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The challenge for single field inflation with BICEP2 result

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A B S T R A C T

The detection of B-mode power spectrum by the BICEP2 collaboration constrains the tensor-to-scalar ratio $r = 0.20^{+0.07}_{-0.05}$ for the lensed-ΛCDM model. The consistency of this big value with the *Planck* results requires a large running of the spectral index. The large values of the tensor-to-scalar ratio and the running of the spectral index put a challenge to single field inflation. For the chaotic inflation, the larger the value of the tensor-to-scalar ratio is, the smaller the value of the running of the spectral index is. For the natural inflation, the absolute value of the running of the spectral index has an upper limit.

1. Introduction

The detection of the primordial B-mode power spectrum by the BICEP2 collaboration confirms the existence of primordial gravitational wave, and the observed B-mode power spectrum gives the constraint on the tensor-to-scalar ratio with $r = 0.20^{+0.07}_{-0.05}$ at 1σ level for the lensed-ΛCDM model [1]. Furthermore, $r = 0$ is disfavored at 7.0σ level. The new constraints on r and the spectral index n_s exclude a wide class of inflationary models. For the inflation model with non-minimal coupling with gravity [2], a universal attractor at strong coupling was found with $n_s = 1 - 2/N$ and $r = 12/N^2$. This model is inconsistent with the BICEP2 result $r \gtrsim 0.1$ at 2σ level because the BICEP2 constraint on r requires the number of e-folds $N = \sqrt{12/r} \lesssim \sqrt{120} \approx 11$ which is not enough to solve the horizon problem. If we require $N = 50$, then $r = 0.0048$, so the model is excluded by the BICEP2 result. For the small-field inflation like the hilltop inflation with the potential $V(\phi) = V_0[1 - (\phi/\mu)^p]$ [3,4], $r \sim 0$, so the model is excluded by the BICEP2 result.

Without the running of the spectral index, the combination of *Planck* + WP + highL data gives $n_s = 0.9600 \pm 0.0072$ and $r_{0.002} < 0.0457$ at the 68% confidence level for the ΛCDM model [5,6] which is in tension with the BICEP2 result. When the running of the spectral index is included in the data fitting, the same combination gives $n_s = 0.957 \pm 0.015$, $n'_s = dn_s/d\ln k = -0.022^{+0.020}_{-0.021}$ and $r_{0.002} < 0.263$ at the 95% confidence level [5,6]. To give a consis-

tent constraint on r for the combination of *Planck* + WP + highL data and the BICEP2 data, we require a running of the spectral index $n'_s < -0.002$ at the 95% confidence level. For the single field inflation, the spectral index n_s for the scalar perturbation deviates from the Harrison–Zel'dovich value of 1 in the order of 10^{-2}, so n'_s is in the order of 10^{-3}. The explanation of large r and n'_s is a challenge to single field inflation. In light of the BICEP2 data, several attempts were proposed to explain the large value of r [7–30]. In this Letter, we use the chaotic and natural inflation models to explain the challenge.

2. Slow-roll inflation

The slow-roll parameters are defined as

$$\epsilon = \frac{M_{pl}^2 V_\phi^2}{2V^2}, \tag{1}$$

$$\eta = \frac{M_{pl}^2 V_{\phi\phi}}{V}, \tag{2}$$

$$\xi = \frac{M_{pl}^4 V_\phi V_{\phi\phi\phi}}{V^2}, \tag{3}$$

where $M_{pl}^2 = (8\pi G)^{-1}$, $V_\phi = dV(\phi)/d\phi$, $V_{\phi\phi} = d^2V(\phi)/d\phi^2$ and $V_{\phi\phi\phi} = d^3V(\phi)/d\phi^3$. For the single field inflation, the spectral indices, the tensor-to-scalar ratio and the running are given by

