_\Xi = -(6)$–$(-3)$ MeV in the Woods–Saxon potential to demonstrate the $\Xi^-$ production spectra in the nuclear $(K^-, K^+)$ reactions. There remains a full uncertainty about the nature of doubly strange $(S = -2)$ dynamics caused by the $\Xi N$ and $\Xi N$–$\Lambda\Lambda$ interaction in nuclei at the present stage. More experimental information is earnestly desired.

The $(K^-, K^+)$ reaction is one of the most promising ways of studying double strange systems such as $\Xi$ hypernuclei for the forthcoming J-PARC experiments [3]. One expects that these experiments will confirm the existence of $\Xi$ hypernuclei and establish properties of the $\Xi$–nucleus potential, e.g., binding energies and widths. This reaction can also populate a $\Lambda\Lambda$ hypernucleus through a conventional DCX two-step mechanism as $K^- p \rightarrow \pi^0\Lambda\Lambda$ followed by $\pi^0 p \rightarrow K^+\Lambda$ [7–9], as shown in Fig. 1(a). Such an inclusive $K^-$ spectrum in the $\Lambda\Lambda$ bound region is rather clean with much less background experimentally. Early theoretical predictions for two-step $^{16}\text{O}(K^-, K^+)$ reactions at the incident momentum $p_{K^+} = 1.1$ GeV/c and scattering angle $\theta_{lab} = 0^\circ$ [7,8] have indicated small cross sections for the $\Lambda\Lambda$ states, for example, $\sim 0.1$ nb/sr for the $0^+(s_2^2)\Lambda\Lambda$ ground state and $\sim 2$ nb/sr for the $2^+(s_2^2)\Lambda\Lambda$ excited state in $^{12}\text{C}$ when we took $0.61$ mb/sr and $0.32$ mb/sr as the laboratory cross sections at $0^\circ$, respectively. It should be noticed that another exotic production of $\Lambda\Lambda$ hypernuclei in the $(K^-, K^+)$ reactions is a one-step mechanism, $K^- p \rightarrow K^+\Xi^- \rightarrow \Xi^- \Lambda\Lambda$ via $\Xi^-$ doorways caused by a $\Xi^-$–$\Lambda\Lambda$ tran-
sition, as shown in Fig. 1(b). The $ΞN$–$ΛΛ$ coupling induces the $Ξ−$ admixture and the $ΛΛ$ energy shift $ΔE_{ΛΛ} = E_{ΛΛ}(1Σ2Z) − 2E_{Λ}(1Σ2Z)$ in the $ΛΛ$-nuclear states $[10–14]$, and its coupling strength is also related to widths of $Ξ$-hypernuclei states $[15, 16]$. For a viewpoint of $S = −2$ studies, it is very important to extract quantitative information concerning the $ΞN$–$ΛΛ$ coupling from spectroscopy of the $Ξ$ and $ΛΛ$ hypernuclei $[17,18]$. In this Letter, we study theoretically production of a doubly strange hypernucleus in the DCX $(K^+, K^−)$ reaction on an $^{16}$O target at $p_K = 1.8$ GeV/c and $θ_{40} = 0°$ within a distorted-wave impulse approximation (DWIA). Thus we focus on the $ΛΛ$–$Ξ$ spectrum for $^{16}$A and $^{15}$C in the $Ξ−$ bound region considering the one-step mechanism, $K^+ p → K^+ Ξ−$ via $Ξ−$ doorways caused by the $ΞN$–$ΛΛ$ coupling in the nuclear $(K^+, K^−)$ reaction, rather than the two-step mechanism as $K^− p → Λ0 Ξ−$ followed by $π^0 p → K^+ Λ$ $[7,8]$. These different mechanisms are well separated kinematically. The forward cross section for the $K^− p → K^+ Ξ−$ elementary process is at its maximum at $p_K = 1.8–1.9$ GeV/c, whereas the $K^− p → Λ0 Ξ−$ reaction at $p_K = 1.1$ GeV/c leads to the maximal cross section for the $π^0 p → K^+ Λ$ process. The present study is the first attempt to evaluate a production spectrum of the $ΛΛ$–$Ξ$ hypernucleus via the $ΞN$–$ΛΛ$ coupling from the inclusive $(K^+, K^−)$ reaction, and to extract the $Ξ−$ admixture probability in the $ΛΛ$ hypernucleus from the spectrum. We also discuss a contribution of the two-step processes in the $(K^+, K^−)$ reactions within the eikonal approximation.

