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Journal of Ocean Engineering and Science 000 (2016) 1–13

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New exact solutions of coupled Boussinesq–Burgers equations by Exp-function method

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Received 3 August 2016; received in revised form 8 September 2016; accepted 18 September 2016
Available online xxx

Abstract

In the present paper, we build the new analytical exact solutions of a nonlinear differential equation, specifically, coupled Boussinesq–Burgers equations by means of Exp-function method. Then, we analyze the results by plotting the three dimensional soliton graphs for each case, which exhibit the simplicity and effectiveness of the proposed method. The primary purpose of this paper is to employ a new approach, which allows us victorious and efficient derivation of the new analytical exact solutions for the coupled Boussinesq–Burgers equations.

Keywords: Exp-function method; Solitary wave solutions; Non-linear evolution equations; Coupled Boussinesq–Burgers equations.

1. Introduction

A well-known model is the coupled Boussinesq–Burgers (BB) equations [1]

$$u_t = -2uu_x + \frac{1}{2}v_x, \tag{1.1}$$

$$v_t = \frac{1}{2}u_{xxx} - 2(uv)_x, \tag{1.2}$$

where x and t represent the normalized space and time respectively. Here $u(x, t)$ represents the horizontal velocity and at the leading order it is the depth averaged horizontal field, while $v(x, t)$ denotes the height of the water surface above the horizontal level at the bottom.

The Boussinesq–Burgers equations emerge in the investigation of fluid flow and represent the proliferation of shallow water waves [1]. Kaup [2] has been investigated IST integrability of Boussinesq–Burgers equation. The multi-phase periodic solutions of BB equations have been has studied in [3,4]. The soliton and multi-soliton of Eqs. (1.1) and (1.2) has been discussed in [5–7].The exact travelling solutions of BB equations have been discussed in [8,9]. El et al. [10,11] removed an initial discontinuity of BB equations by applying Whitham theory of modulations. Kamchatnov et al. [12] described a quasi-classical soliton trains soliton trains of BB equations arising from a large initial pulse. By using Lie symmetry analysis Mhlanga and Khalique [13] describe the travelling wave solution of generalized coupled BB equation. The envelope solition and periodic wave solutions have been studied by Ebadi et al. [14].

Recently a variety of powerful methods [15–18] have been applied in numerous physical models [19–21] and analyzed to obtain the exact solutions to nonlinear evolution equations [22,23].

In the present paper, we implement Exp-function method to find the new exact Soliton Solitary wave solutions of Eqs. (1.1) and (1.2). Exp-function method introduced by He and Wu [24] in 2006, is successfully applied to many kind of nonlinear differential equations.

This method has been successfully implemented on Burgers equations, KdVs–Burgers–Kuramoto equation, two dimensional Bratu-type equation, Generalized Fisher equation, higher order boundary value problem, modified Zakharov–Kuznetsov equation; see [25–28]. This clearly indicates that Exp-function method is easy, concise and an effective method to implement to nonlinear evolution equations arising in mathematical physics. The main advantage of this method over

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http://dx.doi.org/10.1016/j.joes.2016.09.001
2468-0133/

the other methods is that it gives more general solutions with some free parameters.

The rest of the paper is organized as follow: in Section 2, we proposed the algorithm of Exp-function method. Then in Section 3, we implement the proposed method to solve coupled Boussinesq–Burger equations and also we analyzed obtained the solutions by three dimensional graphs in Section 4. Some conclusions are presented in Section 5.

2. Basic idea of Exp-function method

Consider a given nonlinear wave equation

$$N(u, u_t, u_x, u_{xx}, u_{tt}, u_{xt}, \ldots) = 0, \tag{2.1}$$

We seek its wave solutions

$$u = u(\eta), \quad \eta = kx + wt. \tag{2.2}$$

Consequently, (2.1) is reduced to the ordinary differential equation:

$$N(u, wu', ku', k^2u'', w^2u'', kwu'', \ldots) = 0 \tag{2.3}$$

The Exp-function method is based on the assumption that the travelling wave solution can be expressed in the following form:

$$u(\eta) = \frac{\sum_{n=-d}^{c} a_n \exp(n\eta)}{\sum_{m=-q}^{p} b_m \exp(m\eta)} \tag{2.4}$$

where c, d, p and q are positive integers, a_n and b_m are unknown constants.

3. Application of Exp-function to coupled Boussinesq–Burgers equations

Using the transformation $u=u(\eta)$, $v=v(\eta)$, $\eta=kx+wt$ Eqs. (1.1) and (1.2) become the ordinary differential equations,

$$wu' + 2kuu' - \frac{1}{2}kv' = 0, \tag{3.1}$$

$$wv' - \frac{1}{2}k^3u''' + 2k(uv)' = 0, \tag{3.2}$$

On integrating Eqs. (3.1) and (3.2) once with reference to η and assuming that the constant of integration to be zero, we get

$$wu + ku^2 - \frac{1}{2}kv = 0 \tag{3.3}$$

$$wv - \frac{1}{2}k^3u'' + 2kuv = 0 \tag{3.4}$$

According to Exp-function method, we assume that the solution of Eqs. (3.1) and (3.2) can be expressed in the form,

$$u(\eta) = \frac{\sum_{n=-g}^{h} a_n \exp(n\eta)}{\sum_{m=-q}^{p} b_m \exp(m\eta)} = \frac{a_{-g}\exp(-g\eta) + \cdots + a_h \exp(h\eta)}{b_{-q}\exp(-q\eta) + \cdots + b_p \exp(p\eta)}, \tag{3.5}$$

$$v(\eta) = \frac{\sum_{n=-i}^{j} d_n \exp(n\eta)}{\sum_{m=-s}^{t} c_m \exp(m\eta)} = \frac{d_{-i}\exp(-i\eta) + \cdots + d_j \exp(j\eta)}{c_{-s}\exp(-s\eta) + \cdots + c_t \exp(t\eta)}. \tag{3.6}$$

where h, g, p, q, j, i, t and s are positive integers a_n, b_m, d_n and c_m are unknown constants.

For simplicity, we set $h=p=1$, $g=q=1$ and $j=t=1$, $i=s=1$, then Eqs. (3.5) and (3.6) reduced to

$$u(\eta) = \frac{a_{-1}\exp(-\eta) + a_0 + a_1\exp(\eta)}{b_{-1}\exp(-\eta) + b_0 + \exp(\eta)}, \tag{3.7}$$

$$v(\eta) = \frac{d_{-1}\exp(-\eta) + d_0 + d_1\exp(\eta)}{c_{-1}\exp(-\eta) + c_0 + \exp(\eta)}. \tag{3.8}$$

Substituting Eqs. (3.7) and (3.8) into Eqs. (3.3) and (3.4), we have

$$\frac{1}{A}\{\delta_{-3}\exp(-3\eta) + \delta_{-2}\exp(-2\eta) + \delta_{-1}\exp(-\eta) + \delta_0 + \delta_1\exp(\eta) + \delta_2\exp(2\eta) + \delta_3\exp(3\eta)\} = 0, \tag{3.9}$$

$$\frac{1}{B}\{\xi_{-4}\exp(-4\eta) + \xi_{-3}\exp(-3\eta) + \xi_{-2}\exp(-2\eta) + \xi_{-1}\exp(-\eta) + \xi_0 + \xi_1\exp(\eta) + \xi_2\exp(2\eta) + \xi_3\exp(3\eta) + \xi_4\exp(4\eta)\} = 0. \tag{3.10}$$

where

$$A = (b_{-1}\exp(-\eta) + b_0 + \exp(\eta))^2 (c_{-1}\exp(-\eta) + c_0 + \exp(\eta)),$$
$$B = (b_{-1}\exp(-\eta) + b_0 + \exp(\eta))^3 (c_{-1}\exp(-\eta) + c_0 + \exp(\eta)),$$

and δ_{-3}, δ_{-2}, δ_{-1}, δ_0, δ_1, δ_2, δ_3, ξ_{-4}, ξ_{-3}, ξ_{-2}, ξ_{-1}, ξ_0, ξ_1, ξ_2, ξ_4 are defined in Appendix 1.