$$n_s - 1 \approx 2\eta - 6\epsilon, \tag{4}$$

$$r \approx 16\epsilon \approx -8n_t, \tag{5}$$

$$n'_s = dn_s/d\ln k \approx 16\epsilon\eta - 24\epsilon^2 - 2\xi. \tag{6}$$

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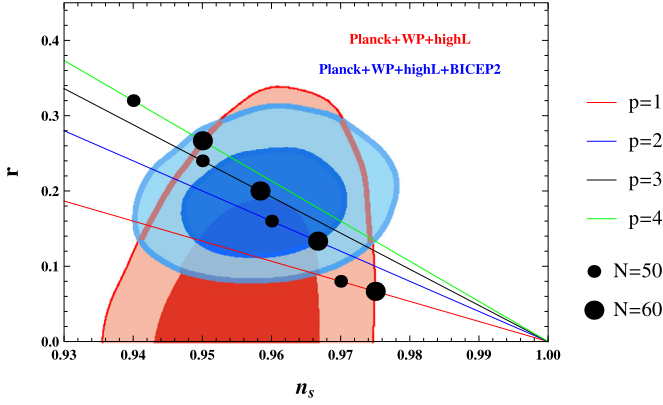
Fig. 1. The n_s–r diagrams for the chaotic inflation with $p = 1$, $p = 2$, $p = 3$ and $p = 4$. The 68% and 95% confidence contours from the *Planck* + WP + highL data [5,6] and the *Planck* + WP + highL + BICEP2 data [1] for the ΛCDM model are also shown.
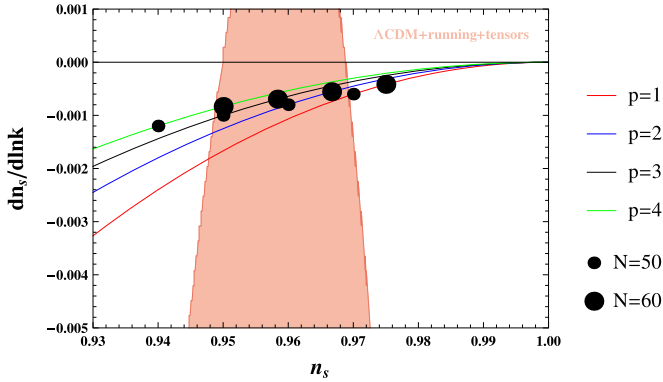

Fig. 2. The n_s–n'_s diagrams for the chaotic inflation with $p = 1$, $p = 2$, $p = 3$ and $p = 4$. The 95% confidence contour for the ΛCDM model from the *Planck* + WP + highL data [5,6] is also shown.

The number of e-folds before the end of inflation is given by

$$N(t) = \int_t^{t_e} H \, dt \approx \frac{1}{M_{pl}^2} \int_{\phi_e}^{\phi} \frac{V(\phi)}{V_\phi(\phi)} d\phi, \tag{7}$$

where the value ϕ_e of the inflaton field at the end of inflation is defined by $\epsilon(\phi_e) = 1$. The scalar power spectrum is

$$\mathcal{P}_\mathcal{R} = A_s \left(\frac{k}{k_*} \right)^{n_s - 1 + n'_s \ln(k/k_*)/2}, \tag{8}$$

where the subscript "$*$" means the value at the horizon crossing, the scalar amplitude

$$A_s \approx \frac{1}{24\pi^2 M_{pl}^4} \frac{\Lambda^4}{\epsilon}. \tag{9}$$

With the BICEP2 result $r = 0.2$, the energy scale of inflation is $\Lambda \sim 2.2 \times 10^{16}$ GeV.

For the chaotic inflation with the power-law potential $V(\phi) = \Lambda^4 (\phi/M_{pl})^p$ [31], the slow-roll parameters are $\epsilon = p/(4N_*)$, $\eta = (p-1)/(2N_*)$ and $\xi = (p-1)(p-2)/(4N_*^2)$. The spectral index $n_s = 1 - (p+2)/(2N_*)$, the running of the spectral index $n'_s = -(2+p)/(2N_*^2) = -2(1-n_s)^2/(p+2) < 0$ and the tensor-to-scalar ratio $r = 4p/N_* = 8p(1-n_s)/(p+2)$. We plot the $n_s - r$ and $n_s - n'_s$ relations in Figs. 1 and 2 for $p = 1$, $p = 2$, $p = 3$ and $p = 4$. In Fig. 1, we also show the points with $N_* = 50$ and $N_* = 60$. From
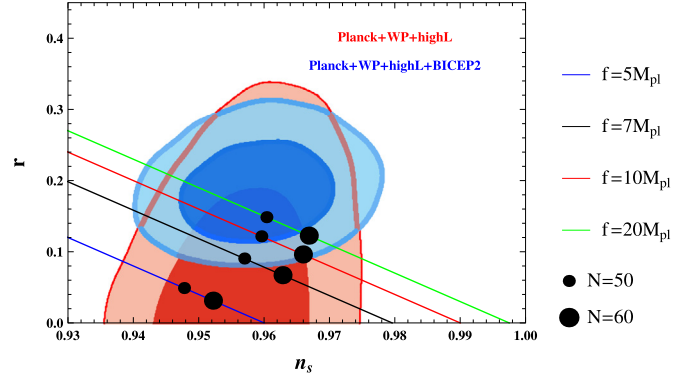

