Joint calibration algorithm for gain-phase and mutual coupling errors in uniform linear array

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Abstract The effect of gain-phase perturbations and mutual coupling significantly degrades the performance of digital array radar (DAR). This paper investigates array calibration problems in the scenario where the true locations of auxiliary sources deviate from nominal values but the angle intervals are known. A practical algorithm is proposed to jointly calibrate gain-phase errors and mutual coupling errors. Firstly, a simplified model of the distortion matrix is developed based on its special structure in uniform linear array (ULA). Then the model is employed to derive the precise locations of the auxiliary sources by one-dimension search. Finally, the least-squares estimation of the distortion matrix is obtained. The algorithm has the potential of achieving considerable improvement in calibration accuracy due to the reduction of unknown parameters. In addition, the algorithm is feasible for practical applications, since it requires only one auxiliary source with the help of rotation platforms. Simulation results demonstrate the validity, robustness and high performance of the proposed algorithm. Experiments were carried out using an S-band DAR test-bed. The results of measured data show that the proposed algorithm is practical and effective in application.

1. Introduction

Digital array radar (DAR) employs a full digital beam-forming (DBF) architecture in the receiving and transmitting system. It has the potential of forming multiple simultaneous beams while providing high anti-interference capabilities. In the last decade, DAR has attracted considerable attention and has been widely used in space surveillance. Most array signal processing algorithms, such as DBF and direction of arrival (DOA), rely crucially on the assumption that the array manifold is perfectly known. However, in actual systems, the array manifold is inevitably affected by gain-phase perturbations and mutual coupling effects. As a result, the performance of DAR may be seriously degraded.

Traditional algorithm for array calibration is to carry out measurements using computational electromagnetic solvers,
which has been applied in some actual radar systems.\textsuperscript{4,5} The algorithm has the problem of time consuming and high demands for testing environments. It may be impractical once the array systems are in operation due to the complex electromagnetic environments.

In order to address the problem, a number of self-calibration algorithms that make use of signal processing technology have been developed. Ref.\textsuperscript{6} estimates DOA parameters and mutual coupling coefficients using the space alternating generalized expectation maximization algorithm. In Ref.\textsuperscript{7}, the mutual coupling effects in the uniform linear array (ULA) are inherently eliminated without any calibration sources, but the algorithm requires some extended elements. Refs.\textsuperscript{8–10} present a category of algorithms that can iteratively estimate the array manifold errors and the DOAs of impinging signals based on the subspace principle. These calibration algorithms usually suffer from low accuracy, high computational complexity and serious ambiguous problems.

Compared with self-calibration algorithms, active calibration algorithms use auxiliary sources to overcome suboptimal convergence problems and have the potential to achieve better calibration accuracy. The algorithm in Ref.\textsuperscript{11} calibrates mutual coupling errors in an arbitrary array using several ideal instrumental elements. But it is difficult to find the ideal elements in practice. A maximum likelihood approach is presented in Refs.\textsuperscript{12,13} to estimate the unknown gain-phase, mutual coupling as well as sensor positions. It has the drawbacks of high computation, and the iterations may not be converge under some conditions. Refs.\textsuperscript{14–16} have proposed a category of eigenstructure algorithm that treats gain-phase and mutual coupling errors as a whole. The closed-form of the distortion matrix is derived with the help of some time-disjoint auxiliary sources. The algorithms have been implemented to improve the performance of actual systems.\textsuperscript{17,18} But they do not consider about the special structure of distortion matrix and have a strict requirement pertaining to the number of auxiliary sources. The algorithm in Ref.\textsuperscript{19} eliminates the repeated entries in the distortion matrix of ULA to reduce the unknown parameters and reaches a better accuracy than the algorithms in Refs.\textsuperscript{14,16}. However, the algorithm requires the precise knowledge of locations of auxiliary sources, which may not be available in some actual applications.