By setting,

$$\delta_{-3} = \delta_{-2} = \delta_{-1} = \delta_0 = \delta_1 = \delta_2 = \delta_3 = 0, \text{ and}$$
$$\xi_{-4} = \xi_{-3} = \xi_{-2} = \xi_{-1} = \xi_0 = \xi_1 = \xi_2 = \xi_3 = \xi_4 = 0\text{'}$$

and solving these system of algebraic equations simultaneously with the aid of symbolic computation system of mathematical software, we obtain the following results,

Case I.

$$a_{-1} = 0, a_0 = -\frac{\sqrt{w}b_0}{\sqrt{2}}, a_1 = 0, b_{-1} = 0, d_{-1} = 0,$$
$$d_0 = -wb_0, d_1 = 0, c_{-1} = b_0^2,$$
$$c_0 = 2b_0, k = \sqrt{2w},$$

Substituting these values into Eqs. (3.7) and (3.8), we get:

$$u(x,t) = -\frac{\sqrt{w}b_0}{\sqrt{2}[b_0 + \cos h(kx+wt) + \sin h(kx+wt)]},$$

$$v(x,t) = -\frac{wb_0}{2b_0 + \cos h(kx+wt) + b_0^2(\cos h(kx+wt) - \sin h(kx+wt)) + \sin h(kx+wt)}.$$

Case II.

$$a_{-1} = 0, a_0 = \frac{\sqrt{w}b_0}{\sqrt{2}}, a_1 = 0,\ \ b_{-1} = 0,\ \ d_{-1} = 0, d_0 = -wb_0,\ \ d_1 = 0,\ \ c_{-1} = b_0^2, c_0 = 2b_0,\ \ k = -\sqrt{2w}$$

Substituting these values into Eqs. (3.9) and (3.10), we get:

$$u(x,t) = \frac{\sqrt{w}b_0}{\sqrt{2}(b_0 + \cos h(kx+wt) + \sin h(kx+wt))},$$

$$v(x,t) = -\frac{wb_0}{2b_0 + \cos h(kx+wt) + b_0^2(\cos h(kx+wt) - \sin h(kx+wt)) + \sin h(kx+wt)}.$$

Case III.

$$a_{-1} = 0, a_0 = 0, a_1 = -\frac{\sqrt{w}}{\sqrt{2}}, b_{-1} = 0, d_{-1} = 0,$$
$$d_0 = -wb_0, d_1 = 0, c_{-1} = b_0^2,$$
$$c_0 = 2b_0, k = \sqrt{2w},$$

Substituting these values into Eqs. (3.7) and (3.8), we get:

$$u(x,t) = -\frac{\sqrt{w}(\cos h(kx+wt) + \sin h(kx+wt))}{\sqrt{2}(b_0 + \cos h(kx+wt) + \sin h(kx+wt))},$$

$$v(x,t) = -\frac{wb_0}{2b_0 + \cos h(kx+wt) + b_0^2(\cos h(kx+wt) - \sin h(kx+wt)) + \sin h(kx+wt)},$$

Case IV.

$$a_{-1} = 0, a_0 = 0, a_1 = \frac{\sqrt{w}}{\sqrt{2}}, b_{-1} = 0, d_{-1} = 0,$$
$$d_0 = -wb_0, d_1 = 0, c_{-1} = b_0^2,$$
$$c_0 = 2b_0, k = -\sqrt{2w},$$

Substituting these values into Eqs. (3.7) and (3.8), we get:

$$u(x,t) = \frac{\sqrt{w}(\cos h(kx+wt) + \sin h(kx+wt))}{\sqrt{2}(b_0 + \cos h(kx+wt) + \sin h(kx+wt))},$$

$$v(x,t) = -\frac{wb_0}{2b_0 + \cos h(kx+wt) + b_0^2(\cos h(kx+wt) - \sin h(kx+wt)) + \sin h(kx+wt)}.$$

Case V.

$$a_{-1} = \frac{\sqrt{w}b_{-1}}{\sqrt{2}}, a_0 = \frac{1}{4}[\sqrt{2w}b_0 - \sqrt{2}M_1], a_1 = 0, d_{-1} = 0,$$

$$d_0 = \frac{-w^2 b_0 + w^{3/2} M_1}{2w}, d_1 = 0,$$

$$c_{-1} = \frac{[3w^{5/2} b_{-1} b_0 - w^{5/2} b_0^3 - w^2 (b_{-1} - b_0^2) M_1}{-w^{5/2} b_0 + w^2 M_1},$$

$$c_0 = \frac{2[2w^{5/2} b_{-1} - w^{5/2} b_0^2 + w^2 b_0 M_1]}{-w^{5/2} b_0 + w^2 M_1},\ k = -\sqrt{2w},$$

Substituting these values into Eqs. (3.7) and (3.8), we get:

$$u(x,t) = \frac{\left[2\sqrt{w} b_{-1} + (\sqrt{w} b_0 - M_1)(\sin h(kx+wt) + \cos h(kx+wt))\right]}{2\sqrt{2}(b_{-1} + b_0(\cos h(kx+wt) + \sin h(kx+wt)) + \cos h(2kx+2wt) + \sin h(2kx+2wt))},$$

$$v(x,t) = -\left[\sqrt{w}\left(-4wb_{-1} + 2wb_0^2 - 2\sqrt{w} b_0 M_1\right)(\cos h(kx+wt) + \sin h(kx+wt))\right] \times \frac{1}{(2\left(-3\sqrt{w} b_{-1} b_0 + \sqrt{w} b_0^3 + (b_{-1} - b_0^2) M_1 + \left(-4\sqrt{w} b_{-1} + 2\sqrt{w} b_0^2 - 2b_0 M_1\right)(\cos h(kx+wt) + \sin h(kx+wt)) + \left(\sqrt{w} b_0 - M_1\right)(\cos h(2kx+2wt) + \sin h(2kx+2wt))\right)\right)},$$

where $M_1 = \sqrt{-4wb_{-1} + wb_0^2}$.

