INTERNATIONAL CONFERENCE ON NEW HORIZONS IN EDUCATION
INTE2012

Misconceptions In Geometry And Suggested Solutions For
Seventh Grade Students

Aşen Özerem
Faculty of Education, Near East University PhD. Student

Abstract

The principal aim of this study is to find the weaknesses of secondary school students at geometry
questions of measures, angles and shapes, transformations and construction and 3-D shapes. The year 7
curriculum contains 4 geometry topics out of 17 mathematics topics. In addition to this, this study aims to
find out the mistakes, 28, 7th grade students made in the last 4 exams including two midterms and two
final exams. To collect data, students were tested on two midterms and two final exams using open–
ended questions on geometry to analyze their problem solving skills and to test how much they acquired
during the year. Frequency tables were used in data analysis. To fulfil this aim in the first midterm exam
the subject measures were tested. In the first final exam which followed the first midterm exam in
addition to measures and angles shapes skills were also tested. Following these tests, in the second
midterm we tested the students on transformation and construction. A descriptive methodology and
student interview were used in the study to analyze and interpret the results. The results from this study
revealed that 7th grade secondary school students have a number of misconceptions, lack of background
knowledge, reasoning and basic operation mistakes at the topics mentioned above

Keywords:  mathematics education; student difficulties; geometry questions; misconceptions; geometrical errors; teaching
suggestions for geometry.

INTRODUCTION

The general aim of mathematics is stated as making an individual acquire the mathematical
knowledge needed in daily basis, teaching how to solve problems, making him/her have a method of
solving problems and acquiring reasoning methods (Altun,2008). For this purpose to acquire
mathematical concepts one should be able to visualize the diagrams. In other words, mathematics is the
field in which preconditions are crucial so before the teaching process student backgrounds on the subject
should be tested (Baykul, 1987). Gagne divided the concepts into two as concrete and abstract concepts.
Concrete concepts are learnt starting from the beginning of life by the person himself. However to learn abstract concepts sometimes being taught by others is necessary (Senemoğlu, 2000). In this context, mathematics based learning should be done according to three aims listed above (Baykul, 2002).

- To student acquiring mathematical concepts.
- To understand mathematical operations.
- To help students make connections with the concepts and operations.

According to Piaget and Inhelder (1956), there are certain stages of learning starting from birth. These stages are:

- Stage 0: scribbles (less than 2)
- Stage 1: topological - irregular closed curves to represent circles, squares, etc (2-4 years)
- Stage 2: projective - progressive differentiation of Euclidean shapes (4-7 years)
- Stage 3: Euclidean - ability to draw Euclidean shapes (7-8 years)

Although there are specific age groups in this, it has not been widely accepted. It has been suggested that even younger children can sometimes operate with some Euclidean concepts. It is probable that topological, projective and Euclidean notions all develop over time and their usage becomes increasingly integrated.

Piaget suggested that children looked at the world from a very different perspective than adults did. So scientists started to investigate the reasons behind it by listening carefully what students were saying and doing on a variety of subject-matter tasks. They found surprising facts that students acquire ideas that completed often quite effectively with the concepts presented in the classroom environment. They had a powerful development of conceptions but they were sometimes inconsistent with the accepted mathematical and scientific concepts.

The Van Hiele model (1986) continues to be the best-known theoretical account of students’ learning about shape. The model suggests that children have to take a sequence of levels in a fixed order in their learning about shape. The first three levels in the model are as follows:

the Visualization level (Level 1, also known as the level of recognition) in which students recognize and learn to name certain geometric shapes but are usually only aware of shapes as a whole, and not of their properties or of their components;

The Analysis level (Level 2, also known as descriptive) students begin to recognize shapes by their properties.

The Abstraction level (Level 3, also known as relational), students begin to form definitions of shapes based on their common properties, and to understand some proofs.

