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ORIGINAL ARTICLE

An analytical method for solving exact solutions of the nonlinear Bogoyavlenskii equation and the nonlinear diffusive predator–prey system

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Abstract In this article, we apply the $\exp(-\Phi(\xi))$-expansion method to construct many families of exact solutions of nonlinear evolution equations (NLEEs) via the nonlinear diffusive predator–prey system and the Bogoyavlenskii equations. These equations can be transformed to nonlinear ordinary differential equations. As a result, some new exact solutions are obtained through the hyperbolic function, the trigonometric function, the exponential functions and the rational forms. If the parameters take specific values, then the solitary waves are derived from the traveling waves. Also, we draw 2D and 3D graphics of exact solutions for the special diffusive predator–prey system and the Bogoyavlenskii equations by the help of programming language Maple.

1. Introduction

NLEEs have much significant roles in various areas of applied science and engineering, especially in mathematical biology, biomathematics, population dynamics, nonlinear optics, fluid mechanics, solid-state physics, biophysics, chemical kinetics, protein chemistry, theory of Bose–Einstein condensates, plasma physics and so on (see [1–4]). The exact solutions of NLEEs might provide much corporal information to understand the instrument of the physical models (see [10–15]). Exact solutions of NLEEs also give information about character of differential equations (see [16–42]). Therefore, it has attracted a great deal of attention for procuring exact solutions of NLEEs by utilizing programming languages Maple. Many powerful methods have been presented by diverse group of mathematicians and physicists such as the Sumudu transform method [5–7], the Hermite–Pade approximation method [8], the solitary wave ansatz method [9,10], the complex hyperbolic function method [11,12], (G'/G)-expansion method [13–21], the Fan sub-equation method [22,23], the improved tanh–coth method [24,25], the collocation method [26,27], tanh–sech method [28,29], sine–cosine method [30,31], homogeneous balance method [32,33], and $\exp(-\Phi(\xi))$-expansion method [34–37].

In this paper, the $\exp(-\Phi(\xi))$-expansion method will be tackled to find exact solutions of the following nonlinear Bogoyavlenskii equation [38,48] and the nonlinear diffusive predator–prey system [39,48]:

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(1^0) We consider the nonlinear Bogoyavlenskii equations [38,48] in the form

$$\begin{cases} 4u_t + u_{xxy} - 4u^2u_y - 4u_xv = 0, \\ uu_y = v_x. \end{cases} \tag{1}$$

In [38], the Lax pair and a non-isospectral condition for the spectral parameter was presented. Eq. (1) was again derived by Kudryashov and Pickerling [40] as a member of a (2 + 1) Schwarzian breaking soliton hierarchy, and rational solutions of it were obtained. The equation also appeared in [41] as one of the equations associated with non-isospectral scattering problems. The Painleve property of Eq. (1) is recently checked by Estevez and Prada [42]. Some exact solutions of this equation are also found in [43]. Eq. (1) as the modified version of a breaking soliton equation,

$$4u_{xt} + 8u_xu_{xy} + 4u_yu_{xx} + u_{xxxy} = 0 \tag{2}$$

describes the (2 + 1)-dimensional interaction of a Riemann wave propagation along the y-axis with a long wave the x-axis [38]. To a certain extent, a similar interaction is observed in waves on the surface of the sea. It is well-known that the solution and its dynamics of the equation can make researchers deeply understand the described physical process.

(2^0) We consider the following system of two coupled NLEEs describing the dynamics of the predator–prey system of equations [39,48]:

$$\begin{cases} u_t = u_{xx} - \beta u + (1+\beta)u^2 - u^3 - uv, \\ v_t = v_{xx} + kuv - mv - \delta v^3, \end{cases} \tag{3}$$

where k, δ, m and β are positive parameters. The solutions of predator–prey system of equations have been studied in various aspects (see [39,44,45]). The dynamics of system (3) assumed the following relations between the parameters, namely $m = \beta$ and $k + \frac{1}{\sqrt{\delta}} = \beta + 1$. Under these assumptions, system (3) can be rewritten in the form of

$$\begin{cases} u_t = u_{xx} - \beta u + \left(k + \frac{1}{\sqrt{\delta}}\right)u^2 - u^3 - uv, \\ v_t = v_{xx} + kuv - \beta v - \delta v^3. \end{cases} \tag{4}$$

The prey–predator model incorporating diffusion is of profound interest as it takes into account the heterogeneity of both the environment and the populations involved. The spatial pattern formation even in the absence of environmental heterogeneity is another interesting phenomenon associated with the diffusion models [46]. Existence of exact solutions is pivotal to better understand about the processes involved.

In this paper, we offer a $\exp(-\Phi(\xi))$-expansion method to solve the NLEEs in mathematical physics and engineering. To illustrate the originality, consistency and advantages of the projected method, the nonlinear Bogoyavlenskii equations and the following system of two coupled NLEEs describing the dynamics of the predator–prey system of equations are solved and further new families of exact solutions are found. The principal improvement of the functional method in this article over the (G'/G)-expansion method is that it provides more new exact solutions including additional free parameters. All the solutions obtained by the (G'/G)-expansion method are obtained through the applied method as a particular case and we get various new solutions as well. The exact solutions have its huge significance to expose the inner mechanism of the physical phenomena. Apart from the physical relevance, the close-form solutions of nonlinear partial differential equations help the numerical solvers to compare the correctness of their results and assist them in the stability analysis.

This paper is organized as follows: In Section 2, we give the description of the modified simple equation method. In Section 3, we use this method to find the exact solutions of the nonlinear diffusive predator–prey system (2) and the Bogoyavlenskii equations (3) pointed out above. In Section 4, the graphical illustrations of the solutions to the nonlinear Bogoyavlenskii equation and the nonlinear diffusive predator–prey system of equations are given. At the end, in Section 5, a conclusion is given.

2. Description of the $\exp(-\Phi(\xi))$-expansion method

Here we briefly describe the main steps of the $\exp(-\Phi(\xi))$-expansion method. Consider a general NLEE is of the form

$$P(u, u_t, u_x, u_{xx}, u_{tt}, u_{tx}, \ldots) = 0, \tag{5}$$

where $u = u(x,t)$ is an unknown function and P is polynomial in $u = u(x,t)$ and its partial derivatives, in which higher order derivatives and nonlinear terms are involved. In order to solve Eq. (5) by this method, one has to resort the following steps:

Step 1. To find the traveling wave solution of (5), introduce the wave variable $\xi = x \pm ct$, so that $u(x,t) = u(\xi)$. Based on this,

$$\frac{\delta}{\delta t} = -c\frac{\delta}{\delta \xi}, \quad \frac{\delta^2}{\delta t^2} = c^2\frac{\delta^2}{\delta \xi^2}, \quad \frac{\delta}{\delta x} = \frac{\delta}{\delta \xi}, \quad \frac{\delta^2}{\delta x^2} = \frac{\delta^2}{\delta \xi^2}, \tag{6}$$

and so on for other derivatives. With the help of (6), the NLEE (5) changes to an ODE as

$$\Re(u, u', u'', u''', \ldots) = 0, \tag{7}$$

where u', u'', etc. denote derivative of u with respect to ξ and $\Re$ is a polynomial of u. Now integrate the ODE (7) as many times as possible and set the constants of integration to be zero.