In practical situations, it may be difficult to access the precise directions of auxiliary sources. However, it is possible to determine the angle intervals between them using additional equipment, such as rotating platforms. This paper focuses on the problems of joint calibration of gain-phase and mutual coupling errors in the above scenario. The proposed algorithm achieves high accuracy and behaves robustly when the incident angles of auxiliary sources are not known precisely.

The paper is organized as follows. In Section 2, the signal model of ULA is demonstrated and the problem of array calibration is illustrated. In Section 3, the proposed algorithm for array calibration in the presence of gain-phase errors and mutual coupling errors is developed. Computer simulations and experimental results of measured data are presented and analyzed in Section 4, followed by conclusions in Section 5.

### 2. Signal model and problem formulation

Consider a ULA consisting of $N$ omnidirectional antenna elements with the space $d$ between neighboring elements. There are $M$ narrowband signals $s_1(t), s_2(t), \ldots, s_M(t)$ located in the far-field region. The signals impinging on the ULA from different directions of $\phi_1, \phi_2, \ldots, \phi_M$, with respect to the normal line of the ULA. The signals are incoherent with each other with a wavelength of $\lambda$. The additive noise is zero-mean, random process with a variance of $\sigma^2$. The outputs of the array can be written as

$$x(t) = As(t) + n(t)$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_N(t)]^T$, $s(t) = [s_1(t), s_2(t), \ldots, s_M(t)]^T$, $n(t) = [n_1(t), n_2(t), \ldots, n_N(t)]^T$ are the output vector, signal vector and noise vector, respectively. $A = [a(\phi_1), a(\phi_2), \ldots, a(\phi_M)]$ is the ideal array manifold matrix, where $a(\phi) = [1, e^{-j2\pi d \sin \phi}, \ldots, e^{-j2\pi (N-1)d \sin \phi}]^T$ denotes the ideal steering vector of the $i$th signal.

Taking gain-phase perturbations and mutual coupling effects into consideration, the outputs can be modified as

$$x(t) = CTAs(t) + n(t)$$

where $T = \text{diag}(\tau_1, \tau_2, \ldots, \tau_N)$ is a diagonal matrix and $\tau_i$ denotes gain-phase errors of the $i$th element. $C \in \mathbb{C}^{N \times N}$ is the mutual coupling matrix (MCM).

Since the structure of $C$ is highly dependent on the physical structure of the array, it can be considered as a banded symmetric Toeplitz matrix in the case of ULA.\textsuperscript{7} Indeed, mutual coupling effects tend to be reciprocal to the distance between elements and may be negligible for the elements separated by a few wavelengths. Therefore, $C$ may be expressed as

$$C = \begin{cases} C(i,j) = c_{i-j}, & \text{for } i = j = 1, 2, \ldots, N \\ 0 < |c_p| < \cdots < |c_1| < |c_1| = 1 \\ c_0 = 0, & \text{for } i > P \end{cases}$$

where $c_i$ is the mutual coupling coefficient between the first and the $i$th element. $P$ is the number of non-zero complex coefficients in the first row of the MCM.

The covariance matrix of array output vector is defined as

$$R_{x}(t) = E[x(t)x(t)^H] = CTAR_{s}(t)A^H T^H C^H + \sigma^2 I_N$$

where $R_{s}(t) = E[s(t)s(t)^H]$ is the covariance matrix of signals, which is nonsingular when the signals are incoherent. $I_N$ is the $N \times N$ identity matrix.

Performing eigen-decomposition on the output covariance matrix, it can be written as

$$R_{x}(t) = \sum_{i=1}^{M} \xi_i e_i e_i^H + \sum_{n=M+1}^{N} \zeta_n e_n e_n^H = U \Sigma U^H + U_{\text{rest}}^H U_{\text{rest}}$$

