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New Concurrent Modulus Algorithm and Soft Decision Directed Scheme for Blind Equalization

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Abstract

The Constant Modulus Algorithm (CMA) is recognized as the most widely used algorithm in blind channel equalization practice. However, the CMA cost function exhibits local minima, which often leads to ill-convergence. This paper proposes a concurrent equalizer, in which a Soft Decision Directed (SDD) equalizer operates cooperatively with a CMA equalizer, controlled through a non-linear link that depends on the system a priory state. The simulation results show that the proposed equalizer has faster convergence rate and lower steady-state mean square error than the CMA equalizer.

Keywords: Constant modulus algorithm; soft decision directed algorithm; blind equalization.

1. Introduction

In digital communication systems, Inter-Symbol Interference (ISI) due to bandwidth limited channels or multipath propagation and phase rotation due to Doppler frequency shift are two main factors which affect the performance of communication systems seriously. A usual way of dealing with ISI is equalization in the receiver. Equalization without any explicit training sequence is referred to as blind channel equalization. Furthermore, for multipoint communication systems, training is infeasible and blind equalizer provides a practical means for combating the detrimental effects of ISI in such systems. Because of its robustness and be easily implemented, the constant modulus algorithm (CMA) based FIR equalizer is by far the most popular blind equalization scheme [1-3]. The problem of the CMA, however, is that it only achieves moderate convergence rate and steady-state mean square error (MSE), which may not be sufficient for the system to obtain adequate performance [4-7].

For solving that problem, a blind equalization which operates a CMA equalizer concurrently with a soft decision directed (SDD) equalizer is proposed, and it is called CMA+SDD. In this CMA+SDD, the

tap weights vector of CMA equalizer is adjusted at every iteration like using CMA equalizer alone, but the tap weights vector of SDD equalizer is adjusted according to a rule based on the decision of the CMA+SDD output. The CMA+SDD can achieve faster convergence rate and lower steady-state mean square error. The computer simulations results show the efficiency of CMA+SDD.

2. Constant modulus algorithm

Constant modulus algorithm, first introduced in [1], is by far the most popular blind equalization algorithm because of its robustness and because it can be easily implemented [2]. So we start analysis with CMA2-2 [1]. As seen in figure 1, $s(k)$ is the transmitted symbol, $C(z)$ is the impulse response of the channel, $n(k)$ is the channel noise, $x(k)$ is the equalizer input, $y(k)$ is the equalizer output and $\hat{s}(k)$ is the output of the decision device. The equalizer N-taps weight vector is defined by $f(k) = [f_0(k), f_1(k), \cdots, f_{N-1}(k)]^{\mathrm{T}}$. The N-taps input vector is defined by $x(k) = [x(k), x(k-1), \cdots, x(k-N+1)]^{\mathrm{T}}$. Equalizer output can be expressed as

$$y(k) = f^{\mathrm{T}}(k)x(k) \tag{1}$$

The cost function of CMA is defined by

$$J_{CMA}(f) = \mathrm{E}\{[e(k)]^2\} \tag{2}$$

where $\mathrm{E}[\cdot]$ indicates statistical expectation and $e(k)$ is the error function of CMA, defined by

$$e(k) = |y(k)|^2 - R^2 \tag{3}$$

where R^2 is a constant modulus which is defined by

$$R^2 = \mathrm{E}[|s(k)|^4] / \mathrm{E}[|s(k)|^2] \tag{4}$$

Using a stochastic gradient algorithm, the weight vector of CMA equalizer is updated by

$$f(k+1) = f(k) - \mu x^*(k)y(k)(|y(k)|^2 - R^2) \tag{5}$$

where μ is the step size.

The CMA is widely used in practice for its robustness and the capability of opening "initially closed eye", but towards the non-constant modulus such as high-order QAM signals, its convergence rate and residual error are not very good. The one of the main reasons for that is the error function of CMA attempts to drive the equalizer output to a lie on a circle of radius R as seen form figure 2, namely, the error function can not become exactly zero even when the channel is perfectly equalized [8-10].
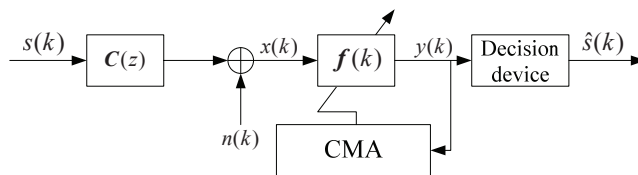

Fig. 1. Structure of the CMA equalizer

3. CMA+SDD

For the non-constant modulus signals, the CMA equalizer is effective in aspects of faster convergence rate and low-complexity. However it will not be zero when even the channel is perfectly equalized. This,

in turn, results in large output error level in steady-state after the equalizer converges completely. Therefore, it is necessary to switch over a conventional DD algorithm to improve the steady state performance, i.e., convergence rate and output error levels, when the eye pattern of the equalizer output is opened to some extent by the CMA. However, as pointed out in [4], in order for such a transfer to be successful, the CMA steady-state MSE should be sufficiently low. In practice, such a low level of MSE may not always be achievable. For example, for denser constellations (such as higher QAM signals), and for a number of practical "bad" channels (such as those with roots near the unit circle in the z-plane) like some underwater acoustic channels, there is a strong probability of a CMA failure in achieving the transfer level.