Many teachers have observed that many young children have numerous misconceptions about geometry when a teacher discusses a geometry proof problem in class, it generally involves oral presentation of a formal proof and body movements pointing at different parts of the figure of the problem. Students must watch, listen, jot notes, and think as a lecture proceeds. They have to refer to many elements of the instruction and incorporate them into their memory (Sweller, 1988). This often causes cognitive overload and poses a negative effect on students’ learning. Numerous researchers have experimented different ways of teaching and found serious problems in geometry learners: incomplete comprehension of the problem and mathematical symbols, producing proofs based on direct visual elements (e.g., Chazan, 1993; Healy & Hoyles, 2000), lacking strategic knowledge in producing proofs, etc. Addressing the difficulties in learning geometry, Duval (1998) and Healy and Hoyles (1998) explained that geometry instruction is often more complex than that of numerical operations or elementary algebra. It is therefore more important that geometry instructions incorporate new and tested approaches such as using visual and multimedia tools in the classroom.

Studying geometry is an important component of learning mathematics because it allows students to analyse and interpret the world they live in as well as equip them with tools they can apply in other areas of mathematics. Therefore, students need to develop an understanding of geometric concepts
as well as gaining adequate geometry related skills. In this project, analyses the development of geometric skills and the use of tools, reproduction of constructions, properties verification, conjecture and research. It can be said that geometry is not used by students from the beginning due to their previous static learning experiences. Another difficulty with some of the students is the geometric language comprehension. After this survey, a seventh grade teacher can analyze students’ geometric mistakes and help them to improve their geometric knowledge. In this paper, we describe some guided research techniques for teachers of seventh grade students’ in a geometry lesson. This article gives the techniques about teaching.

In our sample class, when construction activities are used, they involve developing new ideas and connecting these with students' existing ideas. If students are not in a particular level of Van Hiele model they might not be able to perceive what the teacher sees in a geometric situation so higher levels of understanding is required. Misconceptions arise frequently if learners bypass or skip a level from the model. A teacher should get students to explain how they come to their answers or rules so that s/he can analyze the faulty interaction between the students' extant ideas and the new concept. By this way the teacher can understand the reason behind misconceptions and they can be corrected by challenging or contrasting it with the right conception. Students' prior learning sometimes arises misconceptions either in the classroom or from their interaction with the social and physical world. However the search for the origins of those misconceptions can not be located to the root of an educational problem. If misconceptions are persistent and resistant to change, that means they have got strong experiential foundations.

THE AIM OF THE STUDY:
The aim of this study is to reveal the performances of 7th grade college students at geometry and to show the conceptual difficulties they face while learning. By doing this, the study tried to identify the misconceptions which arise during the learning process of geometry.

SAMPLE
28 seventh grade students consisted of 12 males and 16 females at Turk Maarif Koleji in Cyprus.

METHOD
The purpose of this research is to determine college students’ misconceptions on geometry subject. The descriptive methodology and student interview were used in the study to analyze and interpret the results. The descriptive method was used since the main purpose of this study is to clarify an existing situation. This descriptive research analyzed the perspectives and experiences of 28 students’ exam papers (two midterms and two finals).

THE IMPORTANCE OF TECHNOLOGY IN A GEOMETRY CLASS:
Technology enables both students and teachers to access wide range of tools to use in mathematics. Perkins (1995) offered three stages in the process of understanding in the context of an information and communication technology. These are

- They offer students explanations
- Make relational knowledge available
- Students can possess revisable and extensive web explanations

PROBLEM
The main problem addressed by this of research is the reality of misconceptions that the students already passes or acquire during geometry lessons. These misconceptions are often related to shape perception and three dimensions.
STUDENT INTERVIEW PART:

Students are interviewed and asked four questions to have their opinions taken. The researcher recorded the face to face interviews by taking notes. In order not to create a disturbing environment no recording machine was used. In the students’ statements above, the students’ names are represented by the numbers in the parenthesis. Main subjects of the research and the data of 10 students, which were above randomly chosen 7th grade students, at the end of the term, are:

1. What do you think about Geometry Lessons? Do you find it interesting?

Eight out of ten students love Geometry Lessons and they are interested in the lesson. One of these ten students loves Geometry Lessons however s/he finds the measurements subject challenging. S/he prefers subjects which includes logic and operations. The other 1 accepts the fact that Geometry is necessary but since it is very time consuming, it is not interesting. According to this data, 80 percent of students said they loved Geometry Lessons. Although the students love the lesson, they make mistakes so it can be resulted that more quizzes should be done and student misconceptions need to be more emphasized after the quizzes and addition to this more thought provoking questions should be chosen and the subjects should be related to real life.