In Eq. (5), $\xi_1 \geq \xi_2 \geq \cdots \geq \xi_M$ are the $M$ large eigenvalues of $R_{x}(t)$, and $\zeta_{M+1} = \zeta_{M+2} = \cdots = \zeta_N = \sigma^2$ are small eigenvalues. $\Sigma = \text{diag}(\xi_1, \xi_2, \ldots, \xi_N)$ and $U_{\text{rest}} = \text{diag}(\zeta_{M+1}, \zeta_{M+2}, \ldots, \zeta_N)$ are diagonal matrices. $U = [e_1, e_2, \ldots, e_M] \in \mathbb{C}^{N \times M}$ is composed of the eigenvectors corresponding to the $M$ large eigenvalues, while $U_{\text{rest}} = [e_{M+1}, e_{M+2}, \ldots, e_N] \in \mathbb{C}^{N \times (N-M)}$ contains the rest $N-M$ eigenvectors.
According to the subspace theory, \( \mathbf{U} \) spans the same space with the distortion array manifold matrix \( \mathbf{CTA} \), which is orthogonal to the space spanned by \( \mathbf{U}_{\text{no}} \). Thus the following relation holds true:

\[
\text{span}\{\mathbf{CTA}(\phi)\} = \text{span}\{\mathbf{U}_{\phi}\} \perp \text{span}\{\mathbf{U}_{\text{no}}\}
\]  

(6)

where \( \text{span}\{X\} \) denotes the subspace spanned by \( X \).

In Eq. (6), \( \mathbf{U}_{\phi} \) can be derived from the receiving data of the array. Therefore, the optimal estimation of unknown distortion parameters may be obtained based on the relationships between \( \mathbf{CTA} \) and \( \mathbf{U}_{\phi} \).

3. Proposed algorithm for joint calibration of gain-phase and mutual coupling errors

3.1. Least-squares solutions of distortion matrix

Suppose there are \( M \) far-field auxiliary sources impinging on the array at different times. The incident angles are \( \theta, \theta + \Delta \theta_1, \ldots, \theta + \Delta \theta_{M-1} \). Here we concern about the situation where \( \theta \) is unknown, but the angles’ intervals \( \Delta \theta_1, \Delta \theta_2, \ldots, \Delta \theta_{M-1} \) can be measured precisely. As for the \( m \)th auxiliary source, the covariance matrix of receiving data is denoted as \( \mathbf{R}_m^{(\theta)} \) and the eigenvector corresponding to the maximum eigenvalue of \( \mathbf{R}_m^{(\theta)} \) is denoted as \( \mathbf{e}_m \). It can be determined from Eq. (6) that \( \mathbf{e}_m \) is related to the distortion steering vector by a complex scaling constant. The following equation is obtained by ignoring noise and finite length of snapshots:

\[
\mathbf{CTA}(\theta + \Delta \theta_{m-1}) = \eta_m \mathbf{e}_m \quad m = 1, 2, \ldots, M
\]

(7)

where \( \eta_m \) is an unknown constant. Therefore, a least-squares problem may be formulated with the cost function as

\[
J(C, \Gamma, A, \theta) = \sum_{m=1}^{M} ||\mathbf{CTA}(\theta + \Delta \theta_{m-1}) - \eta_m \mathbf{e}_m||^2
\]

(8)

where \( A = [\eta_1, \eta_2, \ldots, \eta_M]^T \) is denoted as the scaling vector, \( ||\cdot|| \) is the Frobenius norm.

Here we define \( \mathbf{Z} = \mathbf{CT} \) as the array distortion matrix, which contains gain-phase errors and mutual coupling errors. It is considered that there are \( N^2 \) unknown parameters in \( \mathbf{Z} \). However, making use of the special structure of \( \Gamma \) and \( C \), the unknown parameters can be decreased.