Step 2. The solution of (7) can be expressed by a polynomial in $(\exp(-\Phi(\xi)))$ as

$$u(\xi) = \sum_{i=0}^{N} A_i(\exp(-\Phi(\xi)))^i, \tag{8}$$

where A_i $(0 \leqslant i \leqslant N)$, are constants to be determined, such that $A_N \neq 0$ and $\Phi = \Phi(\xi)$ satisfy the following ODE:

$$\Phi'(\xi) = \exp(-\Phi(\xi)) + \mu\exp(\Phi(\xi)) + \lambda, \tag{9}$$

Eq. (9) gives the following solutions, respectively:When $\mu \neq 0,\ \lambda^2 - 4\mu > 0$,

$$\Phi(\xi) = \ln\left(\frac{-\sqrt{(\lambda^2 - 4\mu)}\tanh\left(\frac{\sqrt{(\lambda^2-4\mu)}}{2}(\xi + E)\right) - \lambda}{2\mu}\right); \tag{10}$$

when $\mu \neq 0,\ \lambda^2 - 4\mu < 0$,

$$\Phi(\xi) = \ln\left(\frac{\sqrt{(4\mu-\lambda^2)}\tan\left(\frac{\sqrt{(4\mu-\lambda^2)}}{2}(\xi+E)\right)-\lambda}{2\mu}\right); \tag{11}$$

when $\mu = 0,\ \lambda \neq 0$, and $\lambda^2 - 4\mu > 0$,

$$\Phi(\xi) = -\ln\left(\frac{\lambda}{\exp(\lambda(\xi+E))-1}\right); \tag{12}$$

when $\mu \neq 0, \lambda \neq 0$, and $\lambda^2 - 4\mu = 0$,

$$\Phi(\xi) = \ln\left(-\frac{2(\lambda(\xi+E)+2)}{\lambda^2(\xi+E)}\right); \tag{13}$$

when $\mu = 0, \lambda = 0$, and $\lambda^2 - 4\mu = 0$,

$$\Phi(\xi) = \ln(\xi+E), \tag{14}$$

where $A_N, \ldots, c, \lambda, \mu$ are constants to be determined latter, $A_N \neq 0$. The positive integer N can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in ODE (7), after using (8).

Step 3. We replace relation (8) into Eq. (7) and then we account the function $\exp(-\Phi(\xi))$. As a result of this replacement, we get a polynomial of $\exp(-\Phi(\xi))$. We equate all the coefficients of same power of $\exp(-\Phi(\xi))$ to zero. This process yields a system of algebraic equations whichever can be solved to find $A_N, \ldots, c, \lambda, \mu$. Substituting the values of $A_N, \ldots, c, \lambda, \mu$ into Eq. (8) along with general solutions of Eq. (9) completes the determination of the solution of Eq. (5). We obtain more traveling wave solutions of nonlinear differential equation (6).

After the brief description of method, we now solve the nonlinear Bogoyavlenskii equation and the diffusive predator–prey system of equation using the above method.

3. Applications of the method

Here, we will apply the $\exp(-\Phi(\xi))$-expansion method described in Section 2 to find some new traveling wave solutions and then the solitary wave solutions for the following nonlinear systems of NLEE in applied sciences and engineering.

3.1. The nonlinear Bogoyavlenskii equation

In this subsection, we determine the exact solutions and the solitary wave solutions of system (3). To this end, we use the wave transformation $\xi = x + y - ct$ to reduce system (1) to the following nonlinear system of ordinary differential equations:

$$\begin{cases} -4cu' + u''' - 4u^2u' - 4u'v = 0, \\ \frac{u^2}{2} = v. \end{cases} \tag{15}$$

Substituting the second equation of (15) into the first one, and integrating the resultant equation, we have

$$u'' - 2u^3 - 4cu = 0, \tag{16}$$

with zero constant of integration. Balancing u'' with u^3 in Eq. (16), we get $N = 1$.

Therefore, the solution of Eq. (16) is of the form:

$$u(\eta) = A_0 + A_1(\exp(-\Phi(\xi))), \tag{17}$$

where $A_0,\ A_1$ are constants to be determined such that $A_N \neq 0$, while λ, μ are arbitrary constants.

Substituting Eq. (17) into Eq. (16) and equating the coefficients of $\exp(-\Phi(\xi))^3, \exp(-\Phi(\xi))^2,\ \exp(-\Phi(\xi))^1$, $\exp(-\Phi(\xi))^0$ to zero, we obtain, respectively the following:

$$\exp(-\Phi(\xi))^3:\ 2A_1 - 2A_1^3 = 0, \tag{18}$$

$$\exp(-\Phi(\xi))^2:\ -6A_0A_1^2 + 3A_1\lambda = 0, \tag{19}$$

$$\exp(-\Phi(\xi))^1:\ \lambda^2A_1 + 2A_1\mu - 6A_0^2A_1 - 4cA_1 = 0, \tag{20}$$

$$\exp(-\Phi(\xi))^0:\ 2A_0^3 + A_1\mu\lambda - 4cA_0 = 0. \tag{21}$$

Solving Eqs. (18)–(21) yields

$$\textbf{Cluster1}.\ c = -\frac{1}{8}\lambda^2 + \frac{1}{2}\mu, \quad A_0 = \frac{1}{2}\lambda, \quad A_1 = 1, \tag{22}$$

where λ, μ are arbitrary constants.

$$\textbf{Cluster2}.\ c = -\frac{1}{8}\lambda^2 + \frac{1}{2}\mu, \quad A_0 = -\frac{1}{2}\lambda, \quad A_1 = -1, \tag{23}$$

where λ, μ are arbitrary constants.

For Cluster 1, substituting Eq. (22) into Eq. (17), we obtain

$$u(\xi) = \frac{1}{2}\lambda + (\exp(-\Phi(\xi))), \tag{24}$$

where $\xi = x - \left(-\frac{1}{8}\lambda^2 + \frac{1}{2}\mu\right)t$.

And for Cluster 2, substituting Eq. (23) into Eq. (17), we obtain

$$u(\xi) = -\frac{1}{2}\lambda - (\exp(-\Phi(\xi))), \tag{25}$$

where $\xi = x - \left(-\frac{1}{8}\lambda^2 + \frac{1}{2}\mu\right)t$.

Now substituting Eqs. (10)–(14) into Eq. (24), respectively, we get the following five traveling wave solutions of the nonlinear Bogoyavlenskii equation:

When $\mu \neq 0,\ \lambda^2 - 4\mu > 0$,

$$u_1(\xi) = \frac{1}{2}\lambda - \left(\frac{2\mu}{\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+E)\right)+\lambda}\right),$$

$$v_1(\xi) = \frac{1}{2}\left\{\frac{1}{2}\lambda - \left(\frac{2\mu}{\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+E)\right)+\lambda}\right)\right\}^2,$$

where $\xi = x - \left(-\frac{1}{8}\lambda^2 + \frac{1}{2}\mu\right)t$ and E is an arbitrary constant.

When $\mu \neq 0, \lambda^2 - 4\mu < 0$,

$$u_2(\xi) = \frac{1}{2}\lambda + \left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+E)\right)-\lambda}\right),$$

$$v_2(\xi)=\frac{1}{2}\left\{\frac{1}{2}\lambda+\left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+E)\right)-\lambda}\right)\right\}^2,$$

where $\xi=x-\left(-\frac{1}{8}\lambda^2+\frac{1}{2}\mu\right)t$ and E is an arbitrary constant.

