Constitutive modeling of fatigue damage response of asphalt concrete materials with consideration of micro-damage healing

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A B S T R A C T
A continuum mechanics-based viscodamage (VD) constitutive relationship is proposed to model fatigue damage of asphalt concrete. The form for the evolution of the viscodamage function is postulated based on the damage density which was determined from uniaxial constant strain rate tests that were performed at different strain rates. The proposed viscodamage model is coupled with Schapery’s nonlinear viscoelastic (VE), Perzyna’s viscoplastic (VP), and micro-damage healing (H) models to simulate the nonlinear mechanical response of asphalt concrete during fatigue. Numerical algorithms are implemented in the finite element code Abaqus via the user material subroutine UMAT. The proposed model is validated against extensive experimental data including constant strain rate, cyclic displacement controlled, and cyclic stress controlled tests over a range of temperatures, strain rates, loading frequencies, and stress/strain levels/amplitudes. The model predictions show that the VE–VP–VD–H model is capable of predicting the fatigue damage response of asphalt concrete subjected to different loading conditions. The results demonstrate that micro-damage healing occurs not only during the rest period, but also during the cyclic strain controlled tests even in the absence of the resting time.

1. Introduction
Fatigue damage and permanent deformation are the distresses that cause the most concern of asphalt concrete as they affect both safety and serviceability over the performance life of asphalt pavements. The repetitive actions of traffic loading cause the progressive evolution of micro-damage (i.e. micro-cracks and micro-voids) and permanent strain in asphalt concrete pavements leading to fatigue damage and rutting (i.e. permanent deformation), respectively. Rutting is more dominant at higher temperatures while fatigue damage prevails at lower and intermediate temperatures (e.g. Morrison et al., 1994; Uzan, 1996; Tashman et al., 2005). This paper focuses on modeling the fatigue damage response of asphalt concrete at low and intermediate temperatures. Modeling the permanent deformation response of asphalt concrete pavements at high temperatures has been discussed thoroughly in recent papers by the authors (e.g. Masad et al., 2007; Huang et al., 2011a, 2011b; Abu Al-Rub et al., 2012; Darabi et al., in press, 2012d).

Schapery (1969) viscoelastic model and Perzyna (1971) viscoplastic model are among the classical theories that have been used extensively by several researchers to predict the viscoelastic and viscoplastic responses of asphalt concrete materials, respectively (e.g. Seibi et al., 2001; Masad et al., 2005; Huang et al., 2007, 2011b; Saadeh et al., 2007; Abu Al-Rub et al., 2009; Darabi et al., 2011). However, these models should be coupled with micro-damage evolution in order to capture specific phenomena such as fatigue damage, tertiary creep, and post-peak behavior of stress–strain response.

Several researchers (e.g. Kim and Little, 1990; Park et al., 1996; Lee et al., 2000) have coupled Schapery (1975a, b) damage model with VE–VP models using the extended correspondence principle (Schapery, 1984). However, models based on this approach have only been used to predict damage behavior under tensile loading conditions. Therefore, they cannot be used in modeling the damage evolution for general cases when the material exhibits inherent nonlinear viscoelastic–viscoplastic behavior (Abbas et al., 2004; Huang et al., 2007; Delgadillo et al., 2012) and also when the material is subjected to multi-axial state of stresses occurring in the asphalt pavement structures subjected to traffic loadings (e.g. Gibson et al., 2009; Wang and Al-Qadi, 2010).

Models based on continuum damage mechanics (CDM) can be considered as alternatives to model the effect of micro-damage evolution on the mechanical response of materials subjected to multi-axial stress states (e.g. Kachanov, 1958; Rabotnov, 1969; Lemaitre, 1992). The simplicity associated with the coupling of
the damage models to the complex constitutive models as well as the straightforward procedure associated with their numerical implementation techniques are the primary reasons for the popularity of continuum damage mechanics-based models. This approach has recently been used by the authors to formulate a coupled VE–VP–VD constitutive model that can be used for predicting the damage response of asphalt concrete under different loading conditions (e.g. Abu Al-Rub et al., 2010; Darabi et al., 2011).

Micro-damage healing is another important factor that should be included in constitutive models to predict the fatigue response of asphalt concrete materials more accurately. During the rest period between the loading cycles, some induced micro-damage heals leading to the recovery of the material strength. The models that do not consider micro-damage healing and the strength recovery during the rest period underestimate the fatigue life of asphalt pavements subjected to realistic cyclic loading (Abu Al-Rub et al., 2010).

2. Objectives and methodologies

The above discussion shows that fatigue damage in asphalt concrete is a result of several interacting mechanisms. On one hand, damage evolution and growth degrades the mechanical properties of the material and shortens the fatigue life. On the other hand, healing of micro-damage surpasses the rate of the damage evolution and contribute to extending the fatigue life. Finally, stress relaxation and the viscoelastic nature of asphalt concrete materials add to the time-dependency of micro-damage evolution in asphalt concrete. These mechanisms are coupled and occur concurrently during general loading conditions. Therefore, the objective of this study is to develop a constitutive model that can be used to realistically predict the fatigue damage response of asphalt concrete subjected to general loading conditions by considering these interacting mechanisms while maintaining a straightforward systematic ability to characterize these interacting mechanisms.

The following steps are taken to achieve this objective:

- Extend the continuum-damage mechanics theory to the continuum damage-healing mechanics theory.
- Discuss the mechanism of micro-damage healing during cyclic loadings and demonstrate that healing doesn’t only occur during rest periods but also residual (internal) compressive stresses can aid in the micro-damage healing of asphalt materials.
- Propose a general continuum-based viscodamage model that can capture the response of asphalt concrete subjected to general loading conditions.
- Present a simple procedure for identification of the damage model parameters.
- Validate the model against different cyclic and monotonic loading conditions at different temperatures.