Taking advantage of the diagonal structure of gain-phase matrix, \( \Gamma \) can be expressed by the diagonal elements:

\[
\Gamma = \begin{bmatrix}
\zeta_1 E_q
\vdots \\
\zeta_N E_q
\end{bmatrix}
\]

\[
E_q(i,j) = \delta(\Gamma(i,j) - \zeta_i) \quad i, j = 1, 2, \ldots, N
\]

where \( \delta(\cdot) \) is defined as

\[
\delta(\kappa) = \begin{cases}
1 & \kappa = 0 \\
0 & \text{Others}
\end{cases}
\]

(10)

As noted from Eq. (3), the MCM of ULA is a banded symmetric Toeplitz matrix and may be transformed to a mutual coupling vector as

\[
\begin{bmatrix}
C = \begin{bmatrix}
c_p E_p \\
c_0 E_0
\end{bmatrix} \\
E_p(i,j) = \delta(C(i,j) - c_0) \quad i, j = 1, 2, \ldots, N
\end{bmatrix}
\]

(11)

According to Eqs. (9) and (11), we obtain

\[
\mathbf{Z} = \left( \begin{array}{c}
p \sum_{p=1}^{P} c_p E_p \\
N \sum_{q=1}^{N} \zeta_q E_q
\end{array} \right) = \sum_{j=1}^{L} g_j F_j
\]

(12)

where \( L = PN, g_j \) and \( F_j \) are determined by

\[
\begin{cases}
g_j = c_p F_j \quad (p-1)N < j \leq pN \\
F_j = E_p E_0^{-1} \quad (p-1)N < j \leq pN
\end{cases}
\]

(13)

where \( j = 1, 2, \ldots, L \) and \( p = 1, 2, \ldots, P \).

It is evident that the simplified form of the distortion matrix in Eq. (12) only contains \( L \) unknown parameters. Due to the reduction of unknown parameters, the estimation accuracy will be improved and the requirements for auxiliary sources will be relaxed as well.

By substituting Eq. (12) into Eq. (8), the cost function can be rewritten as

\[
J(g, A, \theta) = \sum_{m=1}^{M} \left| \sum_{j=1}^{L} g_j F_m(a(\theta + \Delta \theta_{m-1}) - \eta_m \mathbf{e}_m) \right|^2
\]

\[
= \sum_{m=1}^{M} \left| \mathbf{T}_m g - \eta_m \mathbf{e}_m \right|^2
\]

(14)

where

\[
\mathbf{T}_m = [F_m(a(\theta + \Delta \theta_{m-1}), F_m(a(\theta + \Delta \theta_{m-1}), \ldots, F_m(a(\theta + \Delta \theta_{m-1})]] \in \mathbb{C}^{N^2 \times L}
\]

\[
[g = [g_1, g_2, \ldots, g_L]^T \in \mathbb{C}^{L \times 1}
\]

(15)

Optimization of Eq. (14) with respect to \( g \) whilst keeping \( A \) and \( \theta \) unchanged provides a solution for \( g \):

\[
g = \left( \sum_{m=1}^{M} \mathbf{T}_m^H \mathbf{T}_m \right)^{-1} \left( \sum_{m=1}^{M} \eta_m \mathbf{T}_m^H \mathbf{e}_m \right)
\]

(16)

Eq. (16) specifies the least-squares estimation of the distortion vector \( g \) with unknown parameters \( A \) and \( \theta \), the solutions of which will be discussed in the following section.

3.2. Determination of incident angles

Inserting Eq. (16) into Eq. (14) yields the optimization problem:

\[
J(A, \theta) = \sum_{m=1}^{M} \left| \mathbf{T}_m \left( \sum_{j=1}^{L} \mathbf{T}_j^H \mathbf{T}_j \right)^{-1} \left( \sum_{j=1}^{L} \eta_j \mathbf{T}_j^H \mathbf{e}_j \right) - \eta_m \mathbf{e}_m \right|^2
\]

\[
= \sum_{m=1}^{M} \left| \mathbf{T}_m T_0 \left( \sum_{j=1}^{L} \eta_j \mathbf{T}_j^H \mathbf{e}_j \right) - \eta_m \mathbf{e}_m \right|^2
\]

(17)

where \( \mathbf{T}_0 = \left( \sum_{m=1}^{M} \mathbf{T}_m^H \mathbf{T}_m \right)^{-1} \in \mathbb{C}^{L \times L} \).