Fig. 3. The n_s–r diagrams for the natural inflation with $f = 5M_{pl}$, $f = 7M_{pl}$, $f = 10M_{pl}$ and $f = 20M_{pl}$. The 68% and 95% confidence contours from the *Planck* + WP + highL data [5,6] and the *Planck* + WP + highL + BICEP2 data [1] for the ΛCDM model are also shown.
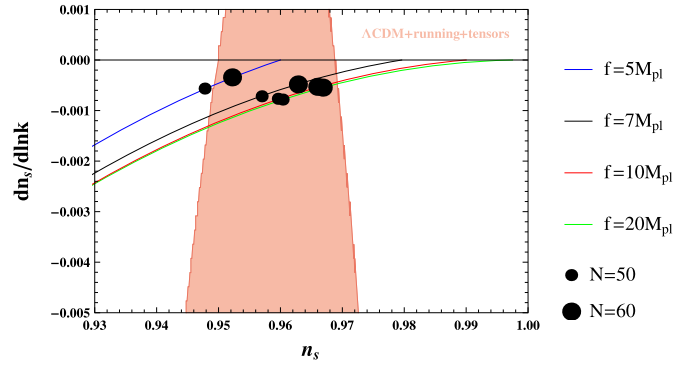

Fig. 4. The n_s–n'_s diagrams for the natural inflation with $f = 5M_{pl}$, $f = 7M_{pl}$, $f = 10M_{pl}$ and $f = 20M_{pl}$. The 95% confidence contour for the ΛCDM model from the *Planck* + WP + highL data [5,6] is also shown.

Figs. 1 and 2, we see that r increases with the power p, but $|n'_s|$ decreases with the power p. Therefore, it is not easy to satisfy both the requirements $r \gtrsim 0.1$ and $n'_s < -0.002$. The chaotic inflation with $2 < p < 3$ is marginally consistent with the observation at the 95% confidence level.

For the natural inflation with the potential $V(\phi) = \Lambda^4[1 + \cos(\phi/f)]$ [32], the slow-roll parameters are

$$\epsilon = \frac{M_{pl}^2}{2f^2} \left[\frac{\sin(\phi/f)}{1 + \cos(\phi/f)} \right]^2, \tag{10}$$

$$\eta = -\frac{M_{pl}^2}{f^2} \frac{\cos(\phi/f)}{1 + \cos(\phi/f)}, \tag{11}$$

$$\xi = -\frac{M_{pl}^4}{f^4} \left[\frac{\sin(\phi/f)}{1 + \cos(\phi/f)} \right]^2 = -\frac{2M_{pl}^2}{f^2} \epsilon. \tag{12}$$

Inflation ends when $\epsilon \sim 1$, so

$$\frac{\phi_e}{f} = \arccos\left[\frac{1 - 2(f/M_{pl})^2}{1 + 2(f/M_{pl})^2} \right], \tag{13}$$

and the number of e-folds before the end of inflation is

$$N_* = \frac{2f^2}{M_{pl}^2} \ln\left[\frac{\sin(\phi_e/2f)}{\sin(\phi_*/2f)} \right]. \tag{14}$$

Combining Eqs. (4)–(6) with (10)–(12), we plot the n_s–r and n_s–n'_s relations for the natural inflation with $f = 5M_{pl}$, $f = 7M_{pl}$, $f = 10M_{pl}$ and $f = 20M_{pl}$ in Figs. 3 and 4. In Fig. 3, we also show the points with $N_* = 50$ and $N_* = 60$. The results show that both r and

$|n'_s|$ increase with the global symmetry breaking scale f. However, there is an upper limit on $|n'_s|$ which is only marginally consistent with the observation at the 95% confidence level. When $f/M_{mp} \gg 1$, the potential can be approximated by $V(\phi) = \Lambda^4(\phi/f - \pi)^2/2$ which is the power-law potential with $p = 2$, this is the reason for the upper limit on n'_s.

3. Conclusions

For a single inflaton field with slow-roll, the tensor-to-scalar ratio $r \approx 16\epsilon$ which is linear with the slow-roll parameter ϵ, but the running of the spectral index n'_s depends on the second order slow-roll parameters, so n'_s is at most in the order of 10^{-3}. The BICEP2 and the *Planck* data constrain $n'_s = -0.0221^{+0.011}_{-0.0099}$ and $r = 0.20^{+0.07}_{-0.05}$ at the 1σ confidence level. Both the chaotic and natural inflation are inconsistent with the observation at the 1σ level. The chaotic inflation with $2 < p < 3$ and the natural inflation with $f \gtrsim 10M_{pl}$ are marginally consistent with the observation at the 95% confidence level. In conclusion, it is a challenge to simultaneously explain r as large as 0.2 and n'_s as large as -0.01 for single field inflation. Unless the *Planck* and the BICEP2 data can be reconciled without large n'_s, the challenge to single field inflation remains.

Acknowledgements

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