3. Continuum damage-healing mechanics

Kachanov (1958), Odqvist and Hult (1961), and Rabotnov (1969) pioneered continuum damage models by introducing the effective (undamaged) stress space concept. They introduced the damage variable $\phi$ as the reduction of the area due to micro-damage, such that:

$$\phi = 1 - \xi = (A - \hat{A})/A = A^D/A$$

(1)

where $A^D$ is the area of micro-damage (i.e. micro-cracks and voids) such that $A^D = A - \hat{A}$; $\hat{A}$ and $A$ being the undamaged area and total cross-section of the material, respectively. The damage density starts from $\phi = 0$ (even for initially damaged materials) and ends with $\phi = 1$ indicating the complete failure. The stresses in the damaged and effective configurations as a function of the damage density can be expressed as:

$$\sigma = (1 - \phi)\bar{\sigma}$$

(2)

where $\sigma$ and $\bar{\sigma}$ are the stress tensors in the damaged and effective configurations, respectively; with superscript “~” designating the effective configuration. The inherent assumption in Eq. (2) is that the damage density is irreversible, such that an increasing function with time is commonly postulated to describe the evolution of the damage density $\phi$ (e.g. Kachanov, 1958; Lemaître, 1992; Krajcinovic, 1996). However, this assumption is not accurate for materials that have the potential to heal and recover parts of their strength and stiffness under specific conditions such as resting periods during the fatigue loading and under hydrostatic pressure.

Recently, Abu Al-Rub et al. (2010) and Darabi et al. (2012a) extended the well-known framework of the continuum damage mechanics to the continuum damage-healing mechanics in order to enhance the ability of these theories to model the micro-damage healing phenomenon in the materials that tend to heal. They introduced the healing natural configuration for self-healing materials as the extension of the effective configuration by incorporating the contribution of the healed micro-damage areas in carrying and transferring loads and the stresses. They expressed the stress tensors in the damaged (nominal) and healing configurations as a function of the damage density $\phi$ and a physically defined healing variable $h$, such that:

$$\sigma = [1 - \phi(1 - h)]\bar{\sigma}; \quad \phi = A^D/A, \quad h = A^H/A^D$$

(3)

where the superimposed “~” designates the healing configuration; and $A^H$ is the area of the healed micro-damaged. The term $h$ represents the healed fraction of the total damaged area ranging from 0 to 1 with $h = 0$ indicating no healing and $h = 1$ indicating that all damage is healed. Comparing Eqs. (2) and (3) makes it possible to define the effective damage density $\phi$ in the continuum damage-healing mechanics as the counterpart of the classical damage density in classical CDM, $\phi$, such that:

$$\phi = \phi(1 - h)$$

(4)

Therefore, $\phi$ can be substituted for $\phi$ in Eq. (2) to consider the effects of both damage and micro-damage healing on mechanical response of materials.

A common argument when using effective stress space in CDM is that once the material becomes damaged, further loading only affects the undamaged material skeleton. This argument implies that the effective stress is only determined as a function of the undamaged area and is independent of the crack shape, geometry, and the configuration around the cracks. However, it has the advantage of simplifying the implementation of such theories and it avoids the complexities associated with direct coupling of damage models to other mechanisms. This paper presents a similar argument stating that once the material becomes damaged, further loading only affects the intact and healed portion of the material (i.e. the healing configuration). Therefore, the constitutive equations are expressed in the healing configuration (i.e. in terms of $\bar{\sigma}$). However, a transformation hypothesis is required in order to relate the strain tensors in the damaged and healing configurations (refer to Darabi et al., 2012a for more details on transformation hypothesis in CDHM). This paper assumes the strain equivalence hypothesis to simplify the implementation of the constitutive models, such that:

$$\varepsilon = \bar{\varepsilon}$$

(5)

where $\varepsilon$ and $\bar{\varepsilon}$ are strain tensors in the damaged and healing configurations, respectively.
4. Micro-damage healing during cyclic strain/displacement controlled loading

This section presents the significance of including the micro-damage healing effect during the cyclic displacement controlled (CDC) loading for viscoelastic materials. During the CDC test, a cyclic displacement is applied at the end of a specimen. The healing mechanism during the CDC test is illustrated with the aid of Fig. 1, which shows part of strain and stress responses during the CDC test in tension at 19 °C (Kim et al., 2008) with a loading frequency of 4 cycles/s. In the CDC test, the stress remains tensile during a large portion of the loading history. During the unloading (i.e. when the strain rate is negative), however, compressive stresses are induced in the specimen. These compressive stresses cause the micro-crack free surfaces to wet (i.e. get close to each other) and undergo healing and crack closure. This effect can be explained based on the fading memory properties of viscoelastic materials and the lag between stress and strain responses, the term $\delta$ as shown in Fig. 1. As explained by Coleman (1964b), a fading memory is referred to the assertion that the deformations experienced in the distant past should have less effect on the present values of the stress than deformations that occurred in the recent past (e.g. Coleman and Noll, 1961; Coleman, 1964a, b).

