Elasto-plastic approach for paper cockling phenomenon: 
On the importance of moisture gradient

P. Lipponen a,*, T. Leppänen a, J. Kouko b, J. Hämäläinen a

a University of Kuopio, Department of Physics, Yliopistonranta 1 F, P.O. Box 1627, FI-70211 Kuopio, Finland
b VTT Technical Research Center of Finland, P.O. Box 1603, FI-40101 Jyväskylä, Finland

Received 27 November 2007; received in revised form 18 February 2008
Available online 29 February 2008

Abstract

Cockling of paper is a common problem occurring in the production, storage and end-use of paper. It is usually induced by a moisture content change. In many cases, cockling is an irreversible phenomenon; i.e. the initial shape is not obtained although the initial moisture content is restored. This kind of moisture content change occurs in copying machines and in the printing process, for example. In this paper, we present a continuum mechanical model, which is used to study the irreversible cockling of paper. In the model, paper is treated as an orthotropic elasto-plastic material and the model takes into account the small-scale variation of fibre orientation. The model is used to show the importance of the through-thickness moisture gradient on the cockling phenomenon during a cyclic moisture content change. The results suggest that the moisture gradient is a crucial factor for the irreversible cockling.

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Keywords: Elastoplasticity; Finite element; Paper; Non-homogenous material; Cockling

1. Introduction

Paper is an orthotropic heterogeneous material which is mainly composed of fibres, fillers and fines (Niskanen, 1998). Due to a moisture content change, undesired out-of-plane deformation, such as cockling (Smith, 1950), may appear in the paper, see Fig. 1. Cockling is a small-scale out-of-plane deformation, which in many cases appears already in the manufacturing process and continues due to the moisture content change. Cockling occurs most in papers with relatively low basis weights—e.g. in copy papers.

Smith (1950) embarked the research of cockling already in the 1950’s studying the relation between dried-in strains and curling, cockling and other phenomena. Smith noticed that cockling is a consequence of the moisture content change, which induces different scales of length and width variations to the paper depending on the moisture expansion behaviour inside the paper. It was also concluded that cockling mainly appears in the drying section of the paper machine due to the shrinkage of paper during changes in the moisture content of
paper. Kajanto (1993) studied cockling via finite element analysis, and noticed that cockling induces from local inhomogeneity in the two-sidedness of paper. The importance of fibre orientation and formation was also discovered. Moreover, Kajanto suggested that cockling originated either from local buckling or curling. Later, Nam and Thorpe (1996) studied cockling in changing moisture conditions by investigating the dimensional stability of three different types of copy paper. The mobility of the cockles during dehydration and rehydration of samples was observed. They concluded that hygroexpansivity appears to be a governing factor for the cockling phenomenon.

Recently, cockling has been studied by Paik and Nam (2001), Ahrens et al. (2005), Hojjatie and Ahrens (2004) and Leppänen et al. (2005). Paik and Nam, and Ahrens et al. studied the importance of drying on the cockling phenomenon. Via experimental studies they showed that the drying rate and uniformity of drying are significant factors affecting the paper cockling. Moreover, Paik and Nam stated that by increasing the applied drying temperature cockling increases on the paper strip. They also concluded that most paper breaks take place on the cockle location during tensile testing. Paik and Nam suggested that paper with good formation and higher bending stiffness can reduce cockling. The influence of formation and basis weight variation was also noticed by Hojjatie and Ahrens (2004). Furthermore, the effect of formation on the strength properties of paper (on millimetre scales) was studied by Ostoja-Starzewski and Castro (2003). Their study showed the dependence between local basis weight and local stiffness and strength of paper. In addition, Ahrens et al. (2005) suggested that drying uniformity is a significant factor in the cockling phenomenon. They concluded that cockling was greater when the dry solids content of the samples was lower, the drying temperature was high and drying was non-uniform.