When $\mu=0,\ \lambda\neq 0$, and $\lambda^2-4\mu>0$,

$$u_3(\xi)=\frac{1}{2}\lambda+\left(\frac{\lambda}{\exp(\lambda(\xi+E))-1}\right),$$

$$v_3(\xi)=\frac{1}{2}\left\{\frac{1}{2}\lambda+\left(\frac{\lambda}{\exp(\lambda(\xi+E))-1}\right)\right\}^2,$$

where $\xi=x-\left(-\frac{1}{8}\lambda^2+\frac{1}{2}\mu\right)t$ and E is an arbitrary constant.

When $\mu\neq 0,\lambda\neq 0$, and $\lambda^2-4\mu=0$,

$$u_4(\xi)=\frac{1}{2}\lambda-\left(\frac{\lambda^2(\xi+E)}{2(\lambda(\xi+E))+2)}\right),$$

$$v_4(\xi)=\frac{1}{2}\left\{\frac{1}{2}\lambda-\left(\frac{\lambda^2(\xi+E)}{2(\lambda(\xi+E))+2)}\right)\right\}^2,$$

where $\xi=x-\left(-\frac{1}{8}\lambda^2+\frac{1}{2}\mu\right)t$ and E is an arbitrary constant.

When $\mu=0,\lambda=0$, and $\lambda^2-4\mu=0$,

$$u_5(\xi)=\frac{1}{2}\lambda+\left(\frac{1}{\xi+E}\right),$$

$$v_5(\xi)=\frac{1}{2}\left\{\frac{1}{2}\lambda+\left(\frac{1}{\xi+E}\right)\right\}^2,$$

where $\xi=x-\left(-\frac{1}{8}\lambda^2+\frac{1}{2}\mu\right)t$ and E is an arbitrary constant.

In this position, inserting Eqs. (10)–(14) into Eq. (25), respectively, we get the following another five traveling wave solutions of the nonlinear Bogoyavlenskii equation.

When $\mu\neq 0,\lambda^2-4\mu>0$,

$$u_6(\xi)=-\frac{1}{2}\lambda+\left(\frac{2\mu}{\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+E)\right)+\lambda}\right),$$

$$v_6(\xi)=\frac{1}{2}\left\{-\frac{1}{2}\lambda+\left(\frac{2\mu}{\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+E)\right)+\lambda}\right)\right\}^2,$$

where $\xi=x-\left(-\frac{1}{8}\lambda^2+\frac{1}{2}\mu\right)t$ and E is an arbitrary constant.

When $\mu\neq 0,\lambda^2-4\mu<0$,

$$u_7(\xi)=-\frac{1}{2}\lambda-\left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+E)\right)-\lambda}\right),$$

$$v_7(\xi)=\frac{1}{2}\left\{-\frac{1}{2}\lambda-\left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+E)\right)-\lambda}\right)\right\}^2,$$

where $\xi=x-\left(-\frac{1}{8}\lambda^2+\frac{1}{2}\mu\right)t$ and E is an arbitrary constant.

When $\mu=0,\lambda\neq 0$, and $\lambda^2-4\mu>0$,

$$u_8(\xi)=-\frac{1}{2}\lambda-\left(\frac{\lambda}{\exp(\lambda(\xi+E))-1}\right),$$

$$v_8(\xi)=\frac{1}{2}\left\{-\frac{1}{2}\lambda-\left(\frac{\lambda}{\exp(\lambda(\xi+E))-1}\right)\right\}^2,$$

where $\xi=x-\left(-\frac{1}{8}\lambda^2+\frac{1}{2}\mu\right)t$ and E is an arbitrary constant.

When $\mu\neq 0,\lambda\neq 0$, and $\lambda^2-4\mu=0$,

$$u_9(\xi)=-\frac{1}{2}\lambda+\left(\frac{\lambda^2(\xi+E)}{2(\lambda(\xi+E))+2)}\right),$$

$$v_9(\xi)=\frac{1}{2}\left\{-\frac{1}{2}\lambda+\left(\frac{\lambda^2(\xi+E)}{2(\lambda(\xi+E))+2)}\right)\right\}^2,$$

where $\xi=x-\left(-\frac{1}{8}\lambda^2+\frac{1}{2}\mu\right)t$ and E is an arbitrary constant.

When $\mu=0,\lambda=0$, and $\lambda^2-4\mu=0$,

$$u_{10}(\xi)=-\frac{1}{2}\lambda-\left(\frac{1}{\xi+E}\right),$$

$$v_{10}(\xi)=\frac{1}{2}\left\{-\frac{1}{2}\lambda-\left(\frac{1}{\xi+E}\right)\right\}^2,$$

where $\xi=x-\left(-\frac{1}{8}\lambda^2+\frac{1}{2}\mu\right)t$ and E is an arbitrary constant.

3.2. A nonlinear diffusive predator–prey system

In this subsection, we determine the exact solutions and the solitary wave solutions of Eq. (4). To this end, we use the wave transformation $\xi=x+y-ct$ to reduce system (3) to the following nonlinear system of ordinary differential equations:

$$\begin{cases} u''+cu'-\beta u+\left(k+\frac{1}{\sqrt{\delta}}\right)u^2-u^3-uv=0,\\ v''+cv'+kuv-\beta v-\delta v^3=0.\end{cases}\tag{26}$$

In order to solve system (26), let us consider the following transformation:

$$v=\frac{1}{\sqrt{\delta}}u.\tag{27}$$

Substituting the transformation (27) into (26), we get

$$u''+cu'-\beta u+ku^2-u^3=0.\tag{28}$$

Balancing u'' with u^3 in Eq. (28), we get $N=1$. Consequently, we get the formal solution:

$$u(\xi)=A_0+A_1(\exp(-\Phi(\xi))),\tag{29}$$

where $A_0,\ A_1$ are constants to be determined such that $A_N\neq 0$, while λ,μ are arbitrary constants.

Substituting Eq. (29) into Eq. (28) and equating the coefficients of $\exp(-\Phi(\xi))^3,\exp(-\Phi(\xi))^2,\ \exp(-\Phi(\xi))^1,\ \exp(-\Phi(\xi))^0$ to zero, we obtain, respectively the following:

$$\exp(-\Phi(\xi))^3:\ 2A_1-2A_1^3=0,\tag{30}$$

$$\exp(-\Phi(\xi))^2:\ -cA_1+kA_1^2+3A_1\lambda-3A_0A_1^2=0,\tag{31}$$

$$\exp(-\Phi(\xi))^1: \ -3A_0^2A_1 + 2A_1\mu - \beta A_1 - cA_1\lambda + \lambda^2 A_1 + 2kA_0A_1 = 0, \tag{32}$$

$$\exp(-\Phi(\xi))^0: \ -A_0^3 + A_1\mu\lambda - \beta A_0 - cA_1\mu + kA_0^2 = 0, \tag{33}$$

Solving Eqs. (30)–(33) yields

Cluster1. $c = -\frac{k}{\sqrt{2}}, \quad \beta = \frac{1}{4}k^2 + 2\mu - \frac{1}{2}\lambda^2,$

$$A_0 = \frac{1}{\sqrt{2}}\lambda + \frac{1}{2}k, \quad A_1 = \sqrt{2}, \tag{34}$$

where λ, μ and k are arbitrary constants.