In order to illustrate the effect of the fading memory, it is hypothetically assumed that the viscoelastic medium contains a single micro-crack of length $a_0$ at point “A” and the damage growth occurs when the tensile strain exceeds a threshold value (i.e. $\varepsilon_{\text{th}}^{\text{damage}}$). Moving from point “A” to point “B” in Fig. 1, the strain remains larger than $\varepsilon_{\text{th}}^{\text{damage}}$ and the induced stresses in the viscoelastic media are also tensile. Therefore, the micro-crack length increases by $\Delta a^d$, such that the crack length at point “B” reaches $a_0 + \Delta a^d$. However, the fading memory effect causes the stresses induced in the material to relax and fade away with time. The increase in the strain level during the preceding loading stage has less effect on the stress level than the decrease in the strain level during the current unloading stage. Obviously, the negative increment in the stress due to the decrease in the strain level during the unloading stage is more than the positive increment in the stress due to the increase in the strain level during the preceding loading stage. Therefore, the viscoelastic medium feels compressive stres-
ses at some point during the unloading stage although the total strain is still tensile (see the stress response in Fig. 1 when moving from point “B” to point “C”). Therefore, the crack length decreases when moving from point “B” to point “C” by the increment of $\Delta \sigma^{in}$. The induced stresses are compressive during a small portion of the loading and deformation history for the tensile CDC test, such that $\Delta \sigma^{in}$ is always greater than $\Delta \sigma^{in}$ (i.e. $\Delta \sigma^{in} > \Delta \sigma^{in}$). Therefore, micro-cracks still propagate and increase in length when moving from point “A” to point “C”. However, accurate estimation of the damage density requires the incorporation of both $\Delta \sigma^{in}$ and $\Delta \sigma^{in}$ components.

As shown in the following sections, the accurate prediction of the fatigue response of asphalt concrete under CDC test requires the inclusion of the micro-damage healing.

5. Viscoelastic and viscoplastic constitutive equations

This section recalls the key equations for the nonlinear Schapery (1969) viscoelastic and Perzyna-type (1971) viscoplastic models that have been used previously to model the mechanical response of asphalt concrete (e.g. Masad et al., 2005; Huang et al., 2007; Abu Al-Rub and Darabi, 2012). However, detailed formulation and numerical implementation of the coupled VE-VP constitutive model will not be presented. The readers are referred to previous papers for detailed presentation of the formulation and numerical implementation techniques (e.g. Heeres et al., 2002; Haj-Ali and Mullan, 2004; Abu Al-Rub et al., 2010; Huang et al., 2011; Darabi et al., 2012b). It should be noted that this paper presents VE–VP equations in the healing configurations to facilitate the coupling of the damage and healing models to the rest of the constitutive model.

Small deformation theory is postulated, such that total strain tensor $\varepsilon$ can be additively decomposed into the viscoelastic strain tensor $\varepsilon^{ve}$ and the viscoplastic strain $\varepsilon^{vp}$, such that:

$$\varepsilon = \varepsilon^{ve} + \varepsilon^{vp}$$

(6)

The Schapery’s one-dimensional single-integral nonlinear viscoelastic model is written as follows:

$$\varepsilon^{ve} = \int_0^t \Delta \sigma^{ve}(t') \psi(t') dt'; \quad \psi = \int_0^t \frac{d\varepsilon}{dT}$$

(7)

where superscripts $s$, $v$, $g$ are nonlinear viscoelastic parameters; $\psi$ is the reduced time; $a_i$ is the time–temperature shift factor; $D_i$ and $\Delta_i$ are the instantaneous and transient creep compliances in the healing natural configuration which are equivalent to their values for the intact state of the material. For numerical purposes, the transient creep compliance can be expressed using the Prony series as:

$$\Delta D^{i} = \sum_{n=1}^{N} D_n [1 - \exp(-\lambda_n \psi)]$$

(8)

where $N$ is the number of terms; and $D_n$ and $\lambda_n$ are the $n^{th}$ coefficients of the Prony series. Decomposing the viscoelastic strain tensor $\varepsilon^{ve}$ into its deviatoric tensor $\varepsilon^{ve}$ and a volumetric component $\varepsilon^{ve,\delta}$ facilitates the presentation of the Schapery’s viscoelastic model for the general three-dimensional problems (Lai and Bakker, 1996), such that:

$$\varepsilon^{ve} = \varepsilon^{ve,\delta} + \frac{1}{3} \varepsilon^{ve,\delta} \delta$$

(9)

where $J$ and $B$ are the shear and bulk compliances of the intact material; $S_{ij} = \frac{1}{2} (\sigma_{ij} - \sigma_{kk} \delta_{ij})$ are the components of the deviatoric stress tensor in the healing configuration; and $\delta_{ij}$ is the Kronecker delta. The deviatoric viscoelastic strain tensor and the volumetric viscoelastic strain can be expressed in terms of $J$ and $B$, such that:

$$\varepsilon^{ve,\delta} = \frac{1}{2} S_{ij} \delta_{ij} + \frac{1}{2} \frac{1}{J} \int_0^t \frac{\Delta \sigma^{ve}}{\delta} d\varepsilon (S_{ij}, \delta_{ij})$$

$$\varepsilon^{ve,\delta} = \frac{1}{3} \frac{1}{B} \int_0^t \frac{\Delta \sigma^{ve,\delta}}{\delta} d\varepsilon (S_{ij}, \delta_{ij})$$

(10)

where $J_0$ and $B_0$ are the instantaneous shear and bulk compliances in the healing configuration, respectively; and $\Delta \sigma^{ve}$ and $\Delta \sigma^{ve,\delta}$ are the transient shear and bulk compliances in the healing configuration, respectively. The compliances $J$ and $B$ are related to the creep compliance $D$ through the following relations:

$$J_0 = 2(1 + v) D_0; \quad \Delta J = 2(1 + v) D; \quad B_0 = 3(1 - 2v) D_0; \quad \Delta B = 3(1 - 2v) \Delta D$$

(11)

where $v$ is the Poisson’s ratio, which is assumed to be time-independent. In this study, it is assumed that the Poisson’s ratio is constant in order to simplify the constitutive model and to reduce the number of testing required for characterization of the viscoelastic response of asphalt concrete materials. The Poisson’s ratio of the viscoelastic materials could be a function of time, temperature, and stress/strain-history (refer to Hilton and Yi, 1998; Hilton, 2011; Kassem et al., in press for more details).