2. Calculations

Let us consider the DCX $(K^+, K^−)$ reaction on the $^{16}$O target at 1.8 GeV/c within a DWIA and examine the production cross sections and wave functions of the doubly strange hypernucleus. To fully describe the one-step process via $Ξ−$ doorways, as shown in Fig. 1(b), we perform nuclear $ΛΛ$–$Ξ$ coupled-channel calculations $[13,14]$, which are assumed to effectively represent the coupling nature in omitting other $ΛΣ$ and $ΣΣ$ channels for simplicity. Here we employ a multichannel coupled wave function of the $ΛΛ$–$Ξ$ nuclear state for a total spin $J_B$ within a weak coupling basis. It is written as

$$\begin{align*}
|Ψ_{J_B}(A, 15C)⟩ &= \sum_{J'J_jj_j} \left[ (Φ_{J_j}^{14C}, Ψ_{j_j}^{14A}(r_{J_j})) J'_J, Ψ_{j_j}^{14A}(r_{J_j}) \right] |J_B⟩ \\
&+ \sum_{J'J_jj_j} \left[ Φ_{J_j}^{15N}, Ψ_{j_j}^{Ξ−}(r_{Ξ}) \right] |J_B⟩ \tag{1}
\end{align*}$$

with $Φ_{J_j}^{15N} = A[Φ_{J_j}^{14C}, Ψ_{J_j}^{15N}(r_{J_j})]$, where $r_{J_j}$ denotes the relative coordinate between the $^{14}$C core-nucleus and the $Λ$ (proton), and $r_{J_j}$ $(r_{Ξ})$ denotes the relative coordinate between the center of mass of the $^{14}$C–$Λ$ $(^{15}$N) subsystem and the $Λ$ ($Ξ$). Thus $Ψ_{J_jj_j}^{14A}$ and $Ψ_{J_jj_j}^{15N}$ describe the relative wave functions of shell model states (that occupy $J_1$, $J_2$ and $J_j$ orbits) for the $Λ$, $Ξ$–$Ξ$ and proton, respectively; $Φ_{J_j}^{14C}$ is a wave function of the $^{14}$C core-nucleus state, and $A$ is the antisymmetrized operator for nucleons. The energy difference between $^{15}$N–$Ξ−$ and $^{14}$C+$Λ + Λ$ channels is $ΔM = M(^{15}$N) + $m_{Ξ−} - M(^{14}$C) - $2m_{Λ} = 18.4$ MeV, where $M(^{15}$N), $M(^{14}$C), $m_{Ξ−}$ and $m_{Λ}$ are masses of the $^{15}$N nucleus, the $^{14}$C nucleus, the $Ξ−$ and $Λ$ hyperons, respectively. We take the $^{15}$N core-nucleus states with $j^P = 1/2^−$ (g.s.) and $3/2^−$ (6.32 MeV), and the $^{14}$C core-nucleus states with $j^P = 0^+$ (g.s.) and $2^+$ (7.01 MeV) that are given in $(0p_{00}^1p_{00}^1, 0p_{00}^10p_{00}^1)$ and $(0p_{00}^10p_{00}^1)$ configurations on $^{16}$O(g.s.) $[7,8]$. Because we assume only natural-parity $π = (−1)^j$ states via $Ξ−$ doorways that are selectively formed by non-spin-flip processes in the forward $K^− p → K^+ Ξ−$ reaction, we consider a spin $S = 0$, $ΛΛ$ pair in the hypernucleus. If the $ΛΛ$ component is dominant in a bound state, we can identify it as a state of the $ΛΛ$ hypernucleus $^{15}$C, in which the $Ξ−$ admixture probability can be estimated by

$$P_{Ξ−} = \sum_{j_jj_j} \langle Φ_{j_jj_j}^{15N} | Ψ_{j_jj_j}^{14A}(r_{J_j}) \rangle.$$

under the normalization of

$$\sum_{j_jj_j} \langle Φ_{j_jj_j}^{15N} | Ψ_{j_jj_j}^{14A}(r_{J_j}) \rangle \sum_{j_jj_j} \langle Ψ_{j_jj_j}^{15N} | Ψ_{j_jj_j}^{14A}(r_{J_j}) \rangle = 1.$$