2. Do you want to change Geometry lessons into something more visual by using computers?

Nine students said that computer use can make the lessons more interesting. It was thought that using computers enable them to visualize and this helps them to learn permanently. One of the students thinks that it is not necessary to use computers. S/he thinks that teacher drawing on the board is more helpful. According to this data, ninety percent of the interviewed students think that computer use in Geometry lessons helps them to learn and remember better.

3. How do you learn and remember the rules and formulae?

Two of the students said that they learn them by writing. One of them said by memorizing. Four of them said by writing and visualizing. One said that by writing and drawing. One said by reading out loud and writing. The ones, who learn by visualizing, claim that when they write and stick papers to some places around the house like their study or wardrobe doors find it easier to learn the rules and the formulae. The last one student said that s/he learns by making logical connections with them and so that she remembers better later by the logic she has created. According to the data, teachers should give students choices of learning rules and formulae so that the students can choose the best for them.

4. Why do you think Geometrical mistakes are made in the exams?

Seven of the students think the cause of the mistakes is hastiness and negligence. One of the student said that the classrooms are crowded so students can not get enough attention from the teachers. One of them said that s/he underestimated drawings so when s/he saw them in the exam s/he was panicked and could not do them properly. The last one thinks s/he did not study enough so got confused in the exam. According to the data, since seventy percent of the students think that the cause of their mistakes are negligence and hastiness, the teachers can suggest them methods to avoid their habits.

The data received after student interviews are directly quoted. During the interviews, it was found that students are satisfied with the method used however teachers can use computers and give more focus on drawings.

DIRECT STUDENT OPINIONS:

Student (1): ‘...I find Geometry lessons enjoyable however more emphasis should be given to drawings and more assignments should be given. I had the most difficulty in the enlargement topic.....’

Student (2): ‘...I didn’t have any difficulties in geometry. After the mathematics topic algebra, it gave me motivation. I enjoyed drawing, using the compass and the protractor. I learn the formula by writing them on papers and white board....’

The curriculum of the 7 grade can be divided into four main categories as measures, angles, transformations and construction and 3D shapes.
FIRST MIDTERM EXAM:
The first midterm exam consisted of 25 questions. 3 out of 25 questions were on geometry subjects.

<table>
<thead>
<tr>
<th>TABLE 1: FIRST MIDTERM EXAM GEOMETRY MISCONCEPTIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mistake Made</td>
</tr>
<tr>
<td>---------------</td>
</tr>
</tbody>
</table>
| While the area of the triangle was found the student forgot to divide the number by two which was on the area formula. (The area formula of triangle is base times height over two and the student forgot dividing it into two) | -Just memorized the formula  
-Can’t visualize the image  
-Lack of reasoning  
-Few authentic in the primary grades | -More exercise on the topic  
-Frequent use of images by more interactive teaching  
-More visual -object use  
-Deduction of the area formula in class |
| Operation mistakes while finding the shaded area from the total | -Lack of spatial/thinking  
-Lack of construction idea  
-Lack of background education on operations | -More exercise  
-More homework  
-Practising the same procedure on paper to make understanding easier  
-More practice should be done on operations during primary school |
| Wrong or missing formulae use. (Ex: area of parallelogram is base times height. The student divided base times height by two) | -Incomplete understanding  
-No concentration  
-Not enough practice of the topic | -Computer based teaching can be used to show students the formulae in more fun and colourful way to make them remember easier. |

FIRST MIDTERM FINAL EXAM RESULTS
In the first midterm final exam, there are 25 questions of which 8 of them are on geometry subjects.