The rate of the viscoplastic strain tensor $\dot{\varepsilon}^{vp}$ is defined through the following non-associative viscoplastic flow rule (Perzyna, 1971), such that:

$$\dot{\varepsilon}^{vp} = \frac{\partial F}{\partial \sigma}$$

(12)

where $F$ is the viscoplastic potential function and $\dot{\varepsilon}^{vp}$ is the viscoplastic multiplier which can be expressed in terms of an overstress function $\Phi$ and the viscoplasticity fluidity parameter $\Gamma^{vp}$, such that:

$$\dot{\varepsilon}^{vp} = \Gamma^{vp}(\Phi(f))^{1/2}; \quad \Phi(f) = \frac{f - \sigma}{\sigma}$$

(13)

where $M$ is the viscoplasticity rate-sensitivity parameter; $f$ is the viscoplastic yield function and $\sigma^0$ is a yield stress quantity used to normalized the overstress function and can be assumed unity; and $\langle \rangle$ is the Macaulay brackets defined by $\langle \Phi \rangle = (\Phi + |\Phi|)/2$.

Due to the non-associative nature of viscoplastic response of asphalt concrete, modified Drucker-Prager-type functions are used for $f$ and $F$ (e.g. Seibi et al., 2001; Dessouky, 2005; Masad et al., 2007), such that:

$$\sigma - \sigma^0 = \frac{\sqrt{3} J_2}{2} \left[ 1 + (1 - \frac{1}{2} \frac{1}{\sigma^0}) \frac{3 J_2}{J_1} \right]$$

(14)

where $\alpha$ and $\beta$ are the pressure-sensitivity material parameters; $L_1 = \sigma_{1}^0$ is the first stress invariant; $\sigma^0$ is the effective shear stress; $J_2 = \frac{1}{2} S_{ij} S_{ij}$ and $J_3 = \frac{1}{2} S_{ij} S_{ij} S_{ij}$ are the second and the third deviatoric stress invariants in the healing configuration, respectively; $k(p)$ is the isotropic hardening function which is a function of the effective (equivalent) viscoplastic strain $p$ (Lemaître and Chaboche, 1990), such that:

$$k(p) = k_0 + k_1 \left[ 1 - \exp(-k_2 p) \right]$$

(15)

where $k_0$, $k_1$, and $k_2$ are material parameters.

This VE–VP constitutive model was calibrated against the Federal Highway Administration’s, FHWA’s, Accelerated Load Facility (ALF) data (e.g. Kim et al., 2008). The dynamic modulus test at different temperatures in tension is used to identify the viscoelastic model parameters as well as the time–temperature shift factors $a_i$. The repeated creep-recovery test at various stress levels in compression was used to identify the viscoplastic model parameters once the viscoelastic model parameters were identified. Readers
are referred to Huang et al. (in press) and Darabi et al. (2012c) for more details on the identification of the VE–VP model parameters. Table 1 lists the tests used to identify the model parameters. The identified VE–VP model parameters are listed in Table 3.

### 6. Viscodamage evolution model

In previous studies, the authors proposed a viscodamage model that can be used to predict the mechanical response of asphalt concrete subjected to different loading conditions (Darabi et al., 2011, 2012b). The damage evolution function was defined as a function of the damage driving force $Y$, the total strain $\varepsilon_{\text{eff}}$, and a history term $(1 - \phi)^2$, such that (Darabi et al., 2011):

$$\dot{\phi} = \Gamma_{\text{vd}} \left( \frac{Y}{Y_0} \right)^q \exp(k_2 \varepsilon_{\text{eff}}^2) \quad (16)$$

where $\Gamma_{\text{vd}}$ is the viscodamage fluidity parameter; $Y_0$ is the reference damage force which can be assumed to be unity; $k$ is a material parameter; and $\varepsilon_{\text{eff}}$ is the effective total strain defined as $\varepsilon_{\text{eff}} = \sqrt{\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2}$. The damage force $Y$ is assumed to have a Drucker–Prager type form, such that $Y_{\text{vd}}$ is defined as:

$$Y_{\text{vd}} = \sqrt{\frac{3}{2}} J_2^{1/2} \left[ 1 + \frac{1}{d_{\text{vd}}} + \left( 1 - \frac{1}{d_{\text{vd}}} \right) \frac{3 J_3^{1/2}}{\sqrt{3 J_2^{1/2}}} \right]^{1/2}$$

(17)

Eqs. (14) and (16) state that damage and viscoplasticity have similar driving forces with different evolution functions. Moreover, Eq. (17) possesses a model parameter (i.e. $d_{\text{vd}}$) distinguishing the damage responses in extension and contraction loading conditions.

Finally, the damage driving force of Eq. (16) is sensitive to the hydrostatic pressure. This damage model has been calibrated and validated against extensive experimental data (e.g. Darabi et al., 2011, 2012b). The calibration was based on using the tertiary part of the creep response of asphalt concrete.

The subsequent sections present a novel approach to identify the proper viscodamage constitutive model for asphalt concrete. The constant strain rate tests in tension at $5^\circ$C were analyzed to identify the damage evolution response. A viscodamage model is proposed based on the determined damage evolution. At $5^\circ$C, one can reasonably assume negligible viscoplastic deformation. To examine this assumption, the identified VE–VP model parameters listed in Table 3 are used to simulate the response of asphalt concrete at $5^\circ$C over a rage of strain rates. The predicted viscoplastic strain over a range of strain rates (i.e. $1 \times 10^{-3} - 1 \times 10^{-5}/s$) is less than $1\%$ of the total applied strain which reasonably verifies the assumption of neglecting the viscoplastic strain at $5^\circ$C.

### Table 1

Summary of the tests used for validation of the viscodamage model.