After we set up the $^{15}$C and $^{15}$N configurations in our model space with Eq. (1), we calculate the wave functions of $Ψ_{j_jj_j}^{14A}(r_{J_j})$ and $Ψ_{j_jj_j}^{15N}(r_{Ξ})$ taking into account their channel coupling. Thus, the complete Green’s function $G(ω)$ $[19]$ describes all information concerning $(^{15}$C+$Λ)$ + $(^{15}$N+$Ξ−)$ coupled-channel dynamics, as a function of the energy transfer $ω$. It is numerically obtained as a solution of the $N$-channels radial coupled equations with a hyperon–nucleus potential $U$ $[20,21]$, which is written in an abbreviated notation as

$$G(ω) = G^{(0)}(ω) + G^{(0)}(ω)UG(ω) \tag{3}$$

with

$$G(ω) = \left( \begin{array}{cc}
G_A(ω) & G_X(ω) \\
G_X(ω) & G_{ΣΣ}(ω)
\end{array} \right), \quad U = \left( \begin{array}{cc}
U_A & U_X \\
U_X & U_{ΣΣ}
\end{array} \right),$$

where $G^{(0)}(ω)$ is a free Green’s function. In our calculations, for example, we deal with $N = 28$ for the $J^P = 1^−$ state. The nuclear optical potentials $U_Y (Y = Σ$ or $Λ)$ can be written as

$$U_Y(r) = V_Y f(r, R, a) + iW_Y f(r, R, a′) + iW_Y^0 g(r, R, a′). \tag{5}$$

where $f$ is the Woods–Saxon (WS) form, $f(r, R, a) = [1 + \exp(r − R/a)]^{−1}$, and $g$ is the derivative of the WS form, $g(r, R, a') = -4a'/(dr/dr)f(r, R, a')$. The spin–orbit potentials are neglected. In $^{15}$N–$Ξ−$ channels, we assume the strength parameter of $V_Z =...
obtained by a two-body interactions [27]. The most interesting object in this calculation [10–16]. It can denote an optical potential for $^{14}$C–Λ as given [21] as

\[ U_{Λ}(r) = \int \rho_{J^p}(r_{Λ}) |U_{CA}(r + \lambda_{Λ}r_{Λ})| \, dr_{Λ}, \]  

(6)

where $\rho_{J^p}(r_{Λ}) = \sum_{i_{1/2}(JM)i_{1/2}^{|J^p|}} |\Psi_{J^p}(r_{Λ})|^2$ and $\lambda_{Λ} = 1 - \nu_{Λ}/(M_{^{14}C} + m_{Λ})$. $U_{CA}$ and $V_{ΛΛ}$ denote an optical potential for $^{15}$C–Λ as given in Eq. (5) and a ΛΛ residual interaction, respectively. Here we neglected $V_{ΛΛ}$ for simplicity. The real part of $U_{CA}$ leads to $B_{Λ} = 12.2$ MeV for the $(0s_{1/2})$ state and $B_{Λ} = 1.6$ MeV for the $(0p_{1/2})$ state in $^{15}$C [25], and its imaginary part exhibits a flux loss of the wave functions through the core excitations of $^{14}$C. We assume $W_{Λ} \approx 1/2W_{N}$ and $W_{D}^{(d)} \approx 1/2W_{N}^{(d)}$ where parameters of $W_{N}$ and $W_{D}^{(d)}$ for nucleon were obtained in Ref. [26] because the well depth of the imaginary potential for Λ is by a factor of 4 weaker than that for nucleon in g–matrix calculations [27].

The ΛΛ–Σ coupling potential $U_{X}$ in off-diagonal parts of $U$ is the most interesting object in this calculation [10–16]. It can be obtained by a two-body $S^{N}–ΛΛ$ potential $v_{S^{N},ΛΛ}(r', r)$ with the $1S_0$, isospin T = 0 state. Here we use a zero-range interaction $v_{S^{N},ΛΛ}(r', r) = v_{S^{N},ΛΛ}^{0} \delta_{S0}(r' - r)$ in a real potential for simplicity, where $v_{S^{N},ΛΛ}^{0}$ is the strength parameter that should be connected with volume integral $\int v_{S^{N},ΛΛ}(r') \, dr' = v_{S^{N},ΛΛ}^{0}$ [13,14,16]. Then the matrix elements can be easily estimated by use of Racah algebra [29]:

