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Modified Kalman Filter for GPS Position Estimation over the Indian Sub Continent

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Abstract

Position accuracy is the measure of a system's capability to provide quality estimates, which in turn depends mainly on measurement noise and the type of algorithm employed. Though many positioning algorithms have emerged, due to its exceptional performance in a wide range of real time applications, the Kalman Filter Estimator (KFE) is used often for their implementation. So this paper concentrates in improving its accuracy further while introducing a new observation matrix in its estimation process and succeeds in performing the same. This paper also extends the modified algorithm for GPS receiver position estimation over the Indian subcontinent. Real time data collected from GPS receiver located at IISC, Bangalore (Lat/Lon: 13.01^0N /77.56^0E), India is used to evaluate the performance of this developed algorithm called Modified Kalman Filter Estimator (MKFE). GPS Statistical Accuracy Measures (SAM) such as Circular Error Probability (CEP), Spherical Error Probability (SEP) etc. are used for performance evaluation. From the results it is observed that the proposed MKFE has faster convergence rate with high accuracy and is suitable for real time defence applications over the Indian subcontinent.

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Keywords: CEP, GPS, KalmanFilter Estimator, Modified Kalman Filter Estimator, MRSE, SAM, SEP

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1. Introduction

The introduction of Global Positioning System (GPS) has brought a revolution in the field of positioning and lead to the development of many sophisticated positioning systems in the fields of defence and civil sectors for guiding of troops in battle field, navigation of ships, landing of aircrafts etc[1]. Even when operated to their full capacity, these systems provide limited accuracy for these applications and hence need to be improved.

The three primary functions on which most of today's defence systems work are Navigation, Tracking and Guiding and it is unimaginable to find a system that doesn't have any relation with these functions. These systems require to determine the two or three dimensional (3D) position information of an object of interest and they use GPS receivers for this purpose. GPS is the constellation of space bodies called satellites which revolve round the earth and provides position information based on range measurements[2]. With a sufficient number of range measurements GPS can provide position estimates to high degree accuracy[3]. The accuracy in turn is the function of type of equipment, geographic area, uncertainty in measurements, navigation algorithm etc. In practice, the measurement uncertainty can never reach zero even though the system noise parameters and biases are modeled effectively[4] and hence need a high accuracy navigation algorithm that makes out an optimal estimate from these uncertain measurements.

The various algorithms used for GPS receiver position estimate includes Least Squares[5] (LS), Weighted Least Squares (WLS), Evolutionary optimizers[6], Kalman filter (KF) etc. The task of tracking and guiding involves estimation of objects future course and this could be only possible when the system dynamics are modeled into the estimator. Out of the available Navigation algorithms the only filter that makes use of the dynamics in estimation is Kalmanfilter[7]. In addition, KF also provides the uncertainty in its estimation whose performance varies with parameters like process noise matrix, measurement noise matrix, observation matrix etc. So this paper concentrates in improving KFE accuracy with a new observation matrix that replaces the conventional matrix in Kalman filter's covariance update equation. The modeling of KF as GPS receiver position estimator and the details about new designed observation matrix lead to the development of MKFE are discussed in subsequent sections.

2. Modified Kalman Filter for GPS Receiver Position Estimation

The GPS receiver either onboard an aircraft or fixed stationary on a building roof as show in Fig.1 uses travel time or Time of Flight (TOF) measurements in determining its position. GPS $TOF_{Si,Rx}$ measurement is the time elapse of a signal to reach at the receiver, R_x, from ith satellite, Si. For acceptable level of accuracy in position estimate GPS requires minimum of four TOF measurements from individual satellites[8]. Every TOF corresponds to a range measurement which is formulated as in Eq.1. Solving these set of equations for unknown receiver position is a highly complex process as they are nonlinear[9].

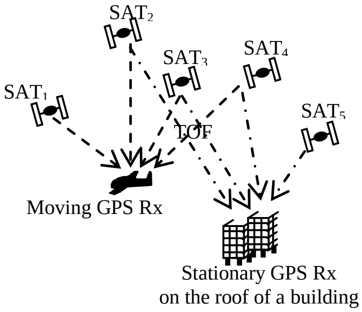

Fig.1. GPS Receiver Placement Scenarios

Hence, Extended Kalman Filter Estimator (EKFE) or KFE with linearised measurement equations is used to estimate the unknown receiver position. The Taylors series first order approximated linear form[10] of Eq.1, computed at $\hat{R}x$ is given in Eq.2 which is used in framing the observation matrix, Ω in KFE.