<table>
<thead>
<tr>
<th>Test</th>
<th>Temperature (°C)</th>
<th>Strain rate (s⁻¹)</th>
<th>Strain amplitude (µε)</th>
<th>Identified model parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complex Modulus test</td>
<td>–10, 10, 35, 55</td>
<td>–</td>
<td>–</td>
<td>$a_1$, $D_1$, $\lambda_0$, Eqs. (7), (8)</td>
</tr>
<tr>
<td>Repeated creep-recovery test at various stress level (RCRT–VS) in compression</td>
<td>55</td>
<td>200</td>
<td>–</td>
<td>$\Gamma_{\text{vp}}$, $M$, $k_0$, $k_1$, $k_2$, $\varkappa_1$, Eqs. (13) and (15)</td>
</tr>
<tr>
<td>Constant strain rate test in tension</td>
<td>5</td>
<td>$7 \times 10^{-5}$</td>
<td>–</td>
<td>$\Gamma_{\text{vd}}$, $\varkappa_2$, $k$, $q$, Eq. (26)</td>
</tr>
<tr>
<td>Cyclic displacement controlled test (CDC) in tension</td>
<td>19</td>
<td>–</td>
<td>1200</td>
<td>$\Gamma_{t}$, $b_1$, $b_2$, $\text{Eq. (29)}$</td>
</tr>
</tbody>
</table>

### Table 2

Summary of the tests used for validation of the viscodamage model.

<table>
<thead>
<tr>
<th>Test</th>
<th>Temperature (°C)</th>
<th>Strain rate (s⁻¹)</th>
<th>Strain amplitude (µε)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniaxial constant strain rate</td>
<td>12</td>
<td>$2.7 \times 10^{-4}$, $4.6 \times 10^{-4}$</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>$5 \times 10^{-4}$, $1.5 \times 10^{-3}$, $4.5 \times 10^{-3}$, $1.35 \times 10^{-2}$</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>$3 \times 10^{-4}$, $3 \times 10^{-3}$</td>
<td>–</td>
</tr>
<tr>
<td>Cyclic stress controlled</td>
<td>5</td>
<td>–</td>
<td>1525</td>
</tr>
<tr>
<td></td>
<td>19</td>
<td>–</td>
<td>250, 750</td>
</tr>
<tr>
<td>Cyclic displacement controlled (CDC)</td>
<td>5</td>
<td>–</td>
<td>1750</td>
</tr>
<tr>
<td></td>
<td>19</td>
<td>–</td>
<td>1500</td>
</tr>
</tbody>
</table>

### Table 3

Identified viscoelastic and viscoplastic model parameters at the reference temperature $T_0 = 10^\circ$C and the time–temperature shift factors.

#### Viscoelastic model parameters

<table>
<thead>
<tr>
<th>$n$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$ (s⁻¹)</td>
<td>$10^4$</td>
<td>$10^2$</td>
<td>10</td>
<td>1</td>
<td>$10^1$</td>
<td>$10^2$</td>
<td>$10^1$</td>
<td>$10^2$</td>
<td>$10^1$</td>
</tr>
<tr>
<td>$D_0$ (kPa)</td>
<td>$1 \times 10^{-8}$</td>
<td>$3 \times 10^{-8}$</td>
<td>$2 \times 10^{-8}$</td>
<td>$1 \times 10^{-7}$</td>
<td>$1.5 \times 10^{-7}$</td>
<td>$9 \times 10^{-7}$</td>
<td>$1 \times 10^{-6}$</td>
<td>$5 \times 10^{-6}$</td>
<td>$6 \times 10^{-6}$</td>
</tr>
<tr>
<td>$D_0$ (kPa)</td>
<td>$3 \times 10^{-8}$</td>
<td>$3 \times 10^{-8}$</td>
<td>$3 \times 10^{-8}$</td>
<td>$3 \times 10^{-8}$</td>
<td>$3 \times 10^{-8}$</td>
<td>$3 \times 10^{-8}$</td>
<td>$3 \times 10^{-8}$</td>
<td>$3 \times 10^{-8}$</td>
<td>$3 \times 10^{-8}$</td>
</tr>
<tr>
<td>Time–temperature shift factors ($T_0 = 10^\circ$C)</td>
<td>$-10$</td>
<td>$10$</td>
<td>35</td>
<td>55</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Gamma$ (°C)</td>
<td>$1.26 \times 10^1$</td>
<td>1</td>
<td>$6.3 \times 10^{-4}$</td>
<td>$10^{-5}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Viscoplastic model parameters

<table>
<thead>
<tr>
<th>$a$</th>
<th>$b$</th>
<th>$\sigma_0^p$ (kPa)</th>
<th>$\Gamma_{\text{vp}}$ (s⁻¹)</th>
<th>$M$</th>
<th>$k_0$ (kPa)</th>
<th>$k_1$ (kPa)</th>
<th>$k_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.2</td>
<td>$100$</td>
<td>$2.4 \times 10^{-8}$</td>
<td>1</td>
<td>50</td>
<td>1800</td>
<td>135</td>
</tr>
</tbody>
</table>
For the constant strain rate test, the strain input can be expressed as follows:

\[ \varepsilon(t) = C t \]  
\[ \text{(18)} \]

where \( C \) is a constant representing the strain rate in the uniaxial constant strain rate test. The uniaxial stress output in the healing configuration can be obtained using the superposition principle for linear viscoelastic materials, such that:

\[ \sigma(t) = E(t) \varepsilon(0) + \int_0^t E(t) \varepsilon(t - \tau) d\tau; \]
\[ \text{(19)} \]

where \( E(t) \) is the relaxation modulus that is already known from the dynamic modulus test results. Substituting the strain input for the uniaxial constant strain rate test (Eq. (18)) into Eq. (19) yields the stress response during the constant strain rate test, such that:

\[ \sigma(t) = C \int_0^t E(\tau) d\tau \]  
\[ \text{(20)} \]

Table 4
Identified viscodamage model parameters at the reference temperature \( T_0 = 10 \, ^\circ\text{C} \).