In Eq. (17), \( \eta_m (m = 1, 2, \ldots, M) \) can be expressed concisely by the vector \( A \). Defining an \( L \times M \) matrix as

\[
\mathbf{T} = [\mathbf{T}_1^H \mathbf{e}_1, \mathbf{T}_2^H \mathbf{e}_2, \ldots, \mathbf{T}_L^H \mathbf{e}_L]^T \quad \text{and the m th principle eigenvector as} \quad \mathbf{e}_m = [e_{1,m}, e_{2,m}, \ldots, e_{N,m}]^T, \text{we have the forms}
\]
\[
\sum_{j=1}^{M} \eta_j T^H_j \mathbf{e}_j = [T^H_{1j} \mathbf{e}_1, \ldots, T^H_{Mj} \mathbf{e}_M] \cdot [\eta_1, \ldots, \eta_M]^T = T \mathbf{A}
\]  
(18)

\[
\eta_m \mathbf{e}_m = \begin{bmatrix}
0_{N \times (M-m)} & 0_{N \times (M-m)} \\
0_{N \times (m-1)} & 0_{N \times (M-m)} \\
\vdots & \vdots \\
0_{N \times (m-1)} & 0_{N \times (M-m)}
\end{bmatrix} \begin{bmatrix}
\eta_1 \\
\eta_2 \\
\vdots \\
\eta_M
\end{bmatrix} = U_m \mathbf{A}
\]  
(19)

where \(0_{p \times q}\) denotes \(p \times q\) null matrix. \(U_m = [0_{N \times (m-1)}, \mathbf{e}_m, \ldots, 0_{N \times (M-m)}]\) is an \(N \times M\) matrix, with the \(m\)th column of \(\mathbf{e}_m\) and other columns of \(0_{N \times 1}\).

Substituting Eqs. (18) and (19) into Eq. (17), the cost function becomes

\[
J(A, \theta) = \sum_{m=1}^{M} \| T_m T_0 T A - U_m A \|_F^2
\]
\[
= \sum_{m=1}^{M} A^H (T_m T_0 T - U_m) \mathbf{H} (T_m T_0 T - U_m) A
\]
\[
= A^H \left( \sum_{m=1}^{M} V_m^H V_m \right) A
\]  
(20)

where \(V_m = T_m T_0 T - U_m \in \mathbb{C}^{N \times M}\).

The solution of the optimization problem in Eq. (20) requires a constraint to avoid the trivial solution. Without loss of generality, we impose the line constraint \(A^H \mathbf{e} = 1\), where \(\mathbf{e} = [1, 0, \ldots, 0]^T \in \mathbb{C}^{M+1}\). Using the method of Lagrange multipliers, the optimization solution of \(A\) is derived as

\[
A = \frac{Q^{-1} \mathbf{e}}{\mathbf{e}^H Q^{-1} \mathbf{e}}
\]  
(21)

where \(Q = \sum_{m=1}^{M} V_m^H V_m\).

Substituting Eq. (21) into Eq. (20), the cost function may be expressed as

\[
J(\theta) = \frac{1}{\mathbf{e}^H Q^{-1} \mathbf{e}}
\]  
(22)

Therefore, the incident angles of calibration sources can be obtained by one-dimensional searching:

\[
\hat{\theta} = \arg \max_{\theta} \left( \mathbf{e}^H Q^{-1} \mathbf{e} \right)
\]  
(23)

With the knowledge of \(\theta\), the scaling vector \(A\) is derived by Eq. (21). Finally, the distortion vector \(\mathbf{g}\) is calculated by Eq. (16) and the distortion matrix \(\mathbf{Z}\) is reconstructed according to Eq. (12).

3.3. Discussion

In the above sections, the proposed algorithm has been described in detail. Referring to Ref.20, a necessary but not sufficient condition for the uniqueness of solutions has been presented: the number of independent equations in the cost function is more than the number of unknown parameters.