\[ U_{X}(r) = \left[ \left( \Phi_{J_{f}^{τ}(15N)} \otimes \left\langle j_{f}^{S} | \right)^{2} \right) \right]_{J_{b}} \int v_{S^{N},ΛΛ}(r_{f}, r') \times \left[ \left\langle \Phi_{J_{g}^{τ}C} | \right)^{2} \otimes \left\langle j_{f}^{S} | \right)^{2} \right]_{J_{b}} \]  

\[ = \sum_{S_{N},ΛΛ} \int \sqrt{\frac{2\tau}{\tau^{2}}} v_{S^{N},ΛΛ}^{0} \delta_{S_{N},ΛΛ} C_{S_{N}}^{0} (J_{f}^{τ}J_{g}^{τ}) F_{S_{N},ΛΛ}^{0} (r), \]  

(7)

where $\Psi_{j_{f}^{τ}} = [Y_{τ} \otimes X_{j_{f}^{τ}}]$ is a spin–orbit wave function and $C_{S_{N},ΛΛ}^{0} (J_{f}^{τ})$ is a purely geometrical factor [29]; $F_{S_{N},ΛΛ}^{0} (r)$ is a form factor. A parentage coefficient for proton removal from $^{15}$N(1/2–, 3/2–) [30] and the center-of-mass correction of a factor $\sqrt{A/(A - T)}$ [31]. The factor $\sqrt{\tau/2}$ comes from the procedure handling a transition between $p^{N}–ΛΛ$ states in the nucleus.

The inclusive $K^{-}p \rightarrow K^{+}Σ^{-}$ double-differential laboratory cross section of the $ΛΛ–Σ$ production in the nuclear $(K^{-}K^{+})$ reaction can be written within the DWIA [32,33] using the Green's function method [19]. In the one-step mechanism, $K^{-}p \rightarrow K^{+}Σ^{-}$ via $Σ^{-}$–doorways, it is given [21] as

\[ \left( \frac{d^2\sigma}{dΩ_{K}dE_{K}} \right)_{lab} = \frac{1}{|J_{A}|} \sum_{\alpha,\bar{α}} \left( \frac{1}{\pi} \right) \times \Im \left[ \int d r' \, d r \, \epsilon_{K}^{*} \epsilon_{K} (r') C_{S_{N},ΛΛ}^{0} (\alpha, \bar{α}, r') F_{S_{N},ΛΛ}^{0} (r) \right] \]  

(8)

for the target with a spin $J_{A}$ and its z-component $M_{z}$, where $|J_{A}| = 2J_{a} + 1$, and a kinematical factor $β$ [34] that expresses the translation from the two-body $K^{-}p$ laboratory system to the $K^{-}^{15}$O laboratory system [2]. The production amplitude $F_{S_{N},ΛΛ}^{0}$ is

\[ F_{S_{N},ΛΛ}^{0} (r) = \tilde{K}_{K^{-}p}^{\pm} Σ^{-}(\pm g_{p}, r') \frac{M_{C}}{M_{B}} \frac{M_{C}}{M_{A}} \]  

\[ \times \langle α|\bar{ψ}_{p}(r)|\bar{Ψ}_{J_{A}}^{0} M_{z} \rangle, \]  

(9)

where $\tilde{K}_{K^{-}p}^{\pm} Σ^{-}(\pm g_{p}, r')$ is a Fermi-averaged amplitude for the $K^{-}p \rightarrow K^{+}Σ^{-}$ reaction in nuclear medium [2], and $g_{p}^{(\pm)}$ and $g_{p}^{(0)}$ are the distorted waves for outgoing $K^{+}$ and incoming $K^{-}$ mesons, respectively; the factors of $M_{B}/M_{C}$ and $M_{C}/M_{A}$ take into account the recoil effects, where $M_{A}$, $M_{B}$ and $M_{C}$ are masses of the target, the final state and the core-nucleus, respectively. $\langle α|\bar{ψ}_{p}(r)|\bar{Ψ}_{J_{A}}^{0} M_{z} \rangle$ is a hole-state wave function for a struck proton in the target, where $α$ denotes the complete set of eigenstates for the system. It should be recognized that the $ΛΛ–Σ$ coupled-channel Green's function with the spreading potential provides an advantage of estimating contributions from sources both as $ΛΛ$ components in $Σ^{-}$–nucleus eigenstates [16] and as $Σ^{-}p$–$ΛΛ$ quasi-scattering processes in the nucleus [15].