Leppänen et al. (2005) studied the cockling phenomenon via continuum mechanical modelling. They agreed with Kajanto (1993) and concluded in their later studies (Leppänen et al., 2006, 2007) that in many cases especially the in-plane shape of cockles is strongly determined by the variation of fibre orientation. This variation impacts on local strength properties and local moisture expansion of paper, see for example Kajanto (1993). Variation of fibre orientation generates both in-plane compressive forces and out-of-plane bending forces. These forces may affect cockling due to moisture content changes at the production and end-use of paper (Leppänen, 2007).

Recently, Kulachenko et al. (2005, 2006) study the fluting phenomenon which is a type of out-of-plane behaviour of paper. Fluting of paper is a problem that occurs in the heatset web offset printing process. In their studies (Kulachenko et al., 2005, 2006) fluting was analysed via finite element modelling. They noticed that when the paper web is perfectly flat before printing, fluting patterns after drying have larger wavelength than those typically observed in fluted samples. It was also mentioned that if initial cockling is introduced to the unprinted sheets, the flute patterns changes. Kulachenko et al. concluded that the fluting with the wave-
length of about 1–2 cm (often observed from printed sheets) are, therefore, likely to originate from initial cockles of unprinted sheets, or driven by the same mechanisms that create them, i.e. non-uniform drying in a small-scale, for example.

In previous paragraphs several potential sources for cockling are suggested, and also the importance of cockling for fluting phenomenon is shortly discussed. Suggested reasons for cockling can be roughly classified into two groups; structural properties (variation of fibre orientation and/or basis weight) and drying effects (uniformity of drying and/or drying rate). The model presented in this paper is used to study the shared effect of structural properties and drying effects by taking into account the variation of fibre orientation and the moisture gradient during moisture content change.

Earlier modelling studies that concentrated on the cockling phenomenon are approached via elastic aspects. A disadvantage in these studies is that they ignore the history dependence of the response, although the studies performed by Paik and Nam (2001) and Ahrens et al. (2005) confirm that these history dependencies are crucial for the cockling of paper. The weakness of elastic models is also clear in many situations in the end-use of paper. For example, in the copying machine paper goes through a cyclic moisture change when the moisture content of paper decreases during copying and increases immediately after passing the copying machine. This cyclic moisture content change may cause irreversible cockling in paper, and elastic models are of course useless when this type of cockling is studied.

There are, however, a few researches carried out quite recently concerning the elasto-plastic response of paper. A three-dimensional, anisotropic constitutive model to predict the in-plane elasto-plastic deformation of paper and paperboard is suggested by Xia et al. (2002). Mäkelä and Östlund (2003) present an orthotropic elasto-plastic constitutive model which is used to describe the static mechanical behaviour of paper. Furthermore, Castro and Ostoja-Starzewski (2003) carry out research where the in-plane behaviour of paper was investigated under uniaxial and biaxial stresses and constitutive equations of paper consistent for both uniaxial and biaxial loading ranges were presented. However, all the above mentioned elasto-plastic studies exclude the elasto-plastic out-of-plane behaviour of paper.

In this paper, we present an elasto-plastic model of paper that can be utilized for many real situations where cockling appears. The model is used to test the effect of different through-thickness moisture gradients on cockling during a cyclic moisture content change.

2. Uniaxial stress–strain measurements

The test apparatus, C-Impact (Kouko et al., 2006), which is a vertical tensile tester, is presented in Fig. 2. A force sensor was located above the upper jaw and an electromagnetic actuator below the lower jaw. The displacement of the lower jaw was measured with a laser distance sensor. The dry solids content of the paper samples was measured with an optical moisture analyzer (PIRMA by VTT), shown in Fig. 2, which was based on near-infrared technology (NIR) (Lehtonen, 2005). The maximum sampling frequency for data acquisition and for the measuring components was 5 kHz (100 Hz for the PIRMA moisture analyzer). The accuracy of dry solids content data was very sensitive to calibration for different paper grades. After the calibration, inaccuracy of the on-line measurement was about ±2 %. The reliability of the moisture calibration was double-checked occasionally during the measurements with a halogen moisture analyzer. The accuracy of the measurement was adequate to specify the target dry solids contents.