<table>
<thead>
<tr>
<th>TABLE 2: The first midterm final exam misconceptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mistakes Made</td>
</tr>
<tr>
<td>---------------</td>
</tr>
</tbody>
</table>
| Wrong formula use (area of triangle, parallelogram...etc) | -Can not understand the term area.  
-no proper understanding of the formulae. | -More warm –up before teaching about shapes  
-Ask students to find the shapes in their real lives. For example a square coffee table, a rectangular notebook, triangular ashtray so that they can understand the shapes better. |
| No given reasons for the answers | -Problem in the second language usage( can not express themselves in the language)  
-Can not give explanations to their answers | -More mathematical term use in the classroom  
-More practice  
-More stress on explanations in the classroom. |
| Lack of assimilation of the angles in parallel lines such as alternate and corresponding angles | -Ignore the importance of angles in parallel lines | -The importance of looking at the angles should be emphasized more in the classroom  
-Colourful images can be used to show alternate and corresponding angles. |
| -Lack of recognition and of perception properties of | -Can not visualize  
-Can not assimilate the | -More emphasis on properties of quadrilaterals and the |
quadrilaterals  
-Can not distinguish the types of quadrilaterals  

properties of quadrilaterals  
-Students put in little effort.  

similarities and differences while teaching  
-The students should be involved more during the similarity and difference stages of learning  

Wrong conversion of metric measurements (such as changing millimetre to metre)  
-No adequate use of conversions in real life  
-Not enough practice or studying  

-Variety of activities can be used in the classroom to show their use in real life (such as showing the metric system on their own rulers)  

Wrong detection of angles in an isosceles triangle  
-Can not connect the background data learned in primary school to new material.  
-Answering the questions spontaneously without reading the rubric of the question.  
-Lack of spatial reasoning  

-More variety of questions on different types of triangles  

Operation mistakes (during area calculations, multiplication, and addition, subtraction or division mistakes).  
-Lack of concentration  
-Underestimate the importance of operations  

-Lectures can be given by student advisors on paying attention techniques  
-More practice on operations  

Mistakes done on angle, side and parallelism properties on special quadrilaterals  
-Insufficient practice and learning  
-Can not assimilate the properties of quadrilaterals  

-colourful materials can be used while teaching properties of quadrilaterals to show the equal angles and sides and parallelism. (equal angles can be shown in red and equal sides in green to emphasize the difference)  

Can not distinguish the concepts of equations and expressions (for example when the side lengths are given in algebraic expressions, students are unable to find the area)  
-Equations and algebraic expressions topic are not learned well.  

-Instead of using x,y,z (which are frightening letters for students, a,b,c can be used more to show the unknown)  

**Table 3**: The second midterm exam misconceptions  

<table>
<thead>
<tr>
<th>Mistake Made</th>
<th>Possible Reason</th>
<th>Suggestions</th>
</tr>
</thead>
</table>
| -While the student was doing enlargement s/he didn't write the coordinate of the center of the enlargement | -The student read the question carelessly  
-The student didn't follow the instructions of the question carefully | -Teacher should emphasize the importance of reading the questions more carefully to give relevant answers.  
-The teacher should summarize the topic to improve the understanding of students |
---The students mixed the names of three dimensional objects. For example: instead of writing cuboid, the student wrote cubic.
---Basic vocabulary mistake. The student started with wrong step so s/he couldn’t finish correctly.
---Details should be shown clearly in the classroom. Revisions and more practice should be done.

---The student found the sum of the interior angles incorrectly and also found the size of each interior and exterior angles incorrectly.
---Learning formulas and definitions inadequately.
---Students should be encouraged to study and practice harder.

---The student shifted the lines while applying reflection and rotation.
---Student can’t use the tracing paper properly and counts the squares on the paper incorrectly.
---The use of tracing paper should be shown in detail by using computer based teaching.