Cluster2. $c = \frac{k}{\sqrt{2}}, \quad \beta = \frac{1}{4}k^2 + 2\mu - \frac{1}{2}\lambda^2,$

$$A_0 = -\frac{1}{\sqrt{2}}\lambda + \frac{1}{2}k, \quad A_1 = -\sqrt{2}. \tag{35}$$

where λ, μ and k are arbitrary constants.

For Cluster 1, substituting Eq. (34) into Eq. (29), we obtain

$$u(\xi) = \frac{1}{\sqrt{2}}\lambda + \frac{1}{2}k + \sqrt{2}(\exp(-\Phi(\xi))), \tag{36}$$

where $\xi = x + \left(\frac{k}{\sqrt{2}}\right)t$.

And for Cluster 2, substituting Eq. (35) into Eq. (29), we obtain

$$u(\xi) = -\frac{1}{\sqrt{2}}\lambda + \frac{1}{2}k - \sqrt{2}(\exp(-\Phi(\xi))), \tag{37}$$

where $\xi = x - \left(\frac{k}{\sqrt{2}}\right)t$.

Now substituting Eqs. (10)–(14) into Eq. (36), respectively, we get the following five traveling wave solutions of the diffusive predator–prey system of equation:

When $\mu \neq 0, \ \lambda^2 - 4\mu > 0,$

$$u_{11}(\xi) = \frac{\lambda}{\sqrt{2}} + \frac{1}{2}k - \sqrt{2}\left(\frac{2\mu}{\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi + E)\right) + \lambda}\right),$$

$$v_{11}(\xi) = \frac{1}{\sqrt{\delta}}\left\{\frac{\lambda}{\sqrt{2}} + \frac{1}{2}k - \sqrt{2}\left(\frac{2\mu}{\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi + E)\right) + \lambda}\right)\right\},$$

where $\xi = x + \left(\frac{k}{\sqrt{2}}\right)t$ and E is an arbitrary constant.

When $\mu \neq 0, \ \lambda^2 - 4\mu < 0,$

$$u_{12}(\xi) = \frac{\lambda}{\sqrt{2}} + \frac{1}{2}k + \sqrt{2}\left(\frac{2\mu}{\sqrt{4\mu - \lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi + E)\right) - \lambda}\right),$$

$$v_{12}(\xi) = \frac{1}{\sqrt{\delta}}\left\{\frac{\lambda}{\sqrt{2}} + \frac{1}{2}k + \sqrt{2}\left(\frac{2\mu}{\sqrt{4\mu - \lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi + E)\right) - \lambda}\right)\right\},$$

where $\xi = x + \left(\frac{k}{\sqrt{2}}\right)t$ and E is an arbitrary constant.

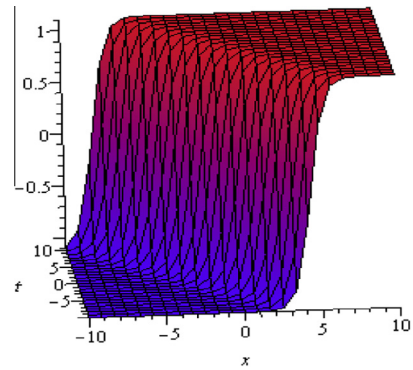

Figure 1 The solitary wave 3D graphics of $u_1(\xi)$ and $u_6(\xi)$ for $\mu = 1, \lambda = 3, E = 1$ and $-10 \leqslant x, t \leqslant 10$.

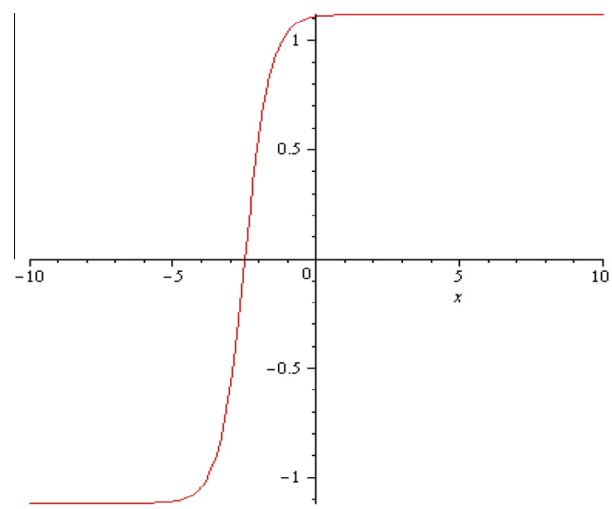

Figure 2 The solitary wave 2D graphics of $u_1(\xi)$ and $u_6(\xi)$ for $t = 1, \mu = 1, \lambda = 3$ and $E = 1$.

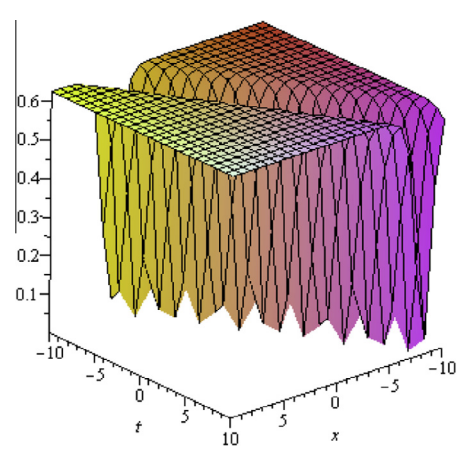

Figure 3 The solitary wave 3D graphics of $v_1(\xi)$ and $v_6(\xi)$ for $\mu = 1, \lambda = 3, E = 1$ and $-10 \leqslant x, t \leqslant 10$.

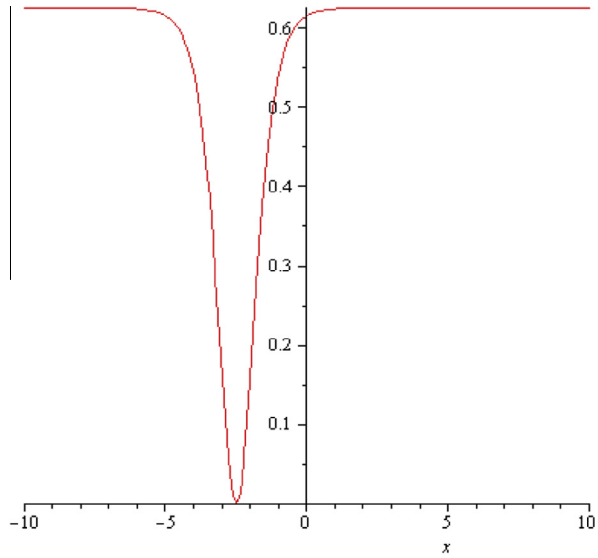

Figure 4 The solitary wave 2D graphics of $v_1(\xi)$ and $v_6(\xi)$ for $t = 1$, $\mu = 1$, $\lambda = 3$ and $E = 1$.

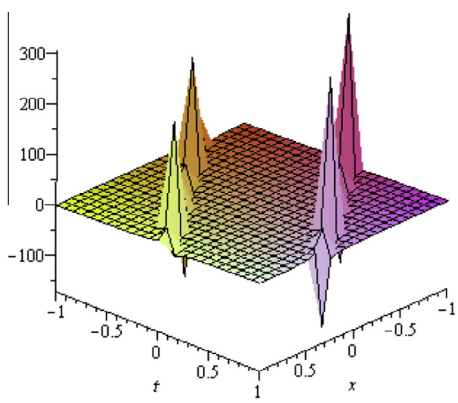

Figure 5 The solitary wave 3D graphics of $u_2(\xi)$ and $u_7(\xi)$ for $\mu = 3$, $\lambda = 1, E = 5$ and $-1 \leqslant x$, $t \leqslant 1$.