In Eq. (14), there are \(2PN\) real parameters containing in \(\mathbf{g}\), \(2M\) real parameters in \(A\) and the unknown incident angle \(\theta\). Thus, it follows that the number of real parameters is \(2PN + 2M + 1\). The number of independent equations, however, is \(2MN\). Therefore, the uniqueness condition requires that

\[
2NM \geq 2PN + 2M + 1
\]  
(24)

Or, equivalently:

\[
M \geq \left[ \frac{P + 2P + 1}{2(N - 1)} \right]
\]  
(25)

where \([\kappa]\) denotes the smallest integer greater than or equal to \(\kappa\).

From Eq. (25), we know that the number of auxiliary sources for the proposed algorithm is mainly dependent on the number of non-zero elements in MCM, instead of the number of antenna elements. As a result, it requires only a few time-disjoint auxiliary sources even for large arrays. It is noted that the algorithm in Ref.21 also makes use of the symmetric Toeplitz property of the MCM. However, the simplified form is established in a different way from Eq. (12). Consequently, the derivations of the corresponding solutions are different either.

The proposed algorithm will be simplified significantly when only mutual coupling errors are considered. In the absence of gain-phase errors, there are only \(P\) unknown parameters in \(\mathbf{g}\). Thus, only two auxiliary sources are demanded and the accuracy will be improved as a result of the reduction of unknown parameters.

In most DAR systems, the antenna arrays are mounted on rotation platforms. The directions of auxiliary sources with respect to the array can be changed by varying the orientations of rotation platforms. In this way, calibration signals from different directions are captured at different times. Since the proposed algorithm demands that calibration sources from different directions are time-disjoint, it requires only one auxiliary source to generate these calibration signals with the help of rotation platforms. Though the incident angles are difficult to measure, the angle intervals may be determined according to the records of rotation platforms. Therefore, the requirements for time-disjoint auxiliary sources are easily satisfied with the help of rotation platforms. As a result, it is convenient to apply the proposed algorithm in practical situations.

4. Simulations and experimental results

4.1. Computer simulations

In this section, some representative simulations are carried out to demonstrate the performance of the proposed algorithm in comparison with previous algorithms. Consider an eight-element omnidirectional ULA with the space between neighboring elements equal to 0.05 m. Each channel is perturbed by gain-phase errors with coefficients of \(\tau = [1, 0.96 - 0.08, 0.96 + 0.09, 1.13 + 0.12, 0.85 + 0.11 j, 1.05 - 0.14 j, 1.06 + 0.04 j, 1.05 - 0.003 j]^T\). As far as the mutual coupling is concerned, we assume that the number of nonzero mutual coupling coefficients is \(P = 3\) and the mutual coupling vector is \(\mathbf{e} = [1, 0.31 + 0.18 j, 0.12 - 0.09 j]^T\). The relative root mean square error (RMSE) of the distortion matrix defined in Eq. (26) is chosen as a measurement of the estimation ac-
racy. In the following simulations, 200 independent Monte Carlo trials were carried out.

\[ e_z = \sqrt{\frac{1}{M_c} \left( \sum_{k=1}^{M_c} \frac{||Z_k - \hat{Z}_k||^2}{||Z_k||^2} \right)} \times 100\% \]  

where \( M_c \) is the number of Monte Carlo trials.

In Simulation 1, the accuracy of the distortion matrix estimation is investigated. Suppose there are eleven auxiliary sources with a frequency of 2.6 GHz located in the far-field region. The direction of the first auxiliary sources is \( \theta = -50^\circ \) and the angle interval between adjacent sources is \( \Delta \theta = 10^\circ \).