The C-Impact test apparatus and data recording were operated by a computer. PID control of the actuator caused small unavoidable inaccuracy to the motion. The criteria for tuning of the PID control were, firstly, constant 1 mm/s displacement rate (error less than 2%) in range between 10 and 90% of target displacement. Secondly, stable target position without oscillation (a short period of damped oscillations was allowed immediately after reaching the target position). Thirdly, the overshoot of displacement was less than 2% at a displacement rate of 1 mm/s. The motion that actuator performed was very repeatable and therefore the measured differences between the samples were not originated from the actuator.

The strip length used was 180 mm and the strip width was 20 mm. There were three press-dry fine paper (pilot) samples with different dry MD/CD tensile ratios 1.5, 1.9 and 2.9. The dry solids contents of the samples were from 53 to 56% and basis weight from 77 to 80 gm⁻². The press-dry paper samples were stored hermetically sealed in a cold storage. The hermetically sealed samples were heated to room temperature before test-
ing, but no other conditioning was performed to the samples. The press-dry paper samples were dried with the C-Impact and strained during drying.

The paper samples were tested in three different in-plane directions. The test angles were MD, MD-45° and CD (MD-90°), where MD = machine direction and CD = cross machine direction. MD-45° means that the strips were cut at an angle bisecting MD and CD.

The paper samples were dried to four specific dry solids contents (one sample to one specific dry solids content) and strained until breaking at that state. The four specific dry solids contents with inaccuracy of the measurement were: (65 ± 2)%, (75 ± 2)%, (85 ± 2)% and (95 ± 2)%.

The elastic modulus was defined by a tangential modulus. The tangential modulus was calculated as part of the tensile stress–strain curve as the first-order derivative of a fitted modified polynomial equation.

3. Elasto-plastic model of paper

The measurements presented in the previous section were performed in the global (MD, CD) coordinate system. In the model, the results obtained from these measurements are applied in the local (1,2) coordinate system, (see Fig. 3). The intensity of local anisotropy $\xi = a/b$ is represented as MD/CD tensile strength ratios of the measured samples. The fibre orientation measurements are provided in Leppänen et al. (2005).

A basic assumption of the model is that the deformation can be divided into an elastic domain and a plastic domain, i.e. plastic properties of materials become manifest only after entering the plastic domain. Thus, the strain tensor $\epsilon$ can be written as

$$\epsilon = \epsilon^{el} + \epsilon^{pl},$$

where $\epsilon^{el}$ and $\epsilon^{pl}$ are elastic and plastic components of strain, respectively. In the case of paper, the transition from the elastic to the plastic region is not sharp. However, the results presented in Niskanen (1998) suggest that the plastic region of paper starts roughly from the point where the stress–strain curve diverges from Young’s modulus. It should be noted that we are using the unit N/m for stress instead of N/m². This is quite common approach in the paper technology, see for example Niskanen (1998), due to uncertainty with measurements of paper thickness.

The yield point, defining the border between the elastic and the plastic region, is determined by utilizing measured stress–strain curves. A 10th degree polynomial was fitted to the stress–strain curves, and the yield
points were fixed to the point at which the derivative of the fit differs from the measured elastic modulus more than 20%. Based on our testing, the selection of 20% is reasonable, see Fig. 4. Moreover, the obtained yield strains are of the same order as those obtained by Htun and de Ruvo (1978).