Because the momentum transfer is very high in the nuclear $(K^{-}K^{+})$ reaction at 1.8 GeV/c, i.e., $q_{S} \sim 360–430$ MeV/c, the distorted waves for outgoing $K^{+}$ and incoming $K^{-}$ in Eq. (9) are calculated with the help of the eikonal approximation [32,35]. As the distortion parameters, we use total cross sections of $σ_{K^{-}N} = 28.9$ mb for $K^{-}N$ scattering and $σ_{K^{+}N} = 19.4$ mb for $K^{+}N$ scattering [6], and $α_{K^{-}N} = α_{K^{+}N} = 0$. We take $15$ μb/sr as the laboratory cross section of $dσ/dΩ = α_{K^{-}p}Σ^{-}$ including the kinematical factor $α$ [9,5]. For the target nucleus $^{15}$O with $J_{A} = 0^{+}$, we assume the wave functions for the proton hole-states in the relative coordinate, which are calculated with central (WS-type) and spin–orbit potentials [22], by fitting to the charge rms radius of 2.72 fm [36]. For the energies (widths) for proton–hole states, we input $12.1$ (0.0), $18.4$ (2.5) and $36$ (10) MeV for $2\sigma_{1/2}^{P}$, $3\sigma_{1/2}^{P}$ and $0\sigma_{1/2}^{P}$ states, respectively.

Three parameters, $V_{S_{N}}$, $W_{2}$ and $v_{S_{N},ΛΛ}^{0}$, are very important for calculating the inclusive spectra with the one-step mechanism. These parameters are strongly connected each other for the shape of the spectrum and its magnitude, as well as for the $Σ^{-}$–$ΛΛ$ binding energies and widths of the $Σ^{-}$ states. Several authors [10,16,14] investigated the effects of the $Σ^{-}K^{+}ΛΛ$ coupling in light nuclei evaluating the volume integrals for $K^{-}$-dependent $S^{N}–ΛΛ$ effective interactions based on Nijmegen potentials [28], in which these values are strongly model dependent; for example, 250,9, 370.2, 501.5, 582.1 and 873.9 MeVfm$^{3}$ for NHC-D, NSC97e, NSC44a, NHC-F and NSC04d potentials ($k_{t} = 1.0$ fm$^{-1}$), respectively [14,28]. The $Σ^{-}p$–$ΛΛ$ conversion cross section of (vo)$Σ^{-}p$–$ΛΛ$ ≃ 7.9 mg also yields to be about 544 MeVfm$^{3}$ [16]. To see the dependence of the spectrum on the $Σ^{-}–ΛΛ$ coupling strength, here, we choose typical values of $v_{S_{N},ΛΛ}^{0} = 250$ and 500 MeV, which approximate the volume integrals of NHC-D and NSC04a, respectively. We take the spreading potential of $\lambda U_{2}$ to be $W_{2} \sim –3$ MeV at the $^{15}$N+$Σ^{-}$ threshold [5,6,14,18]. It should be noticed that this spreading potential expresses nuclear core breakup processes.
caused by the $\Xi^-$ $p \to \Lambda\Lambda$ conversion in the $^{15}$N nucleus, and its effect cannot be invoked in $U_X$.

3. Results and discussion

Now let us discuss the inclusive spectrum in the $^{16}$O($K^- , K^+$) reaction at 1.8 GeV/c (0°) in order to examine the dependence of the spectrum on the parameters of $v_Z$ and $v_{SN,\Lambda\Lambda}$. We consider contributions of the $\Lambda\Lambda$-$\Xi$ nuclear bound and resonance states to the $\Xi^-$ $p \to \Lambda\Lambda$ conversion processes in the $\Xi^-$ bound region.