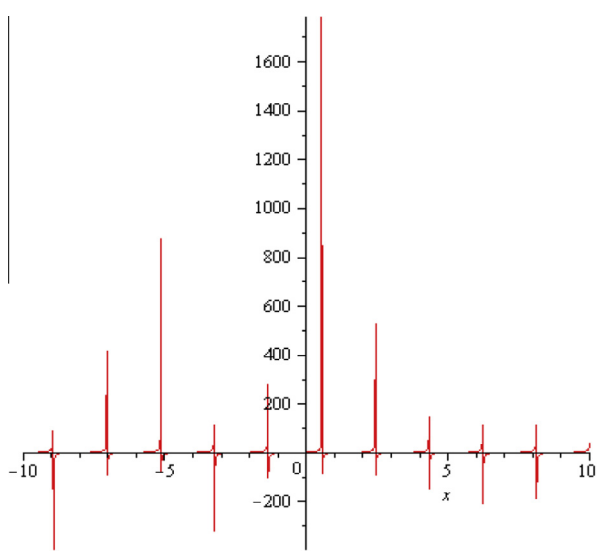

Figure 6 The solitary wave 2D graphics of $u_2(\xi)$ and $u_7(\xi)$ for $t = 1$, $\mu = 3, \lambda = 1$ and $E = 5$.

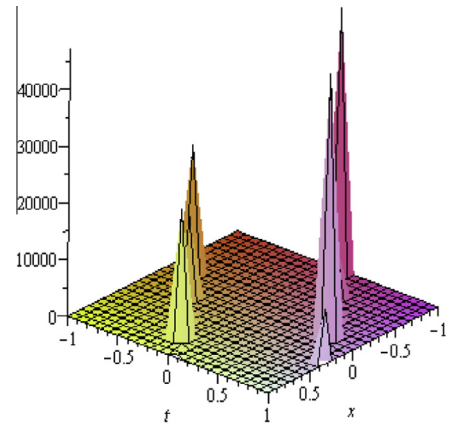

Figure 7 The solitary wave 3D graphics of $v_2(\xi)$ and $v_7(\xi)$ for $\mu = 3$, $\lambda = 1$, $E = 5$ and $-1 \leqslant x$, $t \leqslant 1$.

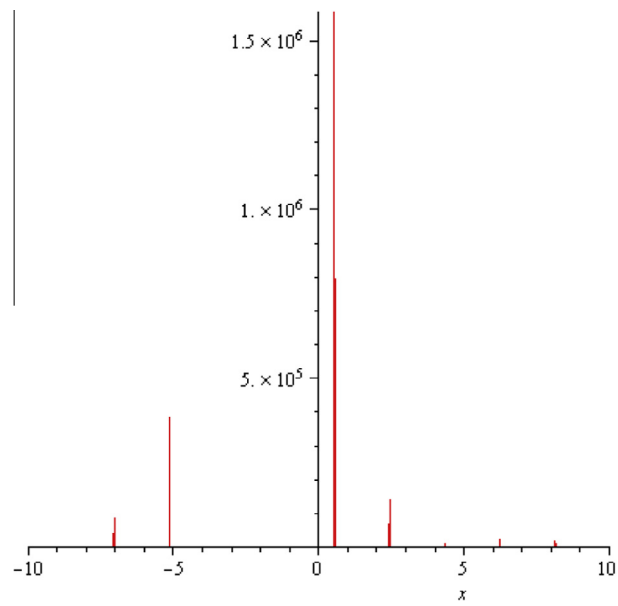

Figure 8 The solitary wave 2D graphics of $v_2(\xi)$ and $v_7(\xi)$ for $t = 1$, $\mu = 3, \lambda = 1$ and $E = 5$.

When $\mu = 0$, $\lambda \neq 0$, and $\lambda^2 - 4\mu > 0$,

$$u_{13}(\xi) = \frac{\lambda}{\sqrt{2}} + \frac{1}{2}k + \sqrt{2}\left(\frac{\lambda}{\exp(\lambda(\xi + E)) - 1}\right),$$

$$v_{13}(\xi) = \frac{1}{\sqrt{\delta}}\left\{\frac{\lambda}{\sqrt{2}} + \frac{1}{2}k + \sqrt{2}\left(\frac{\lambda}{\exp(\lambda(\xi + E)) - 1}\right)\right\},$$

where $\xi = x + \left(\frac{k}{\sqrt{2}}\right)t$ and E is an arbitrary constant.

When $\mu \neq 0$, $\lambda \neq 0$, and $\lambda^2 - 4\mu = 0$,

$$u_{14}(\xi) = \frac{\lambda}{\sqrt{2}} + \frac{1}{2}k - \sqrt{2}\left(\frac{\lambda^2(\xi + E)}{2(\lambda(\xi + E)) + 2)}\right),$$

$$u_{14}(\xi) = \frac{1}{\sqrt{\delta}}\left\{\frac{\lambda}{\sqrt{2}} + \frac{1}{2}k - \sqrt{2}\left(\frac{\lambda^2(\xi + E)}{2(\lambda(\xi + E)) + 2)}\right)\right\},$$

where $\xi = x + \left(\frac{k}{\sqrt{2}}\right)t$ and E is an arbitrary constant.

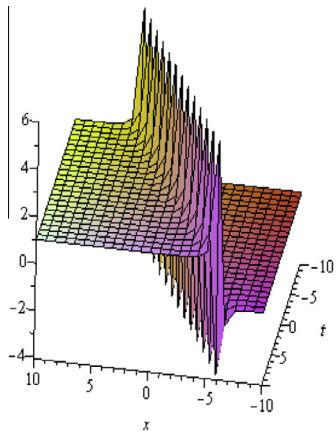

Figure 9 The solitary wave 3D graphics of $u_3(\xi)$ and $u_8(\xi)$ for $\mu = 0$, $\lambda = 2, E = 1$ and $-10 \leqslant x$, $t \leqslant 10$.

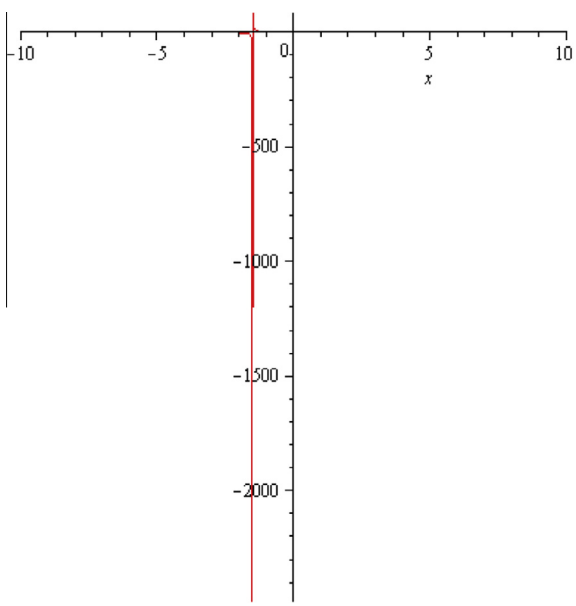

Figure 10 The solitary wave 2D graphics of $u_3(\xi)$ and $u_8(\xi)$ for $t = 1$, $\mu = 0$, $\lambda = 2$ and $E = 1$.