The dependence of yield stress $\sigma^y$ (N/m) and yield strain $\epsilon^y$ (%) on anisotropy $\xi$ and dry solids content $\beta$ was obtained by fitting the non-linear functions

$$\sigma^y = A_1\xi + A_2 e^{A_3\beta} + A_4 + A_5\xi e^{A_6\beta}$$
$$\epsilon^y = B_1\xi + B_2 e^{B_3\beta} + B_4 + B_5\xi e^{B_6\beta}$$

(2) (3)

to the determined yield points. In Eqs. (2) and (3) coefficients $A_i$ and $B_i$ are fitting parameters, which are presented in Table 1. The dependence of the yield point on anisotropy and dry solids content in direction 1 is shown in Fig. 5.
Table 1
Coefficients $A_i, B_i$ and $C_i$ for yield stresses $\sigma_{11}'$, $\sigma_{22}'$ and $\sigma_{45}'$, yield strains $\epsilon_{11}'$, $\epsilon_{22}'$ and $\epsilon_{45}'$, and stress describing the hardening $\sigma^h$

<table>
<thead>
<tr>
<th>$A_1 \cdot 10^6$ (N/m)</th>
<th>$A_2 \cdot 10^{-3}$ (N/m)</th>
<th>$A_3 \cdot 10^{-4}$ (1/%)</th>
<th>$A_4 \cdot 10^6$ (N/m)</th>
<th>$A_5 \cdot 10^6$ (N/m)</th>
<th>$A_6 \cdot 10^{-4}$ (1/%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{11}'$</td>
<td>-62.52</td>
<td>22.23</td>
<td>10.23</td>
<td>-35.92</td>
<td>5.63</td>
</tr>
<tr>
<td>$\sigma_{22}'$</td>
<td>8.14</td>
<td>131.68</td>
<td>8.71</td>
<td>-52.17</td>
<td>8.36</td>
</tr>
<tr>
<td>$\sigma_{45}'$</td>
<td>-68.61</td>
<td>1.99</td>
<td>12.90</td>
<td>5.70</td>
<td>15.84</td>
</tr>
<tr>
<td>$B_1 \cdot 10^{-3}$ (1/%)</td>
<td>$B_2 \cdot 10^{-3}$ (1/%)</td>
<td>$B_3 \cdot 10^{-3}$ (1/%)</td>
<td>$B_4 \cdot 10^{-3}$ (1/%)</td>
<td>$B_5 \cdot 10^{-3}$ (1/%)</td>
<td>$B_6 \cdot 10^{-3}$ (1/%)</td>
</tr>
<tr>
<td>$\epsilon_{11}'$</td>
<td>9.49</td>
<td>25.93</td>
<td>17.53</td>
<td>50.02</td>
<td>0.10</td>
</tr>
<tr>
<td>$\epsilon_{22}'$</td>
<td>-289.41</td>
<td>0</td>
<td>660.79</td>
<td>157.90</td>
<td>226.94</td>
</tr>
<tr>
<td>$\epsilon_{45}'$</td>
<td>-49.09</td>
<td>399.46</td>
<td>-12.76</td>
<td>70.07</td>
<td>1.20</td>
</tr>
<tr>
<td>$C_1 \cdot 10^6$ (N/m)</td>
<td>$C_2 \cdot 10^{-1}$ (N/m)</td>
<td>$C_3 \cdot 10^{-3}$ (1/%)</td>
<td>$C_4 \cdot 10^6$ (N/m)</td>
<td>$C_5 \cdot 10^6$ (N/m)</td>
<td>$C_6 \cdot 10^{-3}$ (1/%)</td>
</tr>
<tr>
<td>$\sigma^h$</td>
<td>-109.95</td>
<td>9.92</td>
<td>7.02</td>
<td>-128.57</td>
<td>19.93</td>
</tr>
</tbody>
</table>

Fig. 5. Dependence of yield strain $\epsilon_{11}'$ and yield stress $\sigma_{11}'$ on anisotropy $\xi$ and dry solids content $\beta$ of paper.