Case VI.

$$a_{-1} = -\frac{\sqrt{w} b_{-1}}{\sqrt{2}}, a_0 = \frac{1}{4}[-\sqrt{2w} b_0 - \sqrt{2} M_1], a_1 = 0, d_{-1} = 0,$$

$$d_0 = \frac{-w^2 b_0 - w^{3/2} M_1}{2w}, d_1 = 0,$$

$$c_{-1} = \frac{\left[-3w^{5/2} b_{-1} b_0 + w^{5/2} b_0^3 - w^2 (b_{-1} - b_0^2) M_1\right]}{w^{5/2} b_0 + w^2 M_1},$$

$$c_0 = \frac{2\left[-2w^{5/2} b_{-1} + w^{5/2} b_0^2 + w^2 b_0 M_1\right]}{\left[w^{5/2} b_0 + w^2 M_1\right]}, k = \sqrt{2w},$$

Substituting these values into Eqs. (3.7) and (3.8), we get:

$$u(x,t) = \frac{\left[-2\sqrt{w} b_{-1} - (\sqrt{w} b_0 + M_1)(\sin h(kx+wt) + \cos h(kx+wt))\right]}{2\sqrt{2}\left[b_{-1} + b_0(\cos h(kx+wt) + \sin h(kx+wt)) + \cos h(2kx+2wt) + \sin h(2kx+2wt)\right]},$$

$$v(x,t) = -\left[\sqrt{w}\left(-4wb_{-1} + 2wb_0^2 + 2\sqrt{w} b_0 M_1\right)(\cos h(kx+wt) + \sin h(kx+wt))\right] \times \frac{1}{[2\left(-3\sqrt{w} b_{-1} b_0 + \sqrt{w} b_0^3 + (-b_{-1} + b_0^2) M_1 + \left(-4\sqrt{w} b_{-1} + 2\sqrt{w} b_0^2 + 2b_0 M_1\right)(\cos h(kx+wt) + \sin h(kx+wt)) + \left(\sqrt{w} b_0 + M_1\right)(\cos h(2kx+2wt) + \sin h(2kx+2wt))\right)]},$$

where $M_1 = \sqrt{-4wb_{-1} + wb_0^2}$.

Case VII.

$$a_{-1} = \frac{\sqrt{w} b_{-1}}{\sqrt{2}}, a_0 = \frac{1}{4}[\sqrt{2w} b_0 + \sqrt{2} M_1], a_1 = 0, d_{-1} = 0,$$

$$d_0 = \frac{-w^2 b_0 - w^{3/2} M_1}{2w}, d_1 = 0,$$

$$c_{-1} = \frac{\left[-3w^{5/2} b_{-1} b_0 + w^{5/2} b_0^3 - w^2 (b_{-1} - b_0^2) M_1\right]}{w^{5/2} b_0 + w^2 M_1},$$

$$c_0 = \frac{2\left[-2w^{5/2} b_{-1} + w^{5/2} b_0^2 + w^2 b_0 M_1\right]}{\left[w^{5/2} b_0 + w^2 M_1\right]}, k = -\sqrt{2w},$$

Substituting these values into Eqs. (3.7) and (3.8), we get:

$$u(x,t) = \frac{\left[2\sqrt{w} b_{-1} + \left(\sqrt{w} b_0 + M_1\right)(\sin h(kx+wt) + \cos h(kx+wt))\right]}{2\sqrt{2}\left[b_{-1} + b_0(\cos h(kx+wt) + \sin h(kx+wt)) + \cos h(2kx+2wt) + \sin h(2kx+2wt)\right]},$$

$$v(x,t) = -\left[\sqrt{w}\left(-4wb_{-1} + 2wb_0^2 + 2\sqrt{w} b_0 M_1\right)(cosh(kx+wt) + sinh(kx+wt))\right] \times \frac{1}{\left[2\left(-3\sqrt{w} b_{-1} b_0 + \sqrt{w} b_0^3 + (-b_{-1} + b_0^2) M_1 + (-4\sqrt{w} b_{-1} + 2\sqrt{w} b_0^2 + \right.\right.}$$
$$2b_0 M_1)(\cos h(kx+wt) + \sin h(kx+wt)) + (\sqrt{w} b_0$$
$$M_1)(\cos h(2kx+2wt) + \sin h(2kx+2wt)))],$$

where $M_1 = \sqrt{-4wb_{-1} + wb_0^2}$.

Case VIII.

$$a_{-1} = -\frac{\sqrt{w} b_{-1}}{\sqrt{2}}, a_0 = \frac{1}{4}[-\sqrt{2w} b_0 + \sqrt{2} M_1], a_1 = 0, d_{-1} = 0,$$

$$d_0 = \frac{-w^2 b_0 + w^{3/2} M_1}{2w}, d_1 = 0,$$

$$c_{-1} = \frac{\left[3w^{5/2} b_{-1} b_0 - w^{5/2} b_0^3 - w^2 (b_{-1} - b_0^2) M_1\right]}{-w^{5/2} b_0 + w^2 M_1},$$

$$c_0 = \frac{2\left[2w^{5/2} b_{-1} - w^{5/2} b_0^2 + w^2 b_0 M_1\right]}{\left[-w^{5/2} b_0 + w^2 M_1\right]}, k = \sqrt{2w},$$

Substituting these values into Eqs. (3.7) and (3.8), we get:

$$u(x,t) = \frac{\left[-2\sqrt{w} b_{-1} - \left(\sqrt{w} b_0 - M_1\right)(\sin h(kx+wt) + \cos h(kx+wt))\right]}{2\sqrt{2}\left[b_{-1} + b_0(\cos h(kx+wt) + \sin h(kx+wt)) + \cos h(2kx+2wt) + \sin h(2kx+2wt)\right]}$$

$$v(x,t) = -\left[\sqrt{w} \times \left(-4wb_{-1} + 2wb_0^2 - 2\sqrt{w} b_0 M_1\right) \times (\cos h(kx+wt) + \sin h(kx+wt))\right] \times \frac{1}{\left[2\left(-3\sqrt{w} b_{-1} b_0 + \sqrt{w} b_0^3 + (b_{-1} - b_0^2) M_1 + (-4\sqrt{w} b_{-1} + 2\sqrt{w} b_0^2 - \right.\right.}$$
$$2b_0 M_1)(\cos h(kx+wt) + \sin h(kx+wt)) + (\sqrt{w} b_0 - M_1)\,(\cos h(2kx+2wt) + \sin h(2kx+2wt)))]$$

where $M_1 = \sqrt{-4wb_{-1} + wb_0^2}$.

Case IX.

$$a_{-1} = 0, a_0 = \frac{1}{4}(\sqrt{2w} b_0 + M_1), a_1 = \frac{\sqrt{w}}{\sqrt{2}}, d_{-1} = 0,$$

$$d_0 = \frac{-w^2 b_0 + w^{3/2} M_1}{2w}, d_1 = 0,$$

$$c_{-1} = \frac{(3w^{5/2}b_{-1}b_0 - w^{5/2}b_0^3 - w^2(b_{-1} - b_0^2)M_1)}{-w^{5/2}b_0 + w^2\sqrt{w(-4b_{-1} + b_0^2)}},$$

$$c_0 = \frac{2(2w^{5/2}b_{-1} - w^{5/2}b_0^2 + w^2b_0M_1)}{(-w^{5/2}b_0 + w^2M_1)}, k = -\sqrt{2w},$$

Substituting these values into Eqs. (3.7) and (3.8), we get:

$$u(x,t) = \left[(\cos h(kx+wt) + \sin h(kx+wt))\left(\sqrt{w}b_0 + M_1\right) + 2\sqrt{w}(\cos h(kx+wt) + \sin h(kx+wt))\right] \times \frac{1}{\left(2\sqrt{2}[b_{-1} + b_0(\cos h(kx+wt) + \sin h(kx+wt)) + \cos h(2kx+2wt) + \sin h(2kx+2wt)]\right)}$$

$$v(x,t) = -\left[\sqrt{w}\left(-4wb_{-1} + 2wb_0^2 - 2\sqrt{w}b_0M_1\right) \times (\cos h(kx+wt) + \sin h(kx+wt))\right] \times \frac{1}{\left[2\left(-3\sqrt{w}b_{-1}b_0 + \sqrt{w}b_0^3 + (b_{-1} - b_0^2)M_1 + \left(-4\sqrt{w}b_{-1} + 2\sqrt{w}b_0^2 - 2b_0M_1\right)(\cos h(kx+wt) + \sin h(kx+wt)) + \left(\sqrt{w}b_0 - M_1\right)(\cos h(2kx+2wt) + \sin h(2kx+2wt))\right)\right]},$$

where $M_1 = \sqrt{-4wb_{-1} + wb_0^2}$.