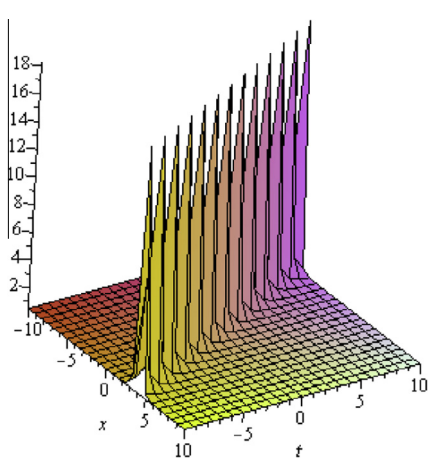

Figure 11 The solitary wave 3D graphics of $v_3(\xi)$ and $v_8(\xi)$ for $\mu = 0$, $\lambda = 2, E = 1$ and $-10 \leqslant x$, $t \leqslant 10$.

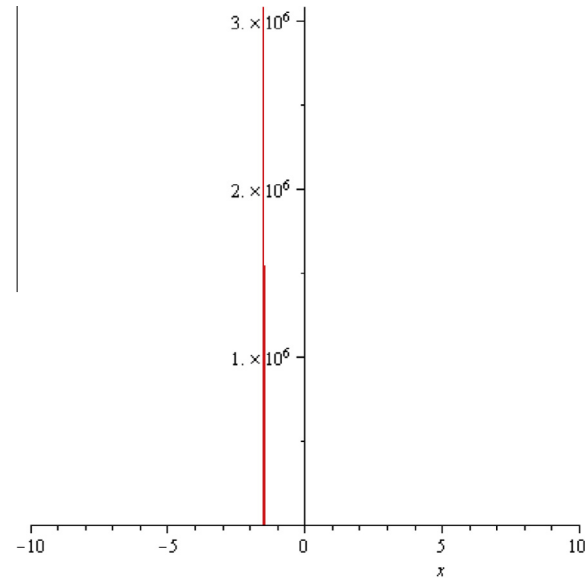

Figure 12 The solitary wave 2D graphics of $v_3(\xi)$ and $v_8(\xi)$ for $t = 1$, $\mu = 0, \lambda = 2$ and $E = 1$.

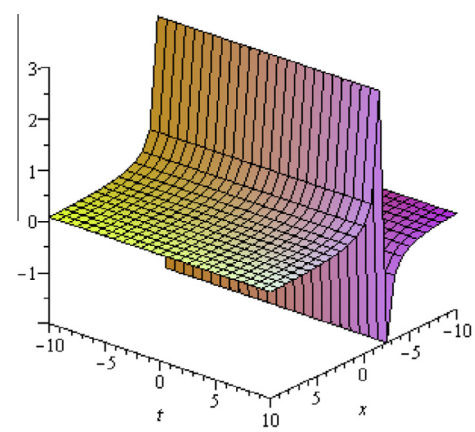

Figure 13 The solitary wave 3D graphics of $u_4(\xi)$ and $u_9(\xi)$ for $\mu = 1$, $\lambda = 2, E = 1$ and $-10 \leqslant x, t \leqslant 10$.

When $\mu = 0, \lambda = 0$, and $\lambda^2 - 4\mu = 0$,

$$u_{15}(\xi) = \frac{\lambda}{\sqrt{2}} + \frac{1}{2}k + \sqrt{2}\left(\frac{1}{\xi + E}\right),$$

$$u_{15}(\xi) = \frac{1}{\sqrt{\delta}}\left\{\frac{\lambda}{\sqrt{2}} + \frac{1}{2}k + \sqrt{2}\left(\frac{1}{\xi + E}\right)\right\},$$

where $\xi = x + \left(\frac{k}{\sqrt{2}}\right)t$ and E is an arbitrary constant.

In this position, inserting Eqs. (10)–(14) into Eq. (37), respectively, we get the following another five traveling wave solutions of the diffusive predator–prey system of equation.

When $\mu \neq 0, \lambda^2 - 4\mu > 0$,

$$u_{16}(\xi) = -\frac{1}{\sqrt{2}}\lambda + \frac{1}{2}k + \sqrt{2}\left(\frac{2\mu}{\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi + E)\right) + \lambda}\right),$$

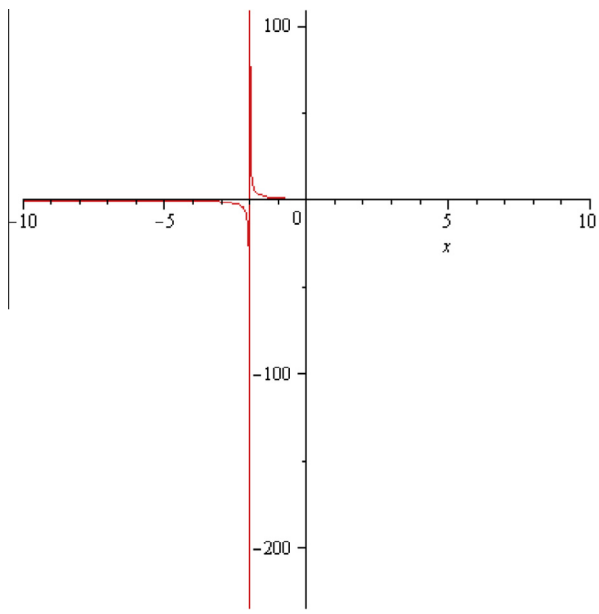

Figure 14 The solitary wave 2D graphics of $u_4(\xi)$ and $u_9(\xi)$ for $t = 1$, $\mu = 1, \lambda = 2$ and $E = 1$.

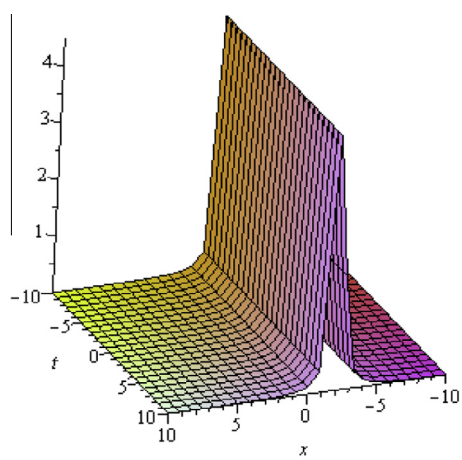

Figure 15 The solitary wave 3D graphics of $u_4(\xi)$ and $u_9(\xi)$ for $\mu = 1$, $\lambda = 2, E = 1$ and $-10 \leqslant x$, $t \leqslant 10$.

$$v_{16}(\xi) = \frac{1}{\sqrt{\delta}}\left\{-\frac{1}{\sqrt{2}}\lambda + \frac{1}{2}k + \sqrt{2}\left(\frac{2\mu}{\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi + E)\right) + \lambda}\right)\right\},$$

where $\xi = x - \left(\frac{k}{\sqrt{2}}\right)t$ and E is an arbitrary constant.