In Fig. 2(a), we show the calculated spectra in the $\Xi^-$ bound region without the $\Lambda\Lambda$-$\Xi$ coupling potential when we use $v_Z = -24$ MeV or $-14$ MeV with the Coulomb potential. The calculated spectra are in agreement with the spectra obtained by previous works [6]. In the case of $v_Z = -24$ MeV, we find that a broad peak of the $[^{15}\text{N}(1/2^-) \otimes s_{Z^-}^-]_1$ quasibound state in $^{16}$C is located at $B_{Z^-} = 13.4$ MeV with a sizable width of $\Gamma = 3.5$ MeV, and a clear peak of the $[^{15}\text{N}(1/2^-) \otimes p_{Z^-}^-]_2$ quasibound state at $B_{Z^-} = 3.7$ MeV with $\Gamma = 3.1$ MeV. Integrated cross sections indicate $\sigma (0^\circ)/d\Omega \approx 28$ nb/sr for the $1^-$ state and 77 nb/sr for the $2^+$ state in $^{16}$C. In the case of $v_Z = -14$ MeV, which is favored in recent calculations [6,13,14,18], we have the $[^{15}\text{N}(1/2^-) \otimes s_{Z^-}^-]_1$-state at $B_{Z^-} = 6.8$ MeV with $\Gamma = 3.8$ MeV and the $[^{15}\text{N}(1/2^-) \otimes p_{Z^-}^-]_2$-state at $B_{Z^-} = 0.5$ MeV with $\Gamma = 1.1$ MeV. The integrated cross sections indicate $\sigma (0^\circ)/d\Omega \approx 6$ nb/sr for the $1^-$ state and 9 nb/sr for the $2^+$ state. Note that the $\Xi^- p \to \Lambda\Lambda$ conversion processes that can be described by the absorption potential $\text{Im} U_{\Xi^-}$ must appear above the $^{15}$N+$\Xi^-$ threshold in $^{16}$C. Nevertheless, the production of these $\Xi^-$ states as well as $\Xi^-$ states coupled to a $^{12}$N(3/2$^+$) nucleus is essential in this model because these states act as doorways when we consider the $\Lambda\Lambda$ states formed in the one-step mechanism. We also expect to extract properties of the $\Xi^-$-nucleus potential such as $v_Z$ and $W_{\Xi^-}$ from the $\Xi^-$ continuum spectra in the $(K^- , K^+)$ reactions on nuclear targets, as already discussed for studies of the $\Xi^-$-nucleus potential in nuclear $(\pi^- , K^+)$ reactions [37,38]. On the other hand, the $\Lambda\Lambda$-$\Xi$ coupling plays an important role in making a production of the $\Lambda\Lambda$ states via $\Xi^-$ doorways below the $^{15}$N+$\Xi^-$ threshold. The positions of their peaks must be slightly shifted downward by the energy shifts $\Delta B_{\Lambda\Lambda}$ due to the coupling potential in Eq. (7). When $v_{SN,\Lambda\Lambda} = 500$ MeV (250 MeV), we obtain $\Delta B_{\Lambda\Lambda} = 1.17$ MeV (0.15 MeV) and the $\Xi^-$ admixture probability $P_{\Xi^-} = 5.24\% (0.87\%)$ in the $[^{14}\text{C}(0^+) \otimes s_{\Lambda\Lambda}^+\Lambda\Lambda]$-excited state and the 0.38 MeV (0.09 MeV) and $P_{\Xi^-} = 0.58\% (0.14\%)$ in the $[^{14}\text{C}(0^+) \otimes s_{\Lambda\Lambda}^+\Lambda\Lambda]$ ground state. The value of $P_{\Xi^-}$ in the $1^-$ state is by a factor of 6–9 as large as that in the 0$^+$ state. These values are strongly connected with the magnitude of the peak for the $\Lambda\Lambda$ state in the spectrum.