$$\mathrm{TOF}_{\mathrm{Si,Rx}} = f(S,\mathrm{Rx}) = \sqrt{(x_{si}-x_{Rx})^2+(y_{si}-y_{Rx})^2+(z_{si}-z_{Rx})^2} \qquad i = 1,2...,\mathrm{no.of.satellites} \tag{1}$$

$$f(S,\mathrm{Rx}) \cong f(S,\hat{\mathrm{R}}\mathrm{x}) + f^{'}(S,\hat{\mathrm{R}}\mathrm{x})(\mathrm{Rx}-\hat{\mathrm{R}}\mathrm{x}) \tag{2}$$

Here, $\mathrm{TOF}_{\mathrm{Si,Rx}}$ is the time of flight between ith satellite, Si and receiver, R_x, **S**=(x_{si},y_{si},z_{si}) is the three dimensional position coordinates of ith satellite, **Rx**=(x_{Rx},y_{Rx},z_{Rx}) is the three dimensional position coordinates of receiver and $f^{'}$ represents the first order derivative of function f(S,Rx).

The Eq.2 can also be represented as

$$f(S,\mathrm{Rx}) - f(S,\hat{\mathrm{R}}\mathrm{x}) \cong f^{'}(S,\hat{\mathrm{R}}\mathrm{x})(\mathrm{Rx}-\hat{\mathrm{R}}\mathrm{x}) \Rightarrow \delta\mathrm{TOF}_{\mathrm{Si,Rx}} \cong \frac{\partial f(S,\mathrm{Rx})}{\partial \mathrm{Rx}}\Big|_{\mathrm{R_X}=\hat{\mathrm{R}}_\mathrm{X}} \delta\mathrm{Rx} \Rightarrow \delta\mathrm{TOF}_{\mathrm{Si,Rx}} \cong \Omega(S,\hat{\mathrm{R}}\mathrm{x})\delta\mathrm{Rx} \tag{3}$$

Here, $\delta\mathrm{TOF}_{\mathrm{Si,Rx}}$ is the error or change in range measurements, δRx is the error or change in receiver position, $\hat{\mathrm{R}}\mathrm{x}$ is the estimated receiver position, and the term $\partial f(S,\mathrm{Rx})/\partial\mathrm{Rx}$ is observation matrix, **Ω** or Jacobian matrix, J. As mentioned previously in order to provide the position estimates and the uncertainty in estimation, KFE uses the observation matrix, **Ω** in two stages[11], prediction and updation which are formulated as below.

Prediction Stage:

$$\mathrm{Rx}^{P}_{t/t-1} = \varphi \mathrm{Rx}^{U}_{t-1/t-1} + e_{\mathrm{Rx}} \tag{4}$$

$$C^{P}_{t/t-1} = \phi \mathrm{Rx}^{P}_{t/t-1} \phi^{T} + \xi \tag{5}$$

Updation Stage:

$$K_G = C^{P}_{t/t-1}\Omega^{T}_{t/t-1}\left(\Omega_{t/t-1}C^{P}_{t/t-1}\Omega^{T}_{t/t-1} + \beta\right)^{-1} \tag{6}$$

$$\mathrm{Rx}^{U}_{t/t} = \mathrm{Rx}^{P}_{t/t-1} + K_G(\mathrm{TOF}_t - \Omega_{t/t-1}\mathrm{Rx}^{P}_{t/t-1}) \tag{7}$$

$$C^{U}_{t/t} = \left(I - K_G\Omega_{t/t-1}\right)C^{P}_{t/t-1} \tag{8}$$

Where, $\mathrm{Rx}^{P}_{t/t-1}$ is the predicted or estimated receiver position at time (t), $\mathrm{Rx}^{U}_{t/t}$ is the updated receiver position estimate at time (t) on post reception of measurements TOF at time (t), $C^{P}_{t/t-1}$ represents the uncertainty in the predicted receiver position at time (t-1), $C^{U}_{t/t-1}$ represents the uncertainty in updated receiver position at time (t), K_G is the Kalman Gain, ϕ represents receiver position state transition matrix, **Ω** is observation matrix, I is the identity matrix, β the measurement error covariance matrix, e_{Rx} and ξ are the process noise and its uncertainty respectively.