Firstly, we assume that the incident angle of the first auxiliary source is known precisely. The solid lines in Figs. 1 and 2 depict the relative RMSE of the distortion matrix versus SNR and snapshots. The Cramer-Rao lower bound (CRLB)\(^9,22\) is also given for comparison. The results show that the proposed algorithm and the algorithm in Ref.\(^{21}\) achieve approximately the same performance and the relative RMSE is close to the CRLB. Both algorithms reach better estimation accuracy than the algorithms in Refs.\(^{14,16}\) due to the utilization of the unique structure of distortion matrix. Secondly, suppose the pre-knowledge of \( \theta \) deviates from the true value with a bias of \( 0.6^\circ \), the relative RMSE are illustrated by the dashed lines in Figs. 1 and 2. When the DOAs of auxiliary sources are not known precisely, the proposed algorithm behaves slightly worse. However, the performance of the other algorithms degrades seriously. Thirdly, the robustness of the algorithms is investigated, as is shown in Fig. 3. With the increase of the bias of \( \theta \), the proposed algorithm can still provide a good estimation of the distortion matrix, but other algorithms lose efficacy quickly.

Simulation 2 concentrates on the performance of DOA estimation in the presence of array errors. Suppose there are two incoherent signals impinging on the ULA from \( -6^\circ \) and \( 3^\circ \), with a SNR of 6 dB and the snapshots of 1024. The DOA estimation accuracy is measured by the RMSE defined in Eq. (27), where \( L \) is the number of signals. Keeping the settings of auxiliary sources the same as simulation 1, the RMSE of the DOA estimation versus the SNR and snapshots of auxiliary sources are shown in Figs. 4–6. With the increase of the SNR and snapshots of auxiliary sources, the distortion matrix may be estimated more precisely, resulting in higher DOA estimation accuracy. It can be seen that the proposed algorithm performs better than others, especially for the situation where the first incident angle is not known precisely. The conclusions are coincident with Simulation 1.

\[ \theta_{\text{RMSE}} = \sqrt{\sum_{k=1}^{M_c} \sum_{l=1}^{L} (\hat{\theta}_{ik} - \theta_{ik})^2 / (M_c L)} \]  

Fig. 1 Relative RMSE of distortion matrix versus SNR (snapshots = 512).

Fig. 2 Relative RMSE of distortion matrix versus snapshots (SNR = 5 dB).

Fig. 3 Relative RMSE of distortion matrix versus angle bias of auxiliary sources (SNR = 5 dB, snapshots = 512).

Fig. 4 RMSE of DOA estimation versus SNR of auxiliary sources (snapshots = 512).
In certain situations, gain-phase errors may be well compensated by some auxiliary instruments such as interior calibration networks. Therefore, only mutual coupling effects are concerned. In simulation 3, the performance of the calibration algorithms for mutual coupling errors is studied. The array parameters are the same as simulation 1 except that there are no gain-phase errors. Two auxiliary sources are located in the far-field region with an angle interval of $\frac{35}{14}$°. The relative RMSE of MCM by the proposed algorithm are shown in Figs. 7 and 8, compared with the rank reduction (RARE) algorithm in Ref.9, the recursive RARE algorithm in Ref.10, the resiliency algorithm in Ref.7 and the CRLB. It can be seen that the proposed algorithm achieves a considerable accuracy and outperforms other algorithms, particularly at low values of SNR and snapshots. Moreover, the proposed algorithm rarely suffers from ambiguous problems and does not require any additional antenna elements.

4.2. Experimental results

The algorithms in the previous literature were mainly based on simulations and few are verified by measured data. In this section, experiments were carried out by an S-band DAR test-bed to evaluate the performance of the proposed algorithm. The test-bed has 8 tapered slot antennas arranged in a line with an adjacent space of 0.05 m. Digital receivers are designed to realize a high level of digitalization. Radio frequency (RF) signals up to 3 GHz are directly sampled by the 10 bits ADCs with a sampling rate of 1.2 Gsps utilizing the band-pass sampling theory. Signal processors based on the Open-VPX standard employ powerful Xilinx Virtex-6 FPGAs and TMS320C6678 DSPs to support system calibration, DBF, DOA and other real-time algorithms.