Case X.

$$a_{-1} = 0, a_0 = \frac{1}{4}(-\sqrt{2w}b_0 - \sqrt{2}M_1), a_1 = -\frac{\sqrt{w}}{\sqrt{2}}, d_{-1} = 0,$$

$$d_0 = \frac{-w^2b_0 + w^{3/2}M_1}{2w}, d_1 = 0,$$

$$c_{-1} = \frac{(3w^{5/2}b_{-1}b_0 - w^{5/2}b_0^3 - w^2(b_{-1} - b_0^2)M_1)}{-w^{5/2}b_0 + w^2M_1},$$

$$c_0 = \frac{2(2w^{5/2}b_{-1} - w^{5/2}b_0^2 + w^2b_0M_1)}{(-w^{5/2}b_0 + w^2M_1)}, k = \sqrt{2w},$$

Substituting these values into Eqs. (3.7) and (3.8), we get:

$$u(x,t) = -\left[(\cos h(kx+wt) + \sin h(kx+wt))\left(\sqrt{w}b_0 + M_1 + 2\sqrt{w}(\cos h(kx+wt) \sin h(kx+wt))\right)\right] + \times \frac{1}{\left(2\sqrt{2}[b_{-1} + b_0(\cos h(kx+wt) + \sin h(kx+wt)) + \cos h(2kx+2wt) + \sin h(2kx+2wt)]\right)}$$

$$v(x,t) = -\left[\sqrt{w}\left(-4wb_{-1} + 2wb_0^2 - 2\sqrt{w}b_0M_1\right)(\cos h(kx+wt) + \sin h(kx+wt))\right] \times \frac{1}{\left[2\left(-3\sqrt{w}b_{-1}b_0 + \sqrt{w}b_0^3 + (b_{-1} - b_0^2)M_1 + \left(-4\sqrt{w}b_{-1} + 2\sqrt{w}b_0^2 - 2b_0M_1\right)(\cos h(kx+wt) + \sin h(kx+wt)) + \left(\sqrt{w}b_0 - M_1\right)(\cos h(2kx+2wt) + \sin h(2kx+2wt))\right)\right]},$$

where $M_1 = \sqrt{-4wb_{-1} + wb_0^2}$.

Please cite this article as: L.K. Ravi et al., New exact solutions of coupled Boussinesq–Burgers equations by Exp-function method, Journal of Ocean Engineering and Science (2016), http://dx.doi.org/10.1016/j.joes.2016.09.001

Case XI.

$$a_{-1}=0,\, a_0=\frac{1}{4}(\sqrt{2w}b_0-\sqrt{2}M_1),\, a_1=\frac{\sqrt{w}}{\sqrt{2}},\, d_{-1}=0,$$

$$d_0=\frac{-w^2b_0-w^{3/2}M_1}{2w},\, d_1=0,$$

$$c_{-1}=\frac{\left[-3w^{5/2}b_{-1}b_0+w^{5/2}b_0^3-w^2(b_{-1}-b_0^2)M_1\right]}{w^{5/2}b_0+w^2M_1},$$

$$c_0=\frac{2\left[-2w^{5/2}b_{-1}+w^{5/2}b_0^2+w^2b_0M_1\right]}{\left[w^{5/2}b_0+w^2M_1\right]},\, k=-\sqrt{2w},$$

Substituting these values into Eqs. (3.7) and (3.8), we get:

$$u(x,t)=\left[(\cos h(kx+wt)+\sin h(kx+wt))\left(\sqrt{w}b_0-M_1\right)+2\sqrt{w}(\cos h(kx+wt)+\sin h(kx+wt))\right]\times\frac{1}{\left[2\sqrt{2}(b_{-1}+b_0(\cos h(kx+wt)+\sin h(kx+wt))+\cos h(2kx+2wt)+\sin h(2kx+2wt))\right]}$$

$$v(x,t)=-\left[\sqrt{w}\left(-4wb_{-1}+2wb_0^2+2\sqrt{w}b_0M_1\right)(\cos h(kx+wt)+\sin h(kx+wt))\right]\times\frac{1}{\left[\left(2\left(-3\sqrt{w}b_{-1}b_0+\sqrt{w}b_0^3+(-b_{-1}+b_0^2)M_1+\left(-4\sqrt{w}b_{-1}+2\sqrt{w}b_0^2+2b_0M_1\right)(\cos h(kx+wt)+\sin h(kx+wt))+\left(\sqrt{w}b_0+M_1\right)(\cos h(2kx+2wt)+\sin h(2kx+2wt))\right)\right)\right]},$$

where $M_1=\sqrt{-4wb_{-1}+wb_0^2}$.

Case XII.

$$a_{-1}=0,\, a_0=\frac{1}{4}(-\sqrt{2w}b_0+\sqrt{2}M_1,\, a_1=-\frac{\sqrt{w}}{\sqrt{2}},\, d_{-1}=0,$$

$$d_0=\frac{-w^2b_0-w^{3/2}M_1}{2w},\, d_1=0,$$

$$c_{-1}=\frac{\left[-3w^{5/2}b_{-1}b_0+w^{5/2}b_0^3-w^2(b_{-1}-b_0^2)M_1\right]}{w^{5/2}b_0+w^2M_1},$$

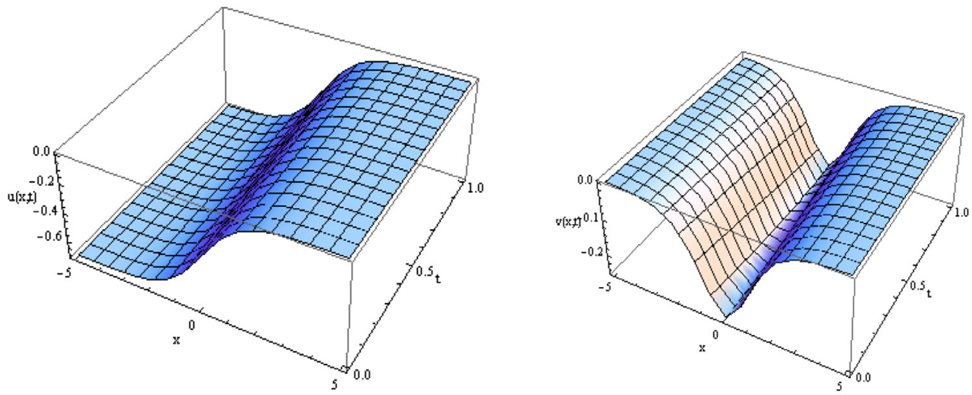
$$c_0=\frac{2\left[-2w^{5/2}b_{-1}+w^{5/2}b_0^2+w^2b_0M_1\right]}{(w^{5/2}b_0+w^2M_1)},\, k=\sqrt{2w},$$