<table>
<thead>
<tr>
<th>Viscodamage model parameters</th>
<th>( \Gamma^{\text{ref}} ) (s(^{-1}))</th>
<th>( Y_0 ) (kPa)</th>
<th>( q )</th>
<th>( k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 2.14 \times 10^{-11} )</td>
<td>1000</td>
<td>2.41</td>
<td>-2.53</td>
<td></td>
</tr>
</tbody>
</table>
It should be noted that healing does not occur during the uniaxial constant strain rate test \((h = 0)\) since the strain increases monotonically and there is no rest period. Using the concept of the continuum damage-healing mechanics, the damage density variable \(\phi\) can be determined as:

\[
\phi(t) = 1 - \frac{\sigma(t)}{\sigma(t)}
\]

where \(\sigma(t)\) is the stress in the damaged configuration which is measured experimentally. This procedure is followed for several uniaxial constant strain rate tests in tension at 5 °C to calculate the damage density variable.

Guided by the evolution function in Eq. (16), a general form is assumed for the damage evolution function as a function of the damage density variable.

\[
\dot{\phi} = \Gamma^{vd} f_1 \left( \frac{Y}{Y_0} \right) f_2 (\varepsilon_{eff})
\]

where \(Y, \varepsilon_{eff}, Y_0,\) and \(\Gamma^{vd}\) are defined in Eqs. (16), (17). The main objective is to identify the forms of functions \(f_1\) and \(f_2\) based on the experimentally calculated damage densities.

Taking the natural logarithm of both sides of Eq. (22) yields:

\[
\ln(\phi) = \ln(\Gamma^{vd}) + \ln \left[ f_1 \left( \frac{Y}{Y_0} \right) \right] + \ln \left[ f_2 (\varepsilon_{eff}) \right]
\]

The rate of the damage density \(\dot{\phi}\) versus the effective total strain \(\varepsilon_{eq}\) can be plotted once the damage density is calculated according to Eq. (21). The first and the third terms in the right-hand-side of Eq. (23) are constants at a fixed effective strain level \(\varepsilon_{eff}\). To identify the function \(f_1\), the values of the normalized damage force in the healing configuration (i.e. \(Y/Y_0\)) at four strain levels (i.e. 0.1, 0.15, 0.2, 0.25, and 0.3%) are plotted versus the damage rate in Fig. 2(a) for several strain rate tests. Fig. 2(b) clearly suggests a power law function for \(f_1\), which agrees with Eq. (16), such that:

\[
f_1 \left( \frac{Y}{Y_0} \right) = \left( \frac{Y}{Y_0} \right)^q
\]

where \(q\) is the slope of the lines as shown in Fig. 2(a). The interception of each strain level line with the vertical axis in Fig. 2(a) is the summation of the first and the third terms in the right side of Eq. (23). This interception at different strain levels is used to identify the viscodamage fluidity parameter \(\Gamma^{vd}\). The values of \(\Gamma^{vd}\) identified at different strain levels are similar suggesting that \(\Gamma^{vd}\) is constant and does not depend on the strain and/or stress levels. The outlined procedure allows the identification of the parameter \(q\) independent of other viscodamage parameters.

Similarly, function \(f_2\) can be identified by plotting \(\phi\) versus the normalized damage force (i.e. \(Y/Y_0\)). The first and second terms on the right hand side of Eq. (23) remain constant at a fixed normalized damage force. To characterize \(f_2\) independently, \(\phi\) is
plotted versus $\varepsilon_{\text{eff}}$ at fixed normalized damage forces, as shown in Fig. 2(b). As shown in Fig. 2(b), the fitted lines at different normalized damage force are parallel, such that $f_2$ can be expressed as:

$$f_2(\varepsilon_{\text{eff}}) = (\varepsilon_{\text{eff}})^k$$

(25)

where $k$ is the slope of $\ln \dot{\phi} - \ln \varepsilon_{\text{eff}}$ curves at different normalized damage forces. The interception of the lines in Fig. 2(b) correspond to the summation of the first two terms in the right side of Eq. (23) from which $\Gamma_{\text{vd}}$ can be identified at different normalized damage force. The viscodamage fluidity parameter $\Gamma_{\text{vd}}$ identified at different strain and normalized damage force levels (Figs. 2(a) and (b)) range between $10^{-12}$ and $6 \times 10^{-12}$/s with the average of $4 \times 10^{-12}$/s.

It should be noted that the viscodamage model presented in this paper (i.e. Eqs. (22), (24), and (25)) is slightly different from the viscodamage model of Darabi et al. (2011) as presented in Eq. (16). The main difference between these two models is that the exponential term of the effective strain in Eq. (16) is substituted with a power law function for the effective strain in Eq. (25).

Eqs. (22), (24), and (25) yield:

$$\dot{\phi} = \Gamma_{\text{vd}} \left( \frac{\dot{\varepsilon}_{\text{eff}}}{\varepsilon_0} \right) q (\varepsilon_{\text{eff}})^k$$

(26)

The dynamic viscodamage loading/unloading surface can be obtained by rearranging Eq. (26), such that:

$$\dot{\chi}_{\text{vd}} = \dot{\chi}_d - Y_0 (\varepsilon_{\text{eff}})^k q \left( \frac{\dot{\phi}}{\Gamma_{\text{vd}}} \right)^i = 0$$

(27)

Fig. 4. Model predictions and experimental measurements for the constant strain rate tests in tension at 12 °C when strain rates are $2.7 \times 10^{-4}$/s; (b) $4.6 \times 10^{-4}$/s.