The hardening behaviour in the model is described by isotropic hardening; i.e. the yield surface changes its size uniformly in all directions such that the yield strength increases or decreases as plastic straining occurs. Hardening is determined by the slope of the stress–strain curve fit at constant 0.5% strain. The dependence of $\sigma^h$, which defines the hardening behaviour, is obtained by fitting a non-linear function

$$\sigma^h = C_1 \xi + C_2 \epsilon_{11}' + C_4 + C_5 \xi \epsilon_{11}'$$

where $C_i$ are fitting parameters, which are presented in Table 1. Fig. 6 illustrates an example of two fitted stress–strain behaviours (yield points and points defining the hardenings) and corresponding measured stress–strain curves.

Due to the orthotropic nature of paper, we have used Hill’s yield function (Hill, 1947) to describe the yield surface. Hill’s yield function, in a plane stress case, can be stated in terms of rectangular Cartesian stress components as

$$f(\sigma) = \sqrt{F \sigma_2^2 + G \sigma_1^2 + H (\sigma_1 - \sigma_2)^2 + 2N \sigma_1 \sigma_2},$$

where parameters $F$, $G$, $H$ and $N$ are obtained by a set of tests of the material in different orientations, and $\sigma_1$, $\sigma_2$ and $\sigma_{12}$ are the components of the stress tensor. The parameter $H$ related to the biaxial stress state is here approximated by Hoffman’s approximation, the suitability of which for paper is tested in Suhling et al. (1985). In their study biaxial measurements were performed to paper of basis weight 205 gm$^{-2}$ and through these measurements applicability for Hoffman’s approximation was inspected. Due to the lack of biaxial tests at this point the Hoffman’s approximation is used, although Suhling et al. (1985) noticed that it overpredicts the observed strengths, at least when 205 gm$^{-2}$ paper was used. By using the same notation as in Hill’s yield function Hoffman’s approximation is written as

$$G = H.$$
Finally, using Hoffman’s approximation and applying the coordinate transformations

\[ r_1 = r_0 \cos^2 \gamma; \]
\[ r_2 = r_0 \sin^2 \gamma; \]
\[ r_{12} = r_0 \sin \gamma \cos \gamma; \]

where \( r_0 \) is the stress applied in the direction defined by \( \gamma \) (direction 1, 2 and 45° correspond \( \gamma = 0°, \gamma = 90° \) and \( \gamma = 45° \), respectively). In Eq. (5) the dependences of Hill’s yield function parameters \( H, F \) and \( N \) on the yield stresses are obtained to be

\[ H = \frac{1}{2}, \]
\[ F = \left( \frac{\sigma_1^y}{\sigma_2^y} \right)^2 - \frac{1}{2}, \]
\[ N = 2 \left( \frac{\sigma_1^y}{\sigma_{45}^y} \right)^2 - \frac{1}{2} \left( \frac{\sigma_1^y}{\sigma_2^y} \right)^2. \]

In the model, the elastic domain is constructed based on the determination of yield points. Elastic moduli of paper in directions 1, 2 and 45° are defined as

\[ E_1 = \frac{\sigma_1^y}{\epsilon_1^y}, \]
\[ E_2 = \frac{\sigma_2^y}{\epsilon_2^y}, \]
\[ E_{45} = \frac{\sigma_{45}^y}{\epsilon_{45}^y}. \]

The shear modulus \( G_{12} \), according to Gibson (1994), can be approximated as

\[ G_{12} = \frac{1}{\left( \frac{1}{E_{45}} - \frac{1}{E_1} - \frac{1}{E_2} + \frac{2\nu_{12}}{E_1} \right)}, \]

where \( \nu_{12} \) is Poisson’s ratio. Poisson’s ratios in directions 1 and 2 are defined, respectively, as