Substituting these values into Eqs. (3.7) and (3.8), we get:

$$u(x,t)=[(\cos h(kx+wt)+\sin h(kx+wt))\left(-\sqrt{w}b_0+M_1-2\sqrt{w}(\cos h(kx+wt)+\sin h(kx+wt))\right)]\times\frac{1}{\left(2\sqrt{2}[b_{-1}+b_0(\cos h(kx+wt)+\sin h(kx+wt))+(\cos h(2kx+2wt)+\sin h(2kx+2wt))]\right)}$$

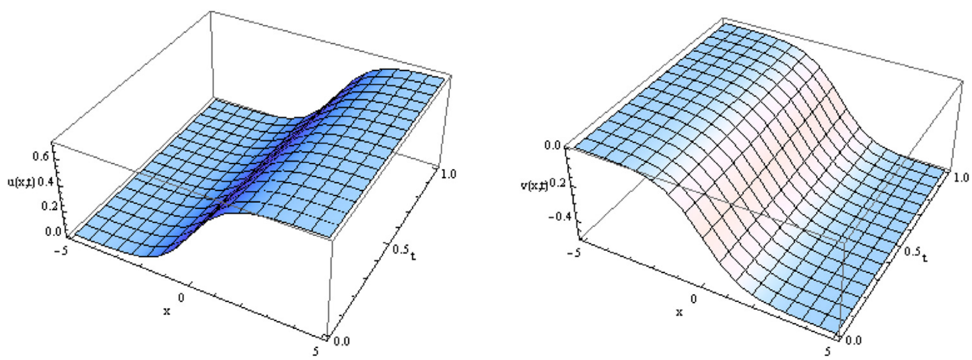
$$v(x,t)=-\left[\sqrt{w}\left(-4wb_{-1}+2wb_0^2+2\sqrt{w}b_0M_1\right)(\cos h(kx+wt)+\sin h(kx+wt))\right]\times\frac{1}{\left[2\left(-3\sqrt{w}b_{-1}b_0+\sqrt{w}b_0^3+(-b_{-1}+b_0^2)M_1+\left(-4\sqrt{w}b_{-1}+2\sqrt{w}b_0^2+2b_0M_1\right)(\cos h(kx+wt)+\sin h(kx+wt))+\left(\sqrt{w}b_0+M_1\right)(\cos h(2kx+2wt)+\sin h(2kx+2wt))\right)\right]},$$

where $M_1=\sqrt{-4wb_{-1}+wb_0^2}$.



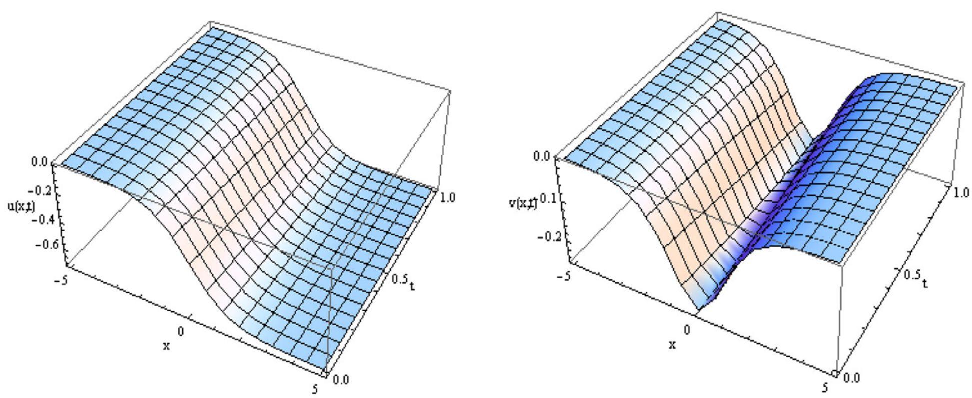
(a) (b)

Fig. 1. 3-D solitary wave solutions graphs of Eqs. (3.7) and (3.8) respectively, in case I, when $b_0 = w = 1$.



(a) (b)

Fig. 2. 3-D solitary wave solutions graphs of Eqs. (3.7) and (3.8) respectively, in case II, when $b_0 = w = 1$.



(a) (b)

Fig. 3. 3-D solitary wave solutions graphs of Eqs. (3.7) and (3.8) respectively, in case III, when $b_0 = w = 1$.

4. The numerical simulations for solutions of BB equation obtained by new proposed method

In this present numerical experiment, the exact solutions of Eqs. (1.1) and (1.2) have been used to draw the graphs as shown in Figs. 1–12.

In the present numerical simulation, the solutions graphs for BB have been presented for visualizing the underlying mechanism of the governing equations. Figs. 1–12 give a great impact on concerning the exchange of amplitude and nature of the solitary waves because of the variant of the parameters. Right here, the 3D graphs describe the behavior of $u(x, t)$ and $v(x, t)$ in space x at time t, which represents the change of amplitude and shape for each obtained solitary wave solutions.

Please cite this article as: L.K. Ravi et al., New exact solutions of coupled Boussinesq–Burgers equations by Exp-function method, Journal of Ocean Engineering and Science (2016), http://dx.doi.org/10.1016/j.joes.2016.09.001

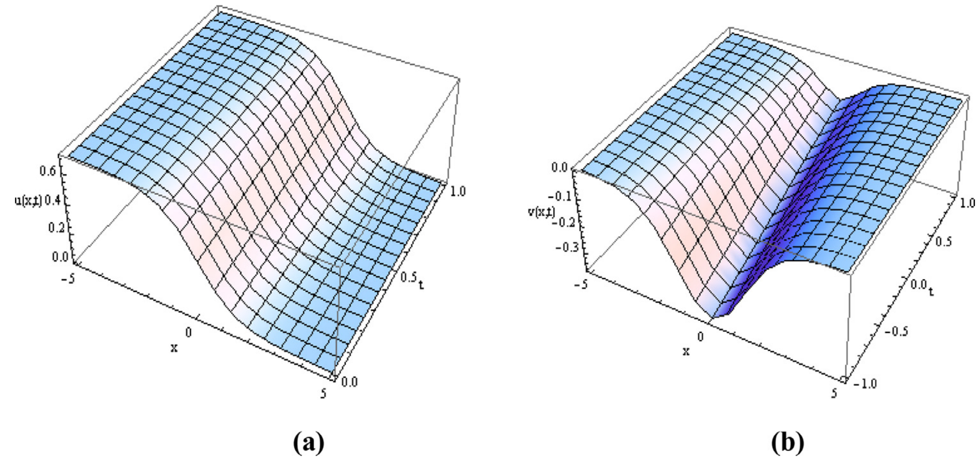


Fig. 4. 3-D solitary wave solutions graphs of Eqs. (3.7) and (3.8) respectively, in case IV, when $b_0 = w = 1$.