Fig. 5. (a) Model predictions and experimental measurements for the constant strain rate tests in tension at different strain rates; (a) stress–strain response at 25 °C; (b) predicted damage density at 25 °C; (c) stress–strain response at 40 °C; (b) predicted damage density at 40 °C. The viscodamage model predictions agree well with the experimental measurements at different strain rates. Also, at the same strain level, the damage density has higher values as the strain rate increases showing that the asphalt concrete materials are more prone to damage as the rate of loading increases.
where $\chi^\text{vd}$ is the viscodamage dynamic growth surface and can also be regarded as the damage initiation and evolution condition, such that the micro-damage evolution occurs only if $\chi^\text{vd} > 0$.

The viscodamage model parameters are identified at 5 °C, whereas, the viscoelastic and viscoplastic model parameters are identified at 10 °C. The temperature-dependency is incorporated in the viscodamage model using the time–temperature shifting. Therefore, the time increment in Eq. (22) is substituted by the reduced time increment (i.e. reduced by the time–temperature shift $\alpha_T$), such that Eq. (22) can be expressed as:

$$\Delta \phi = \Gamma^\text{vd} \left( \frac{Y}{Y_0} \right)^q \left( \frac{\varepsilon_{\text{eff}}}{a_T} \right)^k \frac{\Delta t}{\Delta t_T} = \Gamma^\text{vd} \left( \frac{Y}{Y_0} \right)^q \left( \frac{\varepsilon_{\text{eff}}}{a_T} \right)^k \Delta t$$  \hspace{1cm} (28)

Eq. (28) suggests that using the time temperature shift factor is equivalent to use the viscodamage fluidity parameter normalized by $a_T$. For the consistency purposes, the viscodamage model parameters are shifted to the reference temperature $T_0 = 10$ °C simply by scaling $\Gamma^\text{vd}$, such that $\Gamma^\text{vd}(10 \, \text{°C}) = \Gamma^\text{vd}(5 \, \text{°C}) \times a_T(5 \, \text{°C}, T_0 = 10 \, \text{°C})$. It is assumed that the time–temperature shift factor is the same for all the components (i.e. VE, VP, VD, and H) of the presented constitutive model. The validity of this assumption is investigated in the subsequent sections. Table 4 lists the identified viscodamage model parameters.

### 7. Micro-damage healing model parameters

This section recalls the micro-damage healing model recently proposed by Abu Al-Rub et al. (2010), such that:

$$\dot{h} = \Gamma^h (1 - \phi)^h (1 - h)^b_2$$  \hspace{1cm} (29)

where $\Gamma^h$ is the micro-damage healing fluidity parameter which controls the rate of the micro-damage healing and $b_1$ and $b_2$ are damage and micro-damage healing history parameters, respectively. The model parameters associated with the micro-damage healing model are usually determined using the repeated creep-recovery tests in tension with the rest period between the loading cycles (e.g. Abu Al-Rub et al., 2010). However, such tests are not available in the current experimental database. Therefore, to reduce the number of model parameters, the simplest evolution function for the micro-damage healing model is considered, such that:

$$\dot{h} = \Gamma^h$$  \hspace{1cm} (30)

As emphasized in Section 3 of this paper, micro-damage healing can occur during the cyclic displacement controlled test. This test is, therefore, used to identify the micro-damage healing fluidity model parameter $\Gamma^h$. The displacement input during the cyclic displacement controlled test (CDC) can be written as:

$$\Delta = \Delta_{\text{max}}/2[-1 + \cos(2\pi ft)]$$  \hspace{1cm} (31)

where $f$ is the frequency. The term $\Delta_{\text{max}}$ is the displacement amplitude applied at the end plates, such that $\Delta_{\text{max}}/\ell$ can be considered as the amplitude of the averaged strain applied to the specimen; $\ell$ being the specimen height (i.e. 6 inches). However, the averaged LVDTs' strains should be considered as the input strain for the validation and simulation purposes since it measures the strain in the middle of the specimen where stress and strain can be assumed to be uniformly distributed. Fig. 3(a) shows the LVDT strain input for the CDC test at 19 °C when the amplitude of the averaged specimen’s strain is 1200 με (Kim et al., 2008). Fig. 3(b) shows that the VE–VP–VD model predictions are in agreement with the experimental measurements during the initial loading cycles. However, the coupled VE–VP–VD model significantly underestimates the stress output as the number of loading cycles increases, Fig. 3(c).

As shown in Fig. 3(d), the model predictions using VE–VP components significantly overestimate the stress amplitude while the predictions using VE–VP–VD model significantly underestimate the measured stresses.

Model predictions using the viscodamage model predicts premature failure for the material while the experimental measurements show that the material is capable of sustaining loads for much longer time. As discussed in Section 3, this discrepancy is related to the inability of the model to account for crack closure and recovery effects.
the micro-damage healing during the cyclic strain controlled tests. The fluidity model parameter $C_h$, which characterizes the micro-damage healing rate, is identified using the deviation between the prediction of VE–VP–VD model and the experimental measurements for the CDC test at 19°C when the amplitude of the averaged specimen’s strain is 1200 με. The micro-damage healing fluidity parameter is identified to be $C_h = 6.5 \times 10^{-3}$/s.

Fig. 3(c) shows that the micro-damage healing model significantly enhances the prediction of the stress response as well as the dissipated energy during the intermediate cycles. Fig. 3(d) shows that the model without micro-damage healing underestimates the stress level and predicts premature failure almost by a
factor of four, while the model prediction with the micro-damage healing agrees well with experimental measurements.

8. Model validation

The identified model parameters are used in this section to validate the proposed viscodamage model against uniaxial constant strain rate, cyclic stress controlled, and cyclic strain controlled tests at different temperatures, strain rates, and stress/strain levels/amplitudes. Table 2 lists the tests that have been used for the validation of the proposed viscodamage model. It should be noted that these tests have not been used in the calibration process.

8.1. Uniaxial constant strain rate tests

The uniaxial constant strain rate tests at different strain rates and temperatures are used to further validate the viscodamage model. Fig. 4 shows the VE–VP–VD model predictions as compared to the experimental data at 12 °C for the constant strain rate at different strain rates. Fig. 4 clearly shows that the model is capable of predicting the rate-dependent damage response of asphalt concrete.