Fig. 6. Stress–strain curves determined by the fitted dependencies of yield point and hardening on anisotropy and dry solids content and corresponding measured curves. From left to right: \( \xi = 1.5 \) and 2.9, respectively, and \( \beta = 95\% \) in both cases.
The dependence of $v_{12}$ is roughly based on the measurements presented in Yeh et al. (1991). The dependence for $v_{21}$ is simply obtained by using a Maxwell relation. The effect of moisture expansion is included in the model by quoting Uesaka (1994). The local moisture expansion coefficients $a_1$ and $a_2$ are therefore

$$a_1 = 6 \cdot 10^{-4} - 1.5 \cdot 10^{-4} \sqrt{\xi - 1}\%^{-1},$$

$$a_2 = \sqrt{7 \cdot 10^{-8}(\xi - 1) + 6 \cdot 10^{-4}}\%^{-1}.$$  \hspace{1cm} (19)

$$v_{12} = (1.65 - 0.015\beta)\sqrt{\xi},$$
$$v_{21} = \frac{E_2}{E_1}v_{12}. \hspace{1cm} (18)$$

The heterogeneity of fibre orientation is treated in the same manner as in Leppänen et al. (2005). In the finite element analysis, the paper sample used is divided into elements the in-plane size of which is 2 mm × 2 mm. In the thickness direction, one element is divided into several layers, see Fig. 7. Every layer of each element corresponds to the measured (or artificial) fibre orientation distribution.

The model is solved using ABAQUS/Standard and element SC8R, see Hibbitt and Karlsson (2006, 2004). The element is based on the first-order shear deformation theory (Reddy, 2004) and it uses linear interpolation and reduced integration. It accounts for finite membrane strains and arbitrary large rotations. It also includes the effect of the thickness change. Its eight nodes are located in the corners of the top and bottom surfaces, see Fig. 7. Two layers of nodes allow the use of a different moisture content directly for the top and bottom surfaces. Also the numerical performance of this element, in the case of elastic model and cockling, is extensively tested in Leppänen (2007).

The boundary conditions are imposed in the way that the dimensional change of paper is not constrained in the in-plane direction. That is, paper is free to expand or shrink in the plane of the sheet. The boundary numbers used are presented in Fig. 8. Large-scale out-of-plane deformation is constrained by applying the displacement constraint $u_z = 0$ at edges 1, 2, 3 and 4. This constraint also improves the convergence of the solution. Rigid body motions are constrained by applying $u_{MD} = u_{CD} = 0$ at corner 5 and $u_{MD} = 0$ at corner 6.

**Fig. 7.** Cross-section of one element. Local fibre orientation angle $\theta_i$ and its strength $\xi_i$ correspond to the measured (or artificial) value. The mesh used consists 96 × 96 elements which corresponds 192 mm × 192 mm area. In the thickness direction mesh consists one element layer. This element layer consists several material layers, which are assumed to have an equal thickness.
5. Results

Several simulations were carried out to obtain information about the effect of the through-thickness moisture gradient on the cockling of paper. In these simulations, we performed different cyclic moisture changes, where the average change in the moisture content of paper was 5% in each case, which corresponds quite well to the real circumstances in the copying process of paper (Nam and Thorpe, 1996). All draws were neglected as well as gravitation, i.e. no external forces were applied to the paper. The simulated cycles are presented in Table 2. The simulated out-of-plane results are obtained after the second step, i.e. after the initial moisture content is restored. The uncoated fine paper sample used was produced in a Metso Paper pilot machine. The in-plane size of the sample was 192 mm x 192 mm and the thickness was approximately 100 μm. The basis weight of the sample was 70 gm⁻².

In Fig. 9 the simulated out-of-plane deformation of the CD-directional cross-section (in the middle of the sheet) is presented. As can be seen, the value of the moisture content gradient has a great effect on the simulated out-of-plane deformation. Both amplitude and wavelength of the large-scale deformation pattern depends strongly on the moisture content gradient. Generally, this kind of dependence will appear also with other fiber orientation structures, see Fig. 11. Fig. 11 shows a cross-section from the middle of the sheet when a simplified fiber orientation structure was used. The simplified structure consists of two layers where in both layers all fiber orientation angles are zero degrees. At the top layer there exists a 100 mm x 100 mm area in the middle of the sheet, where anisotropies were defined as the sum of 2.5 and a uniformly distributed random number between 0 and 1. In all other areas anisotropies were defined as the sum of 1.5 and a uniformly distributed random number between 0 and 1.