It is observed from Eq.3 that the observation matrix, **Ω** is the resultant of first order approximation from Taylor's series and as defined in [12,13] a nonlinear function like f(S, Rx) is modifiable if there exist a linear structure which is the function of the predicted state, $\mathrm{Rx}^{P}_{t/t-1}$ or $\hat{\mathrm{R}}\mathrm{x}$ and the actual measurement, $\mathrm{TOF}_{\mathrm{Si,Rx}}$, i.e. if f(S,Rx) is modifiable than Eq.3 can be rewritten as Eq.9.

$$\delta\mathrm{TOF}_{\mathrm{Si,Rx}} = N(\mathrm{TOF}_{\mathrm{Si,Rx}}, \hat{\mathrm{R}}\mathrm{x})\delta\mathrm{Rx} \tag{9}$$

In practice, the range measurements are noisy and hence $\mathrm{TOF}^{*}_{\mathrm{Si,Rx}}$ is used in Eq.9 instead of $\mathrm{TOF}_{\mathrm{Si,Rx}}$.The developed MKFE makes use of the new observation matrix, N instead of **Ω** in Eq.8, while retaining the other KFE equations as same[7]. Kalman Gain, K_G in Eq.6 is the function of **Ω**, $C^{P}_{t/t-1}$ and β, which decides the amount of weight to be imposed on current measurements while updating the receiver position and hence making this equation the function of current measurements, $\mathrm{TOF}^{*}_{\mathrm{Si,Rx}}$ results in poor performance[12]. This is the reason the new observation matrix, N is used in Eq.8 and the resultant new updated covariance matrix for MKFE is given in Eq.10

$$C^{U}_{t/t-1} = \left(I - K_G N_{t/t-1}\right) C^{P}_{t/t-1} \tag{10}$$

3. Results and Discussion

Real time data collected from GPS receiver located at IISC, Bangalore is used in the performance evaluation of KFE and the developed MKFE. The three dimensional position of the receiver is estimated over a period of 23hr 56mins (2872 epochs) with a randomly choose initial position estimate of X: 785m, Y: 746m, Z: 3459m. The data is collected at a sampling interval of 30sec and the position error of epochs the estimator took to reach convergence (i.e., position error in three dimensions < 100m) is plotted in Fig.2. The details pertaining to the convergence epochs are given in Table1. It is observed from the figure and Table1 that the developed algorithm, MKFE converges at faster rate compared to the conventional KFE.

Table.1 Estimator Convergence Performance

GPS-Receiver	KFE	MKFE
IISC, Bangalore	125	108

Also the GPS Statistical Accuracy Measures (SAM) for both the algorithms are calculated for the entire range of data and tabulated in Table 2. Various SAM[14] such as 1(σ) sigma error, Circular Error Probability (CEP), Distance Root Mean Square (DRMS), Spherical Error Probability (SEP), Mean Radial Spherical Error (MRSE), Spherical Accuracy Standard (SAS) are used in the evaluation of algorithms accuracy performance. Table 2 depicts the SAM of both the algorithms on IISC, Bangalore receiver and is given below.

Table.2 SAM of KFE and developed MKFE for IISC, Bangalore GPS receiver

IISC Bangalore GPS Receiver			
Statistical Accuracy Measures		Kalman Filter Estimator (meters)	Modified Kalman Filter Estimator (meters)
Mean	X- Coordinate	27.91	27.65
	Y- Coordinate	56.58	55.18
	Z- Coordinate	13.11	13.09
Deviation	X- Coordinate	9.91	9.29
	Y- Coordinate	28.64	24.38
	Z- Coordinate	6.70	6.32
(1D) 1(σ) Sigma (68%)	X- Coordinate	27.91±9.91	27.65±9.29
	Y- Coordinate	56.58±28.64	55.18±24.38
	Z- Coordinate	13.11±6.70	13.09±6.32
(2D) Horizontal	DRMS (65%)	30.31	26.09
	CEP (50%)	23.31	20.32
	2DRMS (95%)	60.62	52.19
(3D) Horizontal & Vertical	SEP (50%)	23.08	20.40
	MRSE (61%)	31.04	26.85
	SAS (99%)	37.70	33.32

It is obvious from the accuracy measures that for IISC, Bangalore receiver the position estimated by the MKFE will be within 33.32 meters from its true position with a probability of 0.99, where KFE estimates the position within 37.70 meters. This shows the efficiency of developed algorithm over conventional KFE.