Fig. 5(a) presents the viscodamage model predictions as compared to experiments at four strain rates at 25 °C. Fig. 5(a) clearly shows that the viscodamage model predictions agree well with the experimental measurements. Fig. 5(b) shows the increase in the damage susceptibility of the asphalt concrete as the strain rate increases; the damage density has higher values at the same strain level as the strain rate increases. Fig. 5(b) suggests that the model is capable of capturing the commonly observed increase in the damage susceptibility as the strain rate and rate of loading increases. The model is further validated against the uniaxial strain rate tests in tension at 40 °C as shown in Fig. 5(c) and (d). Fig. 4 and 5 present the model capabilities in capturing rate- and temperature-dependent response of asphalt concrete during the monotonic loading.

In this paper, the damage model parameters are determined at 5 °C. The responses at other temperatures are obtained through using the time–temperature shift factor. The time–temperature shift factors for the viscodamage model are assumed to be slightly different from the time–temperature shift factor identified from the dynamic modulus test. However, this difference is negligible such that the asphalt concrete used in this study can be assumed to be a thermorheologically simple material for the coupled VE–VP–VD mechanisms, Fig. 6. This finding is in agreement with the previously reported experimental observations (e.g. Schwartz et al., 2002).

8.2. Cyclic stress controlled tests

In this section, the proposed viscodamage model is validated against the cyclic stress controlled tests in tension. The cyclic stress controlled tests are conducted at 5 °C and 19 °C and for multiple stress amplitudes. This test applies a cyclic stress input with the frequency of 4 cycles/s, such that the stress input can be written as follows:

\[ \sigma = \sigma_{max}/2 \times \left[ -1 + \cos(8\pi t) \right] \]  

(32)

where \( \sigma_{max} \) is the stress amplitude. Fig. 7 schematically presents the stress history for the cyclic stress controlled test where \( t_f \) can be considered as the failure time.

Unlike the cyclic displacement controlled test, the micro-damage healing is not important during the cyclic stress controlled test in tension as both strain and stress responses remain tensile.

Fig. 8(a) and (b) present the VE–VP model predictions and experimental measurements for initial and intermediate loading cycles at 19 °C when the stress amplitude is 750 kPa. As shown in Fig. 8(a), VE–VP model predictions agree well with experimental data for the initial loading cycles. However, as the number of loading cycles increases, VE–VP model significantly underestimates the strain level as shown in Fig. 8(b). This deviation manifests the initiation and evolution of the micro-damage as the number of loading cycles increases. Once the viscodamage and micro-damage healing components are also included in the model, the model predictions agree well with the experiments, Fig. 8(c). Fig. 8(c) also shows that the viscodamage model properly predicts the dissipated energy during the cyclic loading since the predicted hysteresis loop is very similar to the experimental measurements. The strain responses for the cyclic stress controlled tests at 19 °C when the stress amplitude is 250 and 750 kPa are presented in Fig. 8(d). The damage model is further validated at 5 °C when the stress amplitude is 1525 kPa, see Fig. 9. Figs. 8 and 9 show that the model predictions without damage significantly deviate from the experimental data as number of loading cycles increases. However, model predictions agree well with the experimental measurements up to thousands of loading cycles once the viscodamage component of the model is activated.

8.3. Cyclic strain controlled tests

In this subsection, the model is further validated against cyclic controlled displacement test (CDC). To further validate the model, the CDC test is repeated at 19 °C and 5 °C at different strain amplitudes. Fig. 10 shows the stress output for the CDC test at 19 °C and 5 °C when the strain amplitudes are, respectively, 1500 and 1750 \( \mu \)e. Fig. 10 shows that when the micro-damage healing is not included, the model predicts premature failure during the CDC test. However, model predictions using the calibrated VE–VP–VD–H model agrees reasonably well with the experimental measurements. The predictions presented in this section clearly show that the model can reasonably predict the time-, temperature-, and rate-dependent response of asphalt concrete under both monotonic and cyclic loading conditions.

9. Conclusions

In this paper, a viscodamage model that can reasonably predict the fatigue damage response of asphalt concrete materials subjected to different loading conditions is presented. The specific form of the viscodamage evolution function for the asphalt concrete is motivated by the experimentally measured damage density values using different loading conditions.

It is shown that asphalt concrete materials, and in general the viscoelastic materials with healing potential, have a micro-damage healing potential during the cyclic strain controlled loading conditions. This phenomenon is related to the fading memory characteristics of the viscoelastic materials. Therefore, this paper concludes that the existence of a rest period between the loading cycles is not the only condition for the micro-damage healing of the asphalt concrete materials. Residual (internal) compressive stresses can also aid in the micro-damage healing of asphalt materials.

The proposed concept of the continuum damage-healing mechanics significantly simplifies the coupling between the viscoelastic–viscoplastic mechanisms with the viscodamage and micro-damage healing mechanisms. Therefore, expressing the viscoelastic and viscoplastic models in the healing stress space is physically sound and at the same time simplifies the numerical implementation techniques.
The viscoelastic damage model parameters can be identified using a systematic procedure based on the application of uniaxial constant strain rate tests in tension at several strain rates. The viscoelastic, viscoplastic, viscodamage, and micro-damage healing model predictions show that the proposed model can accurately capture the mechanical response of asphalt concrete materials subjected to different loading conditions at a range of temperatures. Model predictions agree well with experimental measurements including uniaxial constant strain rate, cyclic strain/displacement controlled tests at different temperatures, strain rates, and stress/strain levels/amplitudes. These comparisons clearly show the capabilities of the model in capturing the fatigue response of asphalt concrete materials subjected to different loading conditions.

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