Figs. 9 and 11 suggest that the small-scale out-of-plane deformation, which is usually accounted for as cockling, does not appear with the small moisture gradients. This is also affirmed by Figs. 10 and 12.

Table 2

<table>
<thead>
<tr>
<th>Vβ (%)</th>
<th>Top side β (%)</th>
<th>Bottom side β (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>−10</td>
<td>90 → 90 → 90</td>
<td>90 → 100 → 90</td>
</tr>
<tr>
<td>−8</td>
<td>90 → 91 → 90</td>
<td>90 → 99 → 90</td>
</tr>
<tr>
<td>−6</td>
<td>90 → 92 → 90</td>
<td>90 → 98 → 90</td>
</tr>
<tr>
<td>−4</td>
<td>90 → 93 → 90</td>
<td>90 → 97 → 90</td>
</tr>
<tr>
<td>−2</td>
<td>90 → 94 → 90</td>
<td>90 → 96 → 90</td>
</tr>
<tr>
<td>0</td>
<td>90 → 95 → 90</td>
<td>90 → 95 → 90</td>
</tr>
<tr>
<td>2</td>
<td>90 → 96 → 90</td>
<td>90 → 94 → 90</td>
</tr>
<tr>
<td>4</td>
<td>90 → 97 → 90</td>
<td>90 → 93 → 90</td>
</tr>
<tr>
<td>6</td>
<td>90 → 98 → 90</td>
<td>90 → 92 → 90</td>
</tr>
<tr>
<td>8</td>
<td>90 → 99 → 90</td>
<td>90 → 91 → 90</td>
</tr>
<tr>
<td>10</td>
<td>90 → 100 → 90</td>
<td>90 → 90 → 90</td>
</tr>
</tbody>
</table>
and 12 is presented the same cross-sections for moisture content gradients as in Figs. 9 and 11 after wavelengths exceeding 30 mm have been filtered out from the results. Fig. 10 also shows the difference between the simulated large-scale out-of-plane deformation and cockling. Where the amplitude and wavelength of the large-scale out-of-plane deformation depends on the moisture content value, only the amplitude of cockling depends on the moisture content gradient. From Fig. 12, it can be seen that the direction of the moisture content gradient has an effect on simulated cockling; now also the places of the cockles has changed in many cases.
Figs. 13 and 14 present simulated cockling in the case of positive (+6% and +10%) and negative (−6% and −10%) gradients, respectively. Simulated cockling for zero gradient is used as a reference point in both Figs. 13 and 14. The scales in Figs. 13 and 14, respectively, are the same for all three subfigures, but the scales between Figs. 13 and 14 vary, so that in Fig. 13 the scale is from −70 μm to +130 μm and in Fig. 14 from −120 μm to +80 μm. Yet, the vertical distance between black and white is 200 μm in every figure to facilitate the comparison of results.

Fig. 11. Out-of-plane displacement of the CD-directional cross-section in the middle of the sheet when simplified fiber orientation structure is used. Thickness of the sample is 50 μm. Markers correspond the values of moisture content gradients presented in Table 2.

Fig. 12. Out-of-plane displacement of the CD-directional cross-section in the middle of the sheet after the wavelengths exceeding 30 mm have been filtered out from the results presented in Fig. 11. For the clarity, only three cross-sections are presented.