When $\mu \neq 0, \lambda^2 - 4\mu < 0$,

$$u_{17}(\xi) = -\frac{1}{\sqrt{2}}\lambda + \frac{1}{2}k - \sqrt{2}\left(\frac{2\mu}{\sqrt{4\mu - \lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi + E)\right) - \lambda}\right),$$

$$v_{17}(\xi) = \frac{1}{\sqrt{\delta}}\left\{-\frac{1}{\sqrt{2}}\lambda + \frac{1}{2}k - \sqrt{2}\left(\frac{2\mu}{\sqrt{4\mu - \lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi + E)\right) - \lambda}\right)\right\},$$

where $\xi = x - \left(\frac{k}{\sqrt{2}}\right)t$ and E is an arbitrary constant.

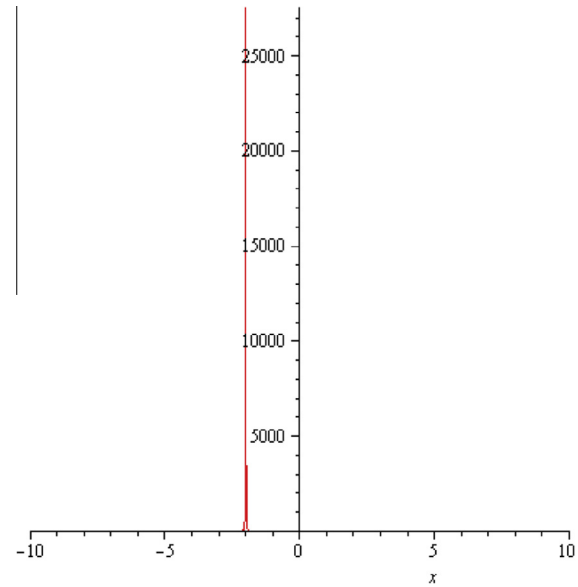

Figure 16 The solitary wave 2D graphics of $u_4(\xi)$ and $u_9(\xi)$ for $t = 1$, $\mu = 1$, $\lambda = 2, E = 1$.

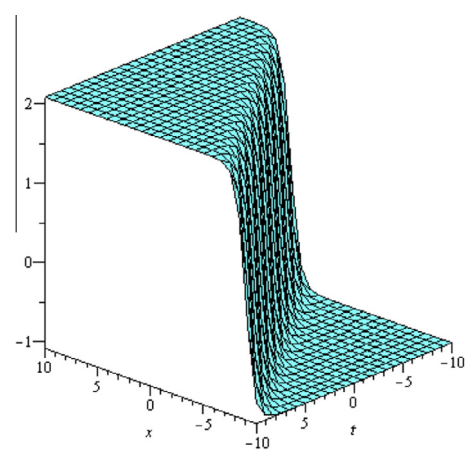

Figure 17 The solitary wave 3D graphics of $u_{11}(\xi)$, $u_{16}(\xi)$, $v_{11}(\xi)$ and $v_{16}(\xi)$ for $\mu = 1$, $\lambda = 3, E = 1, \delta = 2, k = 1$ and $-10 \leqslant x$, $t \leqslant 10$.

When $\mu = 0, \lambda \neq 0$, and $\lambda^2 - 4\mu > 0$,

$$u_{18}(\xi) = -\frac{1}{\sqrt{2}}\lambda + \frac{1}{2}k - \sqrt{2}\left(\frac{\lambda}{\exp(\lambda(\xi + E)) - 1}\right),$$

$$u_{18}(\xi) = \frac{1}{\sqrt{\delta}}\left\{-\frac{1}{\sqrt{2}}\lambda + \frac{1}{2}k - \sqrt{2}\left(\frac{\lambda}{\exp(\lambda(\xi + E)) - 1}\right)\right\},$$

where $\xi = x - \left(\frac{k}{\sqrt{2}}\right)t$ and E is an arbitrary constant.

When $\mu \neq 0, \lambda \neq 0$, and $\lambda^2 - 4\mu = 0$,

$$u_{19}(\xi) = -\frac{1}{\sqrt{2}}\lambda + \frac{1}{2}k + \sqrt{2}\left(\frac{\lambda^2(\xi + E)}{2(\lambda(\xi + E)) + 2)}\right),$$

$$v_{19}(\xi) = \frac{1}{\sqrt{\delta}}\left\{-\frac{1}{\sqrt{2}}\lambda + \frac{1}{2}k + \sqrt{2}\left(\frac{\lambda^2(\xi + E)}{2(\lambda(\xi + E)) + 2)}\right)\right\},$$

where $\xi = x - \left(\frac{k}{\sqrt{2}}\right)t$ and E is an arbitrary constant.

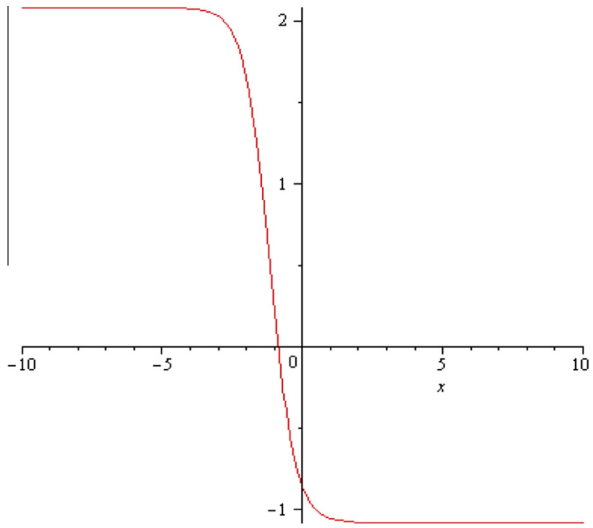

Figure 18 The solitary wave 2D graphics of $u_{11}(\xi)$, $u_{16}(\xi)$, $v_{11}(\xi)$ and $v_{16}(\xi)$ for $t = 1, \mu = 1, \lambda = 3, E = 1, \delta = 2$ and $k = 1$.

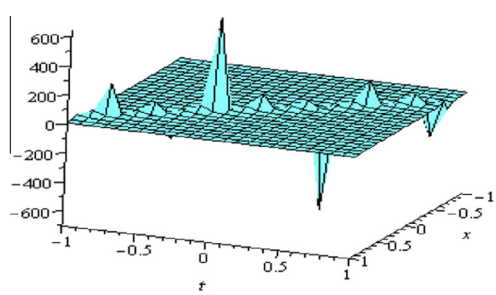

Figure 19 The solitary wave 3D graphics of $u_{12}(\xi)$, and $v_{12}(\xi)$ for $\mu = 3,\ \lambda = 1, E = 2, \delta = 2, k = 1$ and $-1 \leqslant x,\ t \leqslant 1$.

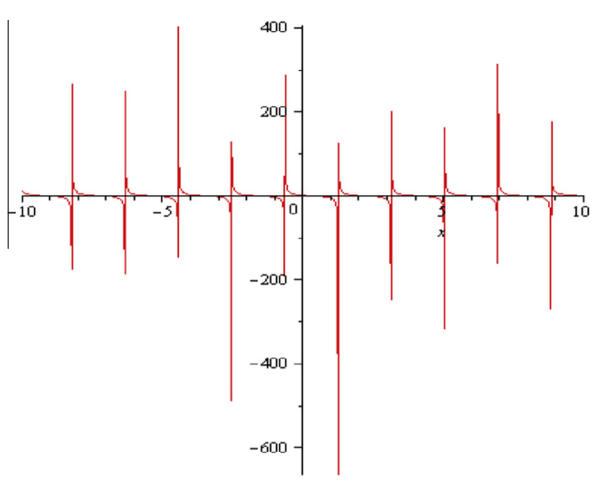

Figure 20 The solitary wave 2D graphics of $u_{12}(\xi)$, and $v_{12}(\xi)$ for $t = 1,\ \mu = 3, \lambda = 1,\ E = 2, \delta = 2$ and $k = 1$.