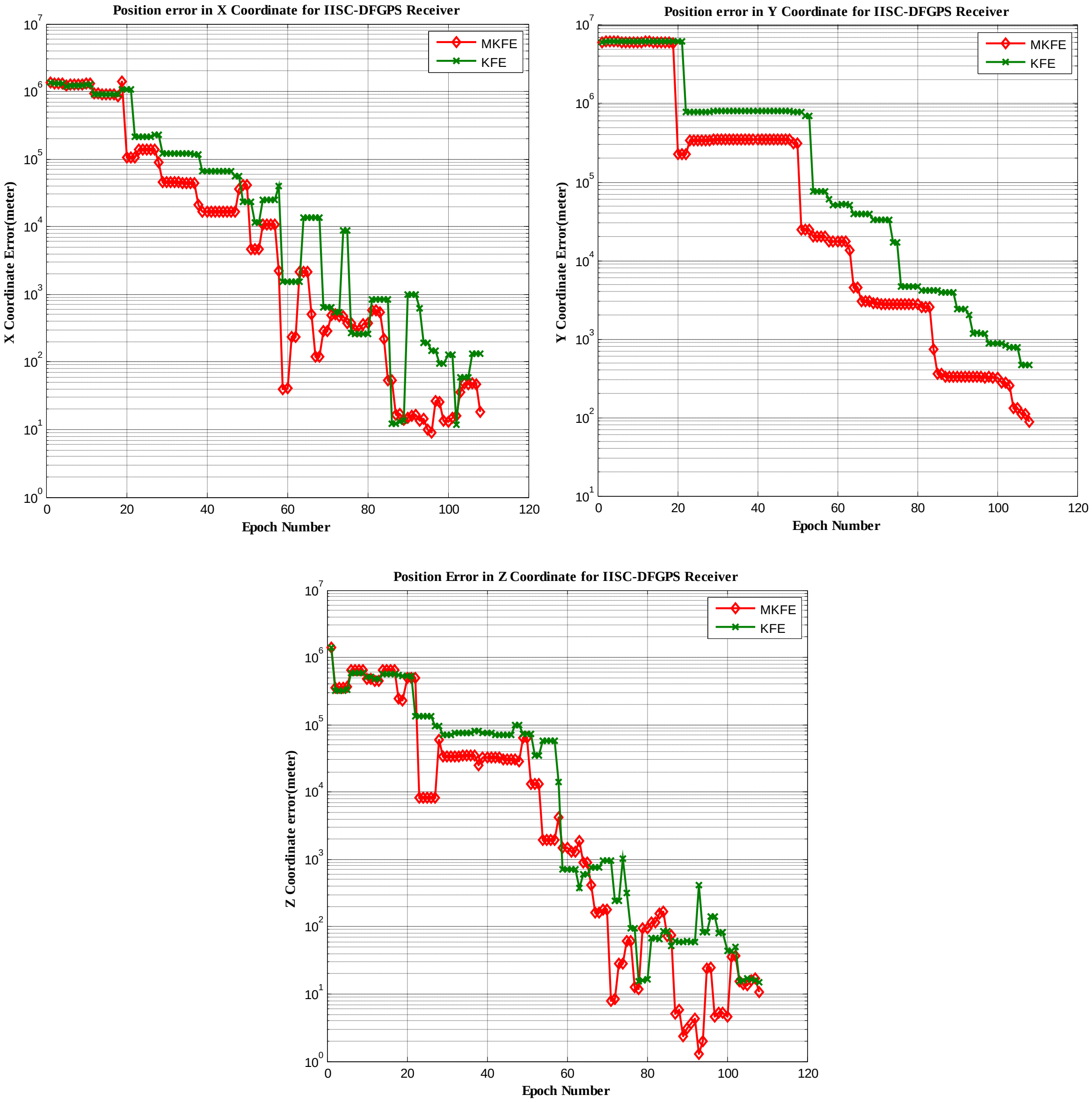

Fig.2 Position Error Vs Epochs (a) Position Error in X Coordinate with KFE and MKFE for IISC, Bangalore GPS receiver (b) Position Error in Y Coordinate with KFE and MKFE for IISC, Bangalore GPS receiver (c) Position Error in Z Coordinate with KFE and MKFE for IISC, Bangalore GPS receiver.

4. Conclusions

A new algorithm for the estimation of unknown system parameters like the GPS receiver position is proposed based on the modified observation matrix of KFE. The observation matrix designed out of first order Taylor's approximation of nonlinear measurement function is modified and used for parameter estimation. The developed algorithm was used to estimate the three dimensional position of the GPS receiver and its performance was evaluated with various SAM. Real time data collected for a period of 23hrs 56mins from the GPS receiver located at IISC, Bangalore is used in performance evaluation. Results demonstrated that MKFE converges in less time with an epoch difference of 17 and has an accuracy difference of 3 meters CEP when compared to conventional KFE. This also showed that the MKFE has faster convergence rate with high accuracy and is suitable for real time defence applications like navigation of ships, landing of CAT I and II aircrafts etc. over the Indian subcontinent.

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