Figs. 13 and 14 present simulated cockling in the case of positive (+6% and +10%) and negative (−6% and −10%) gradients, respectively. Simulated cockling for zero gradient is used as a reference point in both Figs. 13 and 14. The scales in Figs. 13 and 14, respectively, are the same for all three subfigures, but the scales between Figs. 13 and 14 vary, so that in Fig. 13 the scale is from −70 μm to +130 μm and in Fig. 14 from −120 μm to +80 μm. Yet, the vertical distance between black and white is 200 μm in every figure to facilitate the comparison of results.
Figs. 13 and 14 support the notice that the strength and the direction of the moisture gradient affects on the paper’s cockling tendency significantly. In Fig. 15 the standard deviation (SD) of simulated cockling of fine paper sample as a function of moisture gradient is presented. SD of cockling is frequently used to describe

Fig. 13. Simulated cockling using moisture gradients 0%, +6% and +10%. The figure on the left-hand side is from the zero gradient, the middle figure is from the +6% gradient, and the figure on the right-hand side is from the +10% gradient. The in-plane size of the sample is 192 mm × 192 mm and vertical distance between black and white is 200 μm.

Fig. 14. Simulated cockling using moisture gradients 0%, −6% and −10%. The figure on the left-hand side is from the zero gradient, the middle figure is from the −6% gradient, and the figure on the right-hand side is from the −10% gradient. The in-plane size of the sample is 192 mm × 192 mm and vertical distance between black and white is 200 μm.

Fig. 15. SD of simulated cockling as a function of moisture gradient \( \nabla \beta \) from −10 to +10%.

Figs. 13 and 14 support the notice that the strength and the direction of the moisture gradient affects on the paper’s cockling tendency significantly. In Fig. 15 the standard deviation (SD) of simulated cockling of fine paper sample as a function of moisture gradient is presented. SD of cockling is frequently used to describe
the intensity of cockling in paper technology (Leppänen et al., 2006). Furthermore, Fig. 15 indicates that the moisture gradient starts to influence the out-of-plane cockling behaviour strongly after exceeding a certain gradient, which in this case is around ±3%. Fig. 15 also suggests that the dependence of SD of cockling on the moisture gradient is quite linear after this limit value, at least when the absolute value of the gradient is 10% or less.

6. Discussion

The studies performed by Smith (1950), Nam and Thorpe (1996), Ahrens et al. (2003, 2005), Paik and Nam (2001) and many others have shown the importance of drying on the cockling phenomenon. This is also supported by the results presented in this paper. The simulated cockles seem to be strongly dependent on the direction and the strength of the moisture gradient.

The results indicate that the SD of cockling is influenced strongly by the moisture gradient. This fact can be seen from Fig. 15 which reveals that the SD of the simulated cockling increases as the absolute value of the moisture gradient increases. However, the in-plane shape of the cockles remains quite unchanged after the limit value for the moisture content gradient is exceeded, i.e. cockling occurs, see Figs. 10, 13 and 14. This agrees with Leppänen (2007); the in-plane shape of the cockles, which originates from the variation of fibre orientation, are quite stable.

Future work will expand the model’s capabilities to predict the appearance and origin of out-of-plane deformations, including also fluting and curling phenomena. Thus, the model will be a useful tool in evaluating the effect of different factors on paper cockling, fluting and curling tendencies, in both the manufacturing process and during end-use of paper. Furthermore, in order to achieve comprehensive understanding of paper’s tendency to out-of-plane deformations, it might be important to consider modelling also of the visco-plastic effects, i.e. the relaxation phenomenon in the stress–strain behaviour of paper. Moreover, the inclusion of basis weight variations in the model might be necessary (Hojjatie and Ahrens, 2004).

7. Conclusions

The adequacy of the developed orthotropic elasto-plastic material model for a paper sheet in terms of varying conditions in the moisture content of paper was shown. The model has been used to predict the paper’s tendency to cockle in circumstances quite similar to especially copying, but also printing, of paper, for instance.

Beside the moisture gradient, the model uses only two measured structural properties of paper: the local fibre orientation angle and its anisotropy, both of which vary inside the paper sheet. The results indicate that the cockling tendency of paper is highly influenced by the strength and direction of the moisture gradient.

Acknowledgements

Special thanks go to Joonas Sorvari and Matti Malinen for their help related to this work. We are also grateful to UPM-Kymmene Corp. and Metso Paper, Inc. for the financial support of this research project and for providing experimental data for our use.

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