The experiments were conducted in an anechoic chamber (see Fig. 9). The antenna array is placed on the rotation platform. Sinusoidal signals at a frequency of 2.6 GHz are transformed into parallel waves through a reflector. Incident angles were measured according to the records of the rotation platform with a precision of $\frac{0.05}{176}$°. After power on, eleven calibration signals were received, uniformly distributed in $\frac{50}{14}$/°$; 50/14$/°$; 538/138$/°$. Then the platform was rotated from $-50^\circ$ to $49^\circ$ at the step of $1^\circ$, with measured data acquired at each direction for further processing. It is noted that in the anechoic chamber, directions of calibration signals could be measured with the help of the reflector and rotation platform. However, it is difficult to determine the directions in the outside environments where the conditions are not so ideal. In order to investigate the robustness of the proposed algorithm, we impose a bias on the incident angles of calibration signals in some experiments. The proposed algorithm is compared with the algorithm in Ref.14,16, and the gain-phase errors calibration (GPC) algorithm in Ref.21, which only calibrates gain-phase errors but ignores mutual coupling errors.
The first experiment concerns about the DOA estimation in the presence of array manifold errors. Fig. 10 shows the results of DOA estimation of signals from $-27^\circ$. In Fig. 10(a), the records of orientations of the rotation platform are considered as the directions of auxiliary sources. It can be seen that the algorithms in Refs. 14, 16 and the proposed algorithm can successfully estimate the DOA. But the curve of the gain-phase calibration algorithm is not sharp and has a poor accuracy due to mutual coupling effects. Imposing a bias of $2^\circ$ on the directions of the first auxiliary source while maintaining angle intervals of the calibration sources, the proposed algorithm is still able to obtain the correct values. However, the curves of other algorithms skew from true positions in Fig. 10(b).

The second experiment employs the array patterns to evaluate the performance of the calibration algorithms. The array patterns, shown in Fig. 11, are measured by applying the steering vectors with hanning window on signals from directions of $-50^\circ$ to $49^\circ$. Similar to the first experiment a bias of $2^\circ$ is also imposed on the angles of auxiliary sources to investigate the performance of the calibration algorithms. The array patterns steer at $0^\circ$ in Fig. 11(a) and $10^\circ$ in Fig. 11(b), respectively. The gain-phase calibration algorithm results in high sidelobes and
serious distortion of the pattern, since it ignores mutual coupling effects. Although the algorithm in Refs. 14,16 can provide low levels of sidelobes, the patterns deviate from the ideal case. It is worth mentioning that the pattern of the proposed algorithm is closer to the ideal pattern and not affected by the angle bias of auxiliary sources.

In the third experiment, the consistence of element patterns is investigated. Since all the transmitting signals are the same, the element patterns can be measured by the power of receiving signals after calibration. Fig. 12 shows the measured patterns of all the antenna elements, taking the first element as reference and normalized to unity at $\theta = 0^\circ$. Different lines in each subfigure represent the patterns of different elements. Affected by mutual coupling, the element patterns obtained by the gain-phase calibration algorithm differ from each other, which will lead to a deterioration of array performance. Taking mutual coupling errors into consideration, the consistence of element patterns is greatly improved. It is shown that the proposed algorithm outperforms other algorithms again. When incident angles are biased, the proposed algorithm behaves robustly, while the performance of other algorithms degrades.

5. Conclusions

This paper has presented a practical algorithm for array calibration in the presence of gain-phase and mutual coupling errors. The investigations are summarized as follows:

1. The algorithm behaves robustly against angle bias of auxiliary sources. Since the calibration signals are time-disjoint, the algorithm requires only one auxiliary source with the help of rotation platforms.
2. The algorithm achieves high estimation accuracy and rarely suffers from ambiguous problems.
3. Experimental results of measured data show that the algorithm is practical and useful to improve the performance of the DAR test-bed.
4. The authors will devote to extend the algorithm to the calibration of other array manifold imperfections, such as sensor array errors.
5. The authors will try to extend the proposed algorithm to the case when both the incident angles and angle intervals are unknown.

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References


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