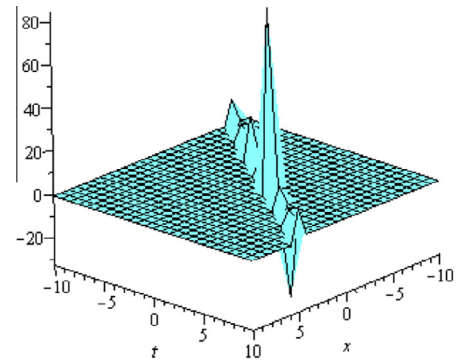

Figure 21 The solitary wave 3D graphics of $u_{17}(\xi)$, and $v_{17}(\xi)$ for $\mu = 3, \lambda = 1,\ E = 2, \delta = 2, k = 1$ and $-1 \leqslant x,\ t \leqslant 1$.

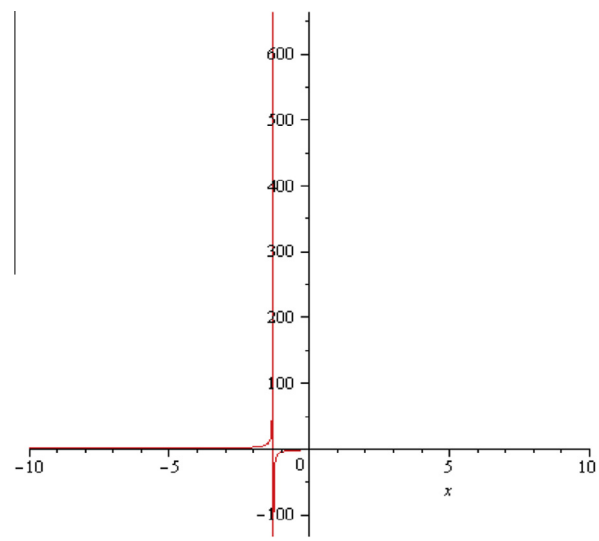

Figure 22 The solitary wave 2D graphics of $u_{17}(\xi)$, and $v_{17}(\xi)$ for $t = 1, \mu = 3,\ \lambda = 1, E = 2, \delta = 2$ and $k = 1$.

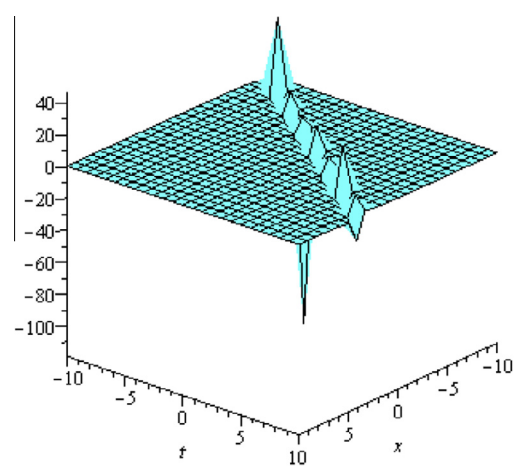

Figure 23 The solitary wave 3D graphics of $u_{19}(\xi)$, and $v_{19}(\xi)$ for $\mu = 1, \lambda = 2,\ E = 2, \delta = 2, k = 1$ and $-10 \leqslant x,\ t \leqslant 10$.

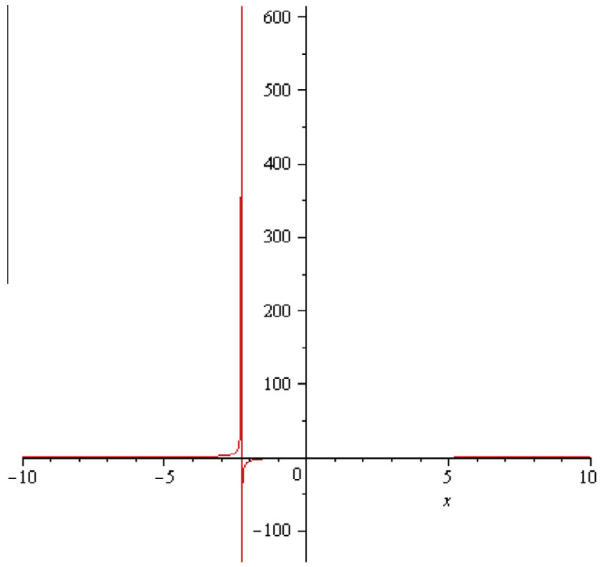

Figure 24 The solitary wave 2D graphics of $u_{19}(\xi)$, and $v_{19}(\xi)$ for $t = 1, \mu = 1, \lambda = 2, E = 2, \delta = 2, k = 1$.

When $\mu = 0, \lambda = 0$ and $\lambda^2 - 4\mu = 0$,

$$u_{20}(\xi) = -\frac{1}{\sqrt{2}}\lambda + \frac{1}{2}k - \sqrt{2}\left(\frac{1}{\xi + E}\right),$$

$$v_{20}(\xi) = \frac{1}{\sqrt{\delta}}\left\{-\frac{1}{\sqrt{2}}\lambda + \frac{1}{2}k - \sqrt{2}\left(\frac{1}{\xi + E}\right)\right\}.$$

where $\xi = x - \left(\frac{k}{\sqrt{2}}\right)t$ and E is an arbitrary constant.

4. Graphical representation of the solutions

Graph is a powerful tool for communication and to describe lucidly the solutions of the problems. A graph is a visual representation of numerical or close-form solutions or other information, often used for comparative purposes. When doing calculation in everyday life we need the basic knowledge of making use of graphs. Therefore, some graphs of the solutions are provided below.

4.1. The nonlinear Bogoyavlenskii equation

The graphical illustrations of the solutions are given below in the figures with the aid of Maple (see Figs. 1–16).

4.2. The nonlinear diffusive predator–prey system of equations

The graphical illustrations of the solutions are given below in the figures with the aid of Maple (see Figs. 17–24).

5. Conclusion

In this paper, the $\exp(-\Phi(\xi))$-expansion method has been fruitfully used to find the exact solutions of NLEEs. As an application, the exact solutions for the nonlinear diffusive predator–prey system of equations and the nonlinear Bogoyavlenskii equations have been constructed using the $\exp(-\Phi(\xi))$-expansion method. On comparing our results obtained in the present article with the well-known results obtained by other authors using different methods, we conclude that our results of the nonlinear diffusive predator–prey system of equations and the nonlinear Bogoyavlenskii equations are new and different from those obtained in [43,47–49]. We also gave the applications of the obtained results in Figs. 1 and 2. In the view of such information, it is seen clearly that this method is highly helpful and practical in terms of giving new solutions such as kink solutions, singular kink solutions, dark soliton solutions, Bright soliton solutions, soliton solutions, singular soliton solutions, multiple soliton-like solutions and triangular periodic solutions. We suggest that the method can also be implemented to different NLEEs.

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