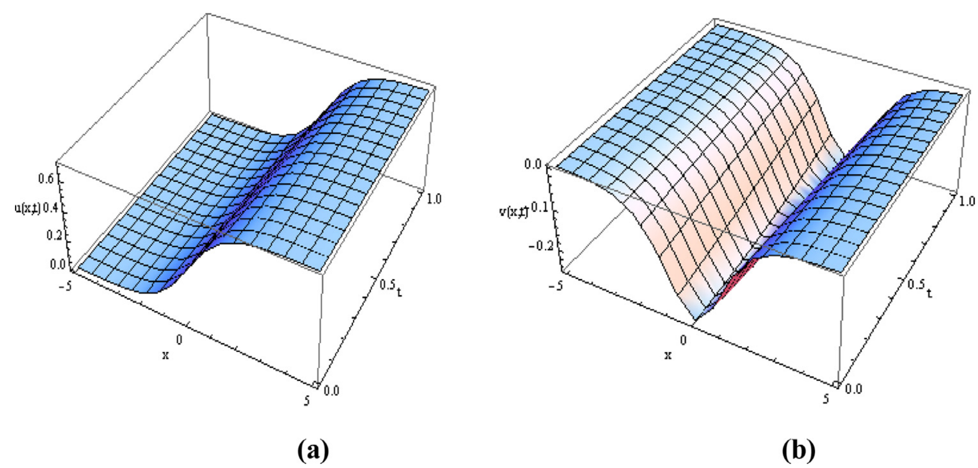


Fig. 5. 3-D solitary wave solutions graphs of Eqs. (3.7) and (3.8) respectively, in case V, when $b_0 = 2$, $b_{-1} = 1$, $w = 1$.

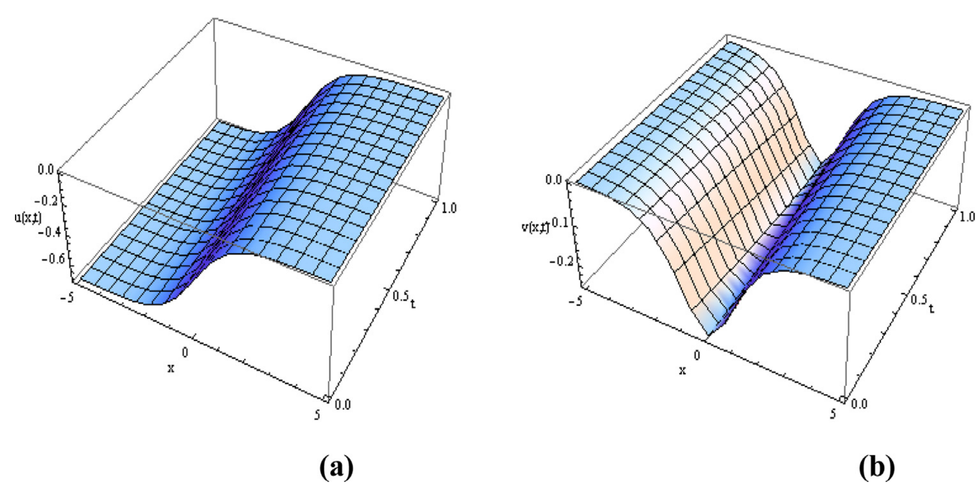


Fig. 6. 3-D solitary wave solutions graphs of Eqs. (3.7) and (3.8) respectively, in case VI, when $b_0 = 2$, $b_{-1} = 1$, $w = 1$.

5. Conclusion

In this paper, we have successfully implemented Exp-function method to derive the exact solitary wave solutions of the coupled Boussinesq–Burgers equations. The obtained results have been graphically demonstrated in order to justify the accuracy and efficiency of the proposed method. The results demonstrate that Exp-function method is straightforward and concise mathematical tool to establish the exact analytical solutions of nonlinear evolution equations. Therefore, we hope that this method

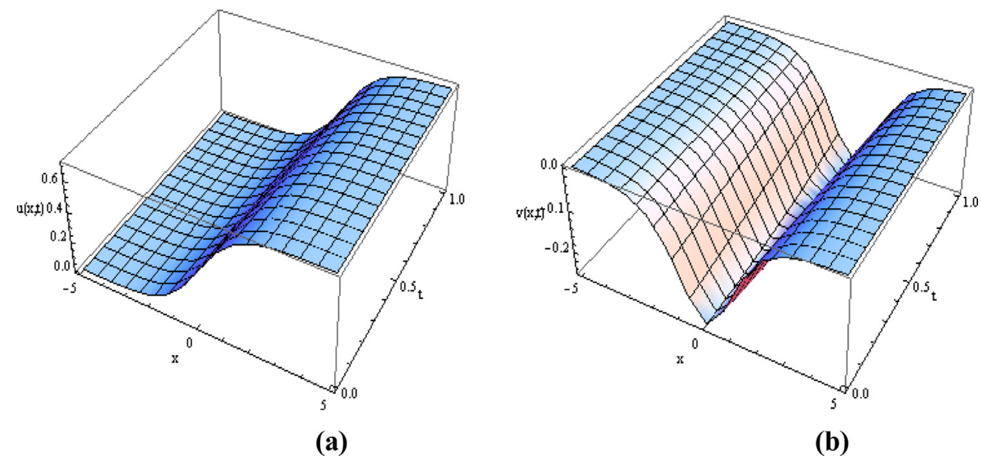


Fig. 7. 3-D solitary wave solutions graphs of Eqs. (3.7) and (3.8) respectively, in case VII, when $b_0 = 0$, $b_{-1} = -1$, $w = 1$.

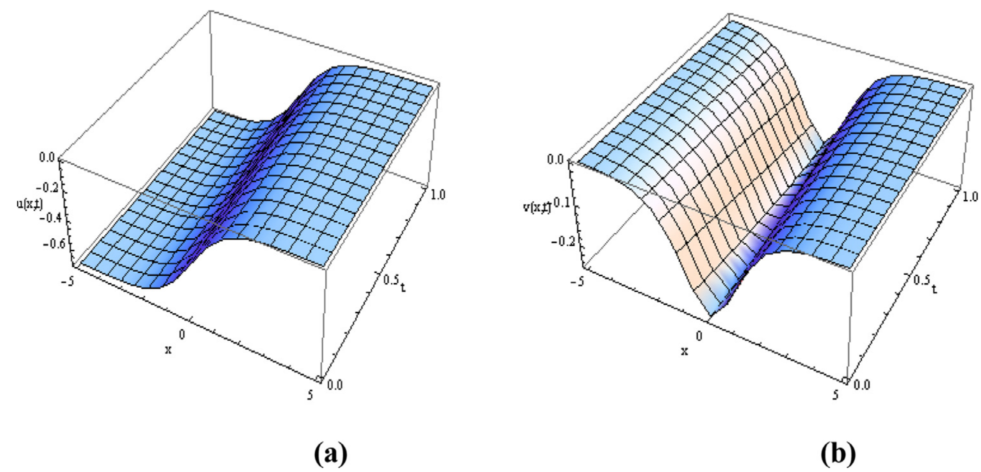


Fig. 8. 3-D solitary wave solutions graphs of Eqs. (3.7) and (3.8) respectively, in case VIII, when $b_0 = 2$, $b_{-1} = 1$, $w = 1$.

