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Chaos as a Part of Logical Structure in Neurodynamics

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Abstract. It is proposed that chaotic attractors incorporated in neural net models can represent *classes* of patterns in the same way in which a set of static attractors represent unrelated patterns. Therefore, chaotic states of neuron activity are associated with higher level cognitive processes such as generalization and abstraction.

Chaotic activity in the human brain is a subject of discussion in many recent publica-tions.[1,2] The interest in the problem was promoted by discovery of strange attractors. This discovery provided a phenomenological framework for understanding electroencephalogram data in regimes of multiperiodic and random signals generated by the brain. An understanding of the role of such chaotic states in the logical structure of the human brain activity would significantly contribute not only to the brain science, but also to theory of advance computing based upon artificial neural networks. In this note we propose a phenomenological approach to the problem: we demonstrate that a chaotic attractor incorporated in neural net models can represent a *class* of patterns, i.e., a collection of all those and only those patterns to which a certain concept applies. Formation of such a class is associated with higher level cognitive processes (generalization). This generalization is based upon a set of unrelated patterns represented by static attractors and associated with the domain of lower level of brain activity (perception, memory). Since a transition from a set of unrelated static attractors to the unique chaotic attractor releases many synaptic interconnections between the neurons, the formation of a class of patterns can be "motivated" by a tendency to minimize the number of such interconnections at the expense of omitting some insignificant features of individual patterns.

Our approach exploits the phenomenology of dissipative nonlinear dynamical systems for computation and information processing performed by neural networks. These systems are modelled by coupled sets of first order differential equations of the form:

$$\dot{x}_i = V_i(x_j, T_{ij}), \; i,j = 1,2,\dots n \tag{1}$$

in which x_i is an n-dimensional vector function of time representing the neuron activity, and T_{ij} is a constant matrix whose elements represent synaptic interconnections between the neurons.

The most important characteristic of the neurodynamical systems(1) is that they are dissipative, i.e., their motions, on the average, contract phase space volumes onto attractors of lower dimensionality than the original space.

So far only point attractors have been utilized in the logical structure of neural network performance: they represent stored vectors (patterns, computational objects, rules). The idea of storing patterns as point attractors of neurodynamics implies that initial configurations of neurons in some neighborhood of a memory state will be attracted to it. Hence, a

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point attractor (or a set of point attractors) is a paradigm for neural net performance based upon the phenomenology of nonlinear dynamical systems. This performance is associated with the domain of lower level brain activity such as perception and memory.

It is easily verifiable that a set of point attractors imposes certain constraints upon the synaptic coefficients T_{ij}. Indeed, for a set of m fixed points $\tilde{x}_i^k (k = 1, 2, \ldots m)$ one obtains $m \times n$ constraints following from Eq. (1):

$$0 = V_i(\tilde{x}_j^k,\ T_{ij}),\ i, j = 1, 2, \ldots n,\ k = 1, 2, \ldots m \tag{2}$$

In order to provide stability of the fixed points $\tilde{x}_j^k$, the synaptic coefficients must also satisfy the following $m \times n$ inequalities:

$$\operatorname{Re} \lambda_i^k < 0, i = 1, 2, \ldots n,\ k = 1, 2, \ldots m \tag{3}$$

in which λ_i^k are the eigenvalues of the matrices $\|\partial f_i / \partial x_j\|$ at the fixed points $\tilde{x}_i^k$.

How can a neural network minimize the number of interconnections T_{ij} without a significant loss of the quality of a prescribed performance?

Let us assume that the vectors $\tilde{x}_j^k$ have some characteristics in common, for instance, their ends are located on the same circle of a radius r_o, i.e., (after proper choice of coordinates):

$$\sum_{i=1}^{2} (\tilde{x}_i^k)^2 = r_o^2,\ k = 1, 2, \ldots m \tag{4}$$

If for the patterns represented by the vectors $\tilde{x}_i^k$ the property (4) is much more important then their angular coordinates $\theta^k (\theta^{k1} \neq \theta^{k2}$ if $k_1 \neq k_2)$, then it is "reasonable" for the neural net to store the circle $r = r_o$ instead of storing m point attractors with at least $2 \times m$ synaptic coefficients T_{ij}. Indeed, in this case the neural net can "afford" to eliminate unnecessary synaptic coefficients by reducing its structure to the simplest form:

$$\dot{r} = r(r - r_o)(r - 2r_o) \quad , \quad \dot{\theta} = \omega = \text{ Const} \tag{5}$$

Eqs. (5) have a periodic attractor

$$r = r_o, \quad \theta = \omega t \tag{6}$$

which generates harmonic oscillations with frequency ω. But what is the role of these oscillations in the logical structure of neural net performance? The transition to the form (5) can be interpreted as a generalization procedure in the course of which a collection of unrelated vectors $\tilde{x}_i^k$ is united into a class of vectors whose lengths are equal to r_o. Hence, in terms of symbolic logic, the circle $r = r_o$ is a logical form for the class of vectors to which the concept (4) applies. In other words, the oscillations (6) represent a higher level cognitive process associated with generalization and abstraction. During these processes, the point describing the motion of Eqs. (5) in the phase space will visit all those and only those vectors whose lengths are equal to r_o; thereby the neural network "keeps in mind" all the members of the class.

Suppose that a bounded set of isolated point attractors which can be united in a class occupies a more complex subspace of the phase space, i.e., instead of the circle (4) the concept defining the class is:

$$\Phi(\tilde{x}_1^k, \tilde{x}_2^k, \ldots \tilde{x}_n^k) = r,\ k = 1, 2, \ldots m \tag{7}$$

Then the formation of the class will be effected by storing a surface:

$$\Phi(x_1, x_2, \ldots x_n) = r \tag{8}$$

as a limit set of the neurodynamics, while all the synaptic coefficients T_{ij} which impose constraints on the velocities along the surface (8)) will be eliminated.

The character of the motion on the limit set depends upon the properties of the surface (8). If (by proper choice of coordinates) this surface can be approximated by a topological product of $(n-1)$ circles (i.e., by an $(n-1)$-dimensional torus) then the motion is quasi-periodic: it generates oscillations with frequencies which are dense in the reals. If the surface (8) is more complex and is characterized by a fractal dimension, the motion on such a limit set must be chaotic: it generates oscillations with continuous spectrum. In both cases the motion is ergodic: the point describing the motion in the phase space sooner or later will visit all the points of the limit set, i.e., the neural net will "keep in mind" all the members of the class.

Thus, it can be concluded that artificial neural networks are capable of performing high level cognitive processes such as formation of classes of patterns, i.e., formation of new logical forms based upon generalization procedures. In terms of the phenomenology of nonlinear dynamics these new logical forms are represented by limit sets which are more complex than point attractors, i.e., by periodic or chaotic attractors. It is shown that formation of classes is accompanied by elimination of a large number of extra synaptic interconnections. This means that these high level cognitive processes increase the capacity of the neural network. The procedure of formation of classes can be initiated by a tendency of the neural network to minimize the number (or the total strength) of the synaptic interconnections without a significant loss of the quality of prescribed performance; such a tendency can be incorporated into the learning dynamics which controls these interconnections[3]. In addition, the phenomenological approach presented above leads to a possible explanation of chaotic activity of the human brain; it suggests that this activity represents the high level cognitive processes such as generalization and abstraction.

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