In Fig. 2(b), we show the calculated spectra with the $\Lambda\Lambda$-$\Xi$ coupling potential when $v_Z = -14$ MeV. We recognize that the shape of these spectra is quite sensitive to the value of $v_{SN,\Lambda\Lambda}$, and it is obvious that no $\Xi N$-$\Lambda\Lambda$ coupling cannot describe the spectrum of the $\Lambda\Lambda$ states below the $^{14}$C+$\Lambda + \Lambda$ threshold. The calculated spectrum for $v_{SN,\Lambda\Lambda} = 500$ MeV has a fine structure of the $\Lambda\Lambda$ excited states in $^{16}$C. We find that significant peaks of the $1^-$ excited states with $^{14}$C+$\Lambda + \Lambda$ threshold. The positions of their peaks must be slightly shifted downward by the energy shifts $\Delta B_{\Lambda\Lambda}$ due to the coupling potential in Eq. (7). When $v_{SN,\Lambda\Lambda} = 500$ MeV (250 MeV), we obtain $\Delta B_{\Lambda\Lambda} = 1.17$ MeV (0.15 MeV) and the $\Xi^-$ admixture probability $P_{\Xi^-} = 5.24\% (0.87\%)$ in the $[^{14}\text{C}(0^+) \otimes s_{\Lambda\Lambda}^+\Lambda\Lambda]$-excited state and the 0.38 MeV (0.09 MeV) and $P_{\Xi^-} = 0.58\% (0.14\%)$ in the $[^{14}\text{C}(0^+) \otimes s_{\Lambda\Lambda}^+\Lambda\Lambda]$ ground state. The value of $P_{\Xi^-}$ in the $1^-$ state is by a factor of 6–9 as large as that in the 0$^+$ state. These values are strongly connected with the magnitude of the peak for the $\Lambda\Lambda$ state in the spectrum.
clear distortion effects [7]. The elementary laboratory cross section \( \langle \sigma \omega d \sigma \rangle_{0} \) is estimated to be 1.57–1.26 mb/sr for \( K^{-} p \rightarrow \pi^{0} A \) and 0.070–0.067 mb/sr for \( \pi^{0} p \rightarrow K^{+} A \) depending on the nuclear medium corrections. This yields

\[
\sum_{f} \left( \frac{d\sigma_{f}^{(2)}}{d\Omega_{f}} \right)_{0} \approx 0.06–0.04 \text{ mb/sr},
\]

which is half smaller than \( \sim 0.14 \text{ mb/sr} \) at 1.1 GeV/c. Considering a high momentum transfer \( q \approx 400 \text{ MeV/c} \) in the \( (K^{-}, K^{+}) \) reactions by comparison with the \( (\pi^{+}, K^{+}) \) reaction [39], we expect that the production probability for the \( \Lambda A \) bound states does not exceed 1% in the quasielastic \( \Lambda A \) production, so that an estimate of the \( \Lambda A \) hypernuclear in the two-step mechanism may be on the order of 0.1–1 nb/s. This cross section is smaller than the cross section for the \( \Lambda A \) \( 1^{-} \) states we mentioned above in the one-step mechanism. Consequently, we believe that the one-step mechanism acts in a dominant process in the \( (K^{-}, K^{+}) \) reaction at 1.8 GeV/c (0°) when \( v_{\text{SN,}\Lambda A}^{0} = 400–600 \text{ MeV} \). This implies that the \( (K^{-}, K^{+}) \) spectrum provides valuable information concerning \( \Xi N – \Lambda A \) dynamics in the \( S = -2 \) systems such as \( \Lambda A \) and \( \Xi \) hypernuclei, which are often discussed in a full coupling scheme [40].

4. Summary and conclusion

We have examined theoretically production of doubly strange hypernuclei in the DCX \( ^{16}O(K^{-}, K^{+}) \) reaction at 1.8 GeV/c within DWIA calculations using coupled-channel Green’s functions. We have shown that the \( \Xi^{-} \) admixture in the \( \Lambda A \) hypernuclei plays an essential role in producing the \( \Lambda A \) states in the \( (K^{-}, K^{+}) \) reaction.

In conclusion, the calculated spectrum for the DCX \( ^{16}O(K^{-}, K^{+}) \) hypernuclei in the one-step mechanism \( K^{-} p \rightarrow K^{+} \Xi^{-} \) via \( \Xi^{-} \) doorways predicts promising peaks of the \( \Lambda A \) bound and excited states in the \( ^{16}O(K^{-}, K^{+}) \) reactions at 1.8 GeV/c (0°). It has been shown that the integrated cross sections for the significant \( 1^{-} \) excited states in \( ^{16}C \) are on the order of 7–12 nb/sr depending on the \( \Xi N – \Lambda A \) coupling strength and also the attraction in the \( \Xi^{-} \)–nucleus potential. The \( \Xi^{-} \) admixture probabilities are on the order of 5–9%. The sensitivity to the potential parameters indicates that the nuclear \( (K^{-}, K^{+}) \) reactions have a high ability for the theoretical analysis of precise wave functions in the \( \Lambda A \) and \( \Xi \) hypernuclei. New information on \( \Lambda A – \Xi \) dynamics in nuclei from the \( (K^{-}, K^{+}) \) data at J-PARC facilities [3] will bring the \( S = -2 \) world development in nuclear physics.

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