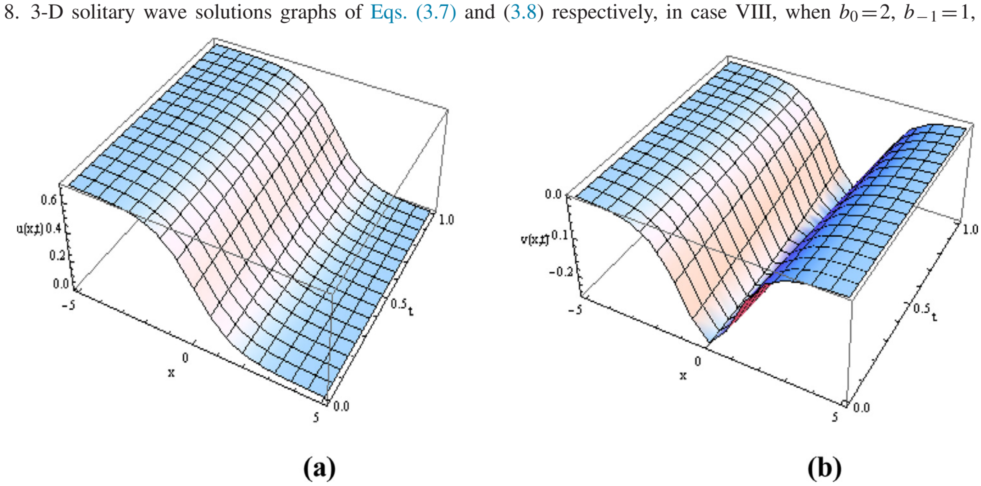


Fig. 9. 3-D solitary wave solutions graphs of Eqs. (3.7) and (3.8) respectively, in case IX, when $b_0 = 2$, $b_{-1} = 1$, $w = 1$.

can be more effectively used to investigate other nonlinear evolution equations which frequently take place in engineering, applied mathematics and physical sciences.

Acknowledgments

The authors would like to express their sincere thanks and gratitude to the anonymous learnt reviewers for their valuable comments and suggestions for the improvement of the paper.

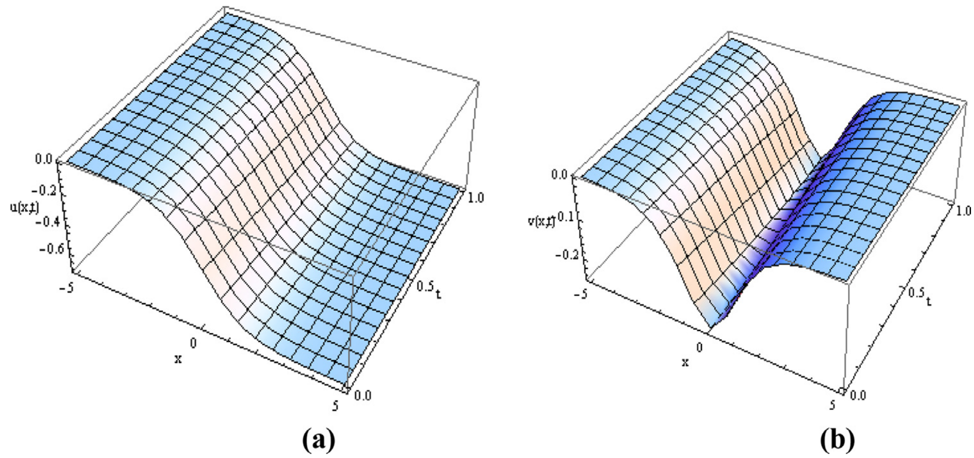


Fig. 10. 3-D solitary wave solutions graphs of Eqs. (3.7) and (3.8) respectively, in case X, when $b_0=2$, $b_{-1}=1$, $w=1$.

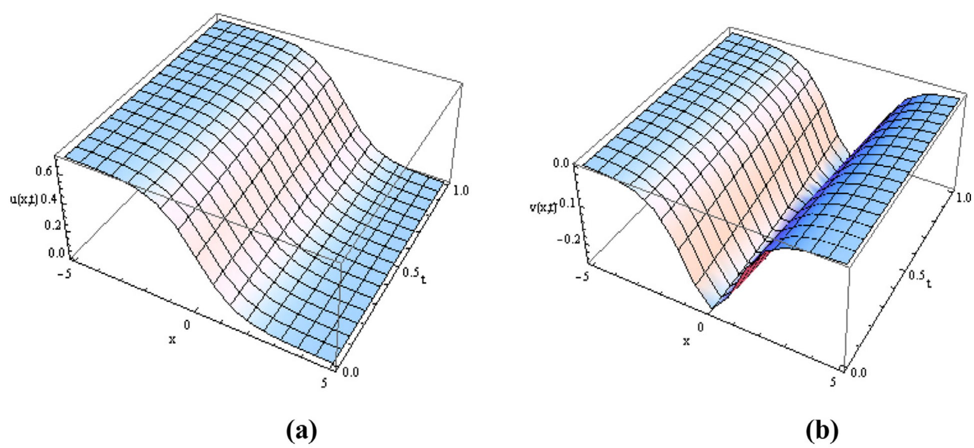


Fig. 11. 3-D solitary wave solutions graphs of Eqs. (3.7) and (3.8) respectively, in case XI, when $b_0=0$, $b_{-1}=-1$, $w=1$.

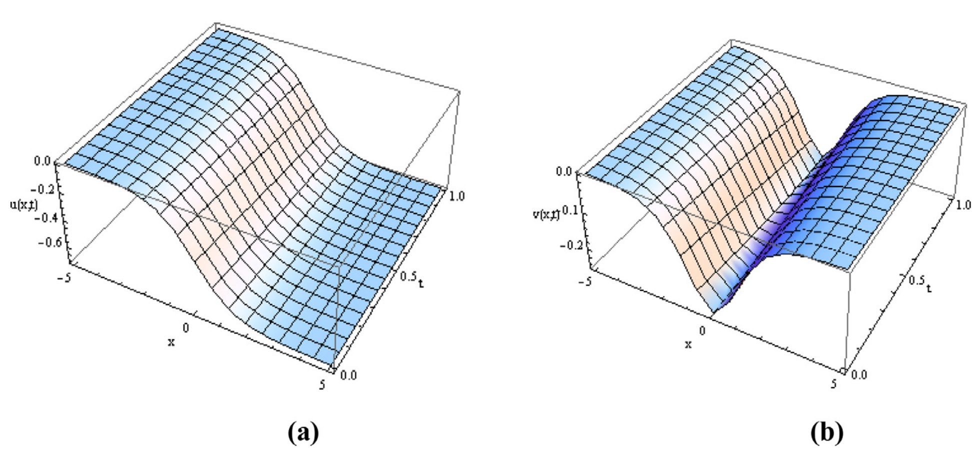


Fig. 12. 3-D solitary wave solutions graphs of Eqs. (3.7) and (3.8) respectively, in case XII, when $b_0=0$, $b_{-1}=-1$, $w=1$.

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Appendix 1

$$\delta_{-3} = ka_{-1}^2c_{-1} + wa_{-1}b_{-1}c_{-1} - \frac{1}{2}kb_{-1}^2d_{-1},$$

$$\delta_{-2} = 2ka_{-1}a_0c_{-1} + wa_0b_{-1}c_{-1} + wa_{-1}b_0c_{-1} + ka_{-1}^2c_0 + wa_{-1}b_{-1}c_0 - kb_{-1}b_0d_{-1} - \frac{1}{2}kb_{-1}^2d_0,$$

$$\delta_{-1} = ka_{-1}^2 + wa_{-1}b_{-1} + wa_{-1}c_{-1} + ka_0^2c_{-1} + 2ka_{-1}a_1c_{-1} + wa_1b_{-1}c_{-1} + wa_0b_0c_{-1} + 2ka_{-1}a_0c_0 + wa_0b_{-1}c_0 + wa_{-1}b_0c_0 - kb_{-1}d_{-1} - \frac{1}{2}kb_0^2d_{-1} - kb_{-1}b_0d_0 - \frac{1}{2}kb_{-1}^2d_1,$$