TABLE 4 Second Term Final exam misconceptions:

<table>
<thead>
<tr>
<th>Mistakes Made</th>
<th>Possible Reasons</th>
<th>Suggestions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Missing information in descriptive questions (for example, when the student was asked to describe transformation s/he did not use specific words like translation, rotation)</td>
<td>-Lack of enough knowledge</td>
<td>-Students should be more careful in the exams while describing transformation. -The teacher should revise transformation more in detail from beginning to end by using visual aid.</td>
</tr>
<tr>
<td>The student did mistakes while enlarging objects and wrong use of coordinates</td>
<td>-Did not understand the process of enlarging. -Confused the coordinates when enlarging objects</td>
<td>-Computer based exercises can be practiced so that students can get a better knowledge on the enlargement topic. -Describe the position and movement more in detail. -2 and 3 dimensional objects should be used to make students able to visualize the images.</td>
</tr>
<tr>
<td>The student multiplied the number by 2 instead of 3 when calculating the volume of a cube</td>
<td>-Did not know the formula -Confused the formula of the cube with the formula of area</td>
<td>-To teach students volume, use visuals of 3-d objects from different perspectives and analyze the idea of volume.</td>
</tr>
<tr>
<td>Wrong use of vocabulary (for example the student used the word “translation” instead of “transformation”)</td>
<td>-Lack of recognition of formal terms</td>
<td>-Formal mathematical terms should be emphasized more during teaching and practising.</td>
</tr>
<tr>
<td>Measurements were wrong in</td>
<td>-Wrong use of protractor</td>
<td>-Make students use of real</td>
</tr>
</tbody>
</table>
In a student learning process there are some key factors such as network, images, words, anecdotes, cases in point, formal principles and finally explanation structures.

CONCLUSION
According to my research, it can be concluded that 7th grade students succeeded in reaching the curriculum objectives. My research aims to make the teachers aware of student misconceptions and general educational issues. The results from the study revealed that seventh year secondary school students have a number of misconceptions and lack of knowledge related to geometry subject.

SUGGESTIONS:
The major problems in mathematics are inadequate thinking and reasoning abilities. The role of the teacher is very crucial to overcome this problem. The teacher should explain students what they
should be careful about in image based questions in detail. In mathematics, teaching should be done in using visual aids. It was found out that students couldn't understand and evaluate mathematics, visual materials and methods which aim at students' five senses should be used to improve understanding. To succeed in geometry learning, it is very important to define objects and their definitions. Students get confused at recognizing the shapes. The reason for this is human perception. To eliminate this problem the teacher should first make students recognize the shapes then teach how to rotate the objects mentally to perceive them more clearly. According to the level of geometric thinking of the students, methods can vary. The teacher should continuously remind students that rotation of an object does not change its shape.

**New practices for geometry lesson General suggestions**

To teach students the names of various shapes, television, books and computer games can be used. Their comprehension of the concept should be improves. Their meta cognitive abilities should be enhanced.

- Teachers should use relevant vocabulary to describe relevant geometric statements and their relationships.
- To assess the validity of geometric arguments a teacher should apply logic. A way of doing this is, analyzing the consequences of using alternative definitions for geometric objects.
- To help students memorize the formulas easier, the formulas can be shown with either proofs using different approaches.

The properties of geometric figures and mathematical thinking should be applied in order to perform and justify basic geometric constructions. Simple straightedge and compass constructions should be performed and explained. In order to increase efficiency and reach aims computer based, visual methods are necessary. To test or create the conjectures of geometric properties or relations geometric computer or calculator packages can be used.

- Geometry sketchpad is a software that can be used for constructing basic geometric figures. It also enables you to edit and with the display menu you can add figures and animate them. It's custom tools let you replay complex geometric constructions in an easy one step way.
- Scheme of work: A detailed scheme which topics and what order topics should be covered.
- Practice book: They provide students plenty of exercises based on the content of the units.
- Powerpoint Representations

**REFERENCES**


Jeavans, A.C, why dynamic geometry software is such an effective tool in mathematics education, Chichester, U.K.

1.