$$\delta_0 = \frac{1}{2}(4ka_{-1}a_0 + 2wa_0b_{-1} + 2wa_{-1}b_0 + 2wa_0c_{-1} + 4ka_0a_1c_{-1} + 2wa_1b_0c_{-1} + 2wa_{-1}c_0 + 2ka_0^2c_0 + 4ka_{-1}a_1c_0 + 2wa_1b_{-1}c_0 + 2wa_0b_0c_0 - 2kb_0d_{-1} - 2kb_{-1}d_0 - kb_0^2d_0 - 2kb_{-1}b_0d_1),$$

$$\delta_1 = wa_{-1} + ka_0^2 + 2ka_{-1}a_1 + wa_1b_{-1} + wa_0b_0 + wa_1c_{-1} + ka_1^2c_{-1} + wa_0c_0 + 2ka_0a_1c_0 + wa_1b_0c_0 - \frac{1}{2}kd_{-1} - kb_0d_0 - kb_{-1}d_1 - \frac{1}{2}kb_0^2d_1,$$

$$\delta_2 = wa_0 + 2ka_0a_1 + wa_1b_0 + wa_1c_0 + ka_1^2c_0 - \frac{1}{2}kd_0 - kb_0d_1,$$

$$\delta_3 = wa_1 + ka_1^2 - \frac{1}{2}kd_1,$$

$$\xi_{-4} = 2ka_{-1}b_{-1}^2d_{-1} + wb_{-1}^3d_{-1},$$

$$\xi_{-3} = -\frac{1}{2}k^3a_0b_{-1}^2c_{-1} + \frac{1}{2}k^3a_{-1}b_{-1}b_0c_{-1} + 2ka_0b_{-1}^2d_{-1} + 4ka_{-1}b_{-1}b_0d_{-1} + 3wb_{-1}^2b_0d_{-1} + 2ka_{-1}b_{-1}^2d_0 + wb_{-1}^3d_0$$

$$\xi_{-2} = 2k^3a_{-1}b_{-1}c_{-1} - 2k^3a_1b_{-1}^2c_{-1} + \frac{1}{2}k^3a_0b_{-1}b_0c_{-1} - \frac{1}{2}k^3a_{-1}b_0^2c_{-1} - \frac{1}{2}k^3a_0b_{-1}^2c_0 + \frac{1}{2}k^3a_{-1}b_{-1}b_0c_0 + 4ka_{-1}b_{-1}d_{-1} + 3wb_{-1}^2d_{-1} + 2ka_1b_{-1}^2d_{-1} + 4ka_0b_{-1}b_0d_{-1} + 2ka_{-1}b_0^2d_{-1} + 3wb_{-1}b_0^2d_{-1} + 2ka_0b_{-1}^2d_0 + 4ka_{-1}b_{-1}b_0d_0 + 3wb_{-1}^2b_0d_0 \; 2ka_{-1}b_{-1}^2d_1 + wb_{-1}^3d_1,$$

$$\xi_{-1} = -\frac{1}{2}k^3a_0b_{-1}^2 + \frac{1}{2}k^3a_{-1}b_{-1}b_0 + 3k^3a_0b_{-1}c_{-1} - \frac{3}{2}k^3a_{-1}b_0c_{-1} - \frac{3}{2}k^3a_1b_{-1}b_0c_{-1} + 2k^3a_{-1}b_{-1}c_0 - 2k^3a_1b_{-1}^2c_0 + \frac{1}{2}k^3a_0b_{-1}b_0c_0 - \frac{1}{2}k^3a_{-1}b_0^2c_0 + 4ka_0b_{-1}d_{-1} + 4ka_{-1}b_0d_{-1} + 6wb_{-1}b_0d_{-1} + 4ka_1b_{-1}b_0d_{-1} + 2ka_0b_0^2d_{-1} + wb_0^3d_{-1} + 4ka_{-1}b_{-1}d_0 + 3wb_{-1}^2d_0 + 2ka_1b_{-1}^2d_0 + 4ka_0b_{-1}b_0d_0 + 2ka_{-1}b_0^2d_0 + 3wb_{-1}b_0^2d_0 + 2ka_0b_{-1}^2d_1 + 4ka_{-1}b_{-1}b_0d_1 + 3wb_{-1}^2b_0d_1,$$

$$\xi_0 = \frac{1}{2}(4k^3a_{-1}b_{-1} - 4k^3a_1b_{-1}^2 + k^3a_0b_{-1}b_0 - k^3a_{-1}b_0^2 - 4k^3a_{-1}c_{-1} + 4k^3a_1b_{-1}c_{-1} + k^3a_0b_0c_{-1} - k^3a_1b_0^2c_{-1} + 6k^3a_0b_{-1}c_0 - 3k^3a_{-1}b_0c_0 - 3k^3a_1b_{-1}b_0c_0 + 4ka_{-1}d_{-1} + 6wb_{-1}d_{-1} + 8ka_1b_{-1}d_{-1} + 8ka_0b_0d_{-1} + 6wb_0^2d_{-1} + 4ka_1b_0^2d_{-1} + 8ka_0b_{-1}d_0 + 8ka_{-1}b_0d_0 + 12wb_{-1}b_0d_0 + 8ka_1b_{-1}b_0d_0 + 4ka_0b_0^2d_0 + 2wb_0^3d_0 + 8ka_{-1}b_{-1}d_1 + 6wb_{-1}^2d_1 + 4ka_1b_{-1}^2d_1 + 8ka_0b_{-1}b_0d_1 + 4ka_{-1}b_0^2d_1 + 6wb_{-1}b_0^2d_1),$$

$$\xi_1 = 3k^3a_0b_{-1} - \frac{3}{2}k^3a_{-1}b_0 - \frac{3}{2}k^3a_1b_{-1}b_0 - \frac{1}{2}k^3a_0c_{-1} + \frac{1}{2}k^3a_1b_0c_{-1} - 2k^3a_{-1}c_0 + 2k^3a_1b_{-1}c_0 + \frac{1}{2}k^3a_0b_0c_0 - \frac{1}{2}k^3a_1b_0^2c_0 + 2ka_0d_{-1} + 3wb_0d_{-1} + 4ka_1b_0d_{-1} + 2ka_{-1}d_0 + 3wb_{-1}d_0 + 4ka_1b_{-1}d_0 + 4ka_0b_0d_0 + 3wb_0^3d_0 + 2ka_1b_0^2d_0 + 4ka_0b_{-1}d_1 + 4ka_{-1}b_0d_1 + 6wb_{-1}b_0d_1 + 4ka_1b_{-1}b_0d_1 + 2ka_0b_0^2d_1 + wb_0^3d_1,$$

$$\xi_2 = -2k^3a_{-1} + 2k^3a_1b_{-1} + \frac{1}{2}k^3a_0b_0 - \frac{1}{2}k^3a_1b_0^2 - \frac{1}{2}k^3a_0c_0 + \frac{1}{2}k^3a_1b_0c_0 + wd_{-1} + 2ka_1d_{-1} + 2ka_0d_0 + 3wb_0d_0 + 4ka_1b_0d_0 + 2ka_{-1}d_1 + 3wb_{-1}d_1 + 4ka_1b_{-1}d_1 + 4ka_0b_0d_1 + 3wb_0^2d_1 + 2ka_1b_0^2d_1,$$

$$\xi_3 = -\frac{1}{2}k^3a_0 + \frac{1}{2}k^3a_1b_0 + wd_0 + 2ka_1d_0 + 2ka_0d_1 + 3wb_0d_1 + 4ka_1b_0d_1,$$

$$\xi_4 = wd_1 + 2ka_1d_1.$$

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