In order to avoid this pitfall in such equalizer with switching operation, which precludes the use of denser constellations for higher transmission speeds, this paper proposes a concurrent operation of the CMA and SDD algorithm. The concurrent operation is achieved by means of a non-linear link between the two algorithms, which avoids the SDD algorithm to destroy the delicate dynamic of the CMA equalizer. Actually, this link is a switch that depends on the decision of system output and that controls the concurrent operation.

In the structure of CMA+SDD, the CMA is dominant part, and the conventional SDD algorithm takes over in tracking part. It is automatically switched between the two parts according to the equalizer output error level with no need to detect the convergence. The key idea of the proposed algorithm is that to avoid the weakness of SDD algorithm, the SDD part of operation is confined to the case of high confidence in correctness of the decision. So we propose the novel method to control the SDD part operation as follows:

Let $\boldsymbol{f} = \boldsymbol{f}_c + \boldsymbol{f}_d$, here $\boldsymbol{f}_c$ is the weight vector of the CMA equalizer which is designed to minimize the CMA cost function (2). While $\boldsymbol{f}_d$ is the weight vector of the DD equalizer which is designed to minimize the decision based MSE

$$J_{DD}(\boldsymbol{f}) = \frac{1}{2}\mathrm{E}\{|\hat{s}(k) - y(k)|^2\} \tag{6}$$

By adjusting $\boldsymbol{f}_d$, where $\hat{s}(k)$ denotes the quantized equalizer output defined by

$$\hat{s}(k) = \arg\min_{s_{il}\in s}|y(k) - s_{il}|. \tag{7}$$

More precisely, at symbol-spaced sample, given

$$y(k) = \boldsymbol{f}_c^T(k)\boldsymbol{x}(k) + \boldsymbol{f}_d^T(k)\boldsymbol{x}(k) \tag{8}$$

the CMA part adapts $\boldsymbol{f}_c$ according to the rule

$$\boldsymbol{f}_c(k+1) = \boldsymbol{f}_c(k) - \mu_c\boldsymbol{x}^*(k)y(k)(|y(k)|^2 - R^2) \tag{9}$$

The DD adaptation follows immediately after the CMA equalizer adaptation but it only takes place if the CMA adjustment is viewed to be a successful one. Let

$$\bar{y}(k) = \boldsymbol{f}_c^T(k+1)\boldsymbol{x}(k) + \boldsymbol{f}_d^T(k)\boldsymbol{x}(k) \tag{10}$$

Then the DD part adjusts $\boldsymbol{f}_d$ according to

$$\boldsymbol{f}_d(k+1) = \boldsymbol{f}_d(k) - \mu_d\boldsymbol{x}^*(k)[y(k) - \hat{s}(k)]\delta(x) \tag{11}$$

where the indicator function

$$\delta(x) = \begin{cases} 1, sign\{real[\hat{s}(k)]\} == sign\{real[\hat{\bar{s}}(k)]\} \,\&\, sign\{image[\hat{s}(k)]\} == sign\{image[\hat{\bar{s}}(k)]\} \\ 0, else \end{cases} \tag{12}$$

where $\hat{\bar{s}}(k)$ is the quantized equalizer output of $\bar{y}(k)$.

It can be seen that f_d is updated only if the equalizer hard decision before and after the CMA adaptation are the same. A potential problem of (hard) decision-directed adaptation is that if the decision is wrong, error propagation occurs which subsequently degrades equalizer adaptation. But in the proposed method if the equalizer hard decisions before and after the CMA adaptation are the same, the decision probably is a right one. The DD adaptation, when is safe to perform, has a much faster convergence speed and is capable of lowering the steady-state MSE, compared with the pure CMA. The adaptive gain μ_d for the DD equalizer can often be chosen much larger than μ_c for the CMA.

4. Simulation study

Here, we present simulation results to demonstrate performances of the proposed MCMA+DD by comparing with CMA and MCMA. And we calculate the mean square error by this expression [2]

$$MSE = (\boldsymbol{h}_\delta - \boldsymbol{Cf})^* (\boldsymbol{h}_\delta - \boldsymbol{Cf})\sigma_s^2 + \boldsymbol{f}^* \boldsymbol{f}\sigma_w^2 \qquad (13)$$

where σ_s^2 is the source variance, σ_w^2 is the noise variance, $\boldsymbol{h}_\delta = \boldsymbol{Cf}$ under the condition of perfect equalization, which is defined as

$$\boldsymbol{h}_\delta = [0,\cdots,0,1,0,\cdots,0]^T \qquad (14)$$

where the nonzero coefficient is in the δ th position.

16-QAM data symbols were transmitted through a channel with impulse response $\boldsymbol{C} = [0.3132,-0.1040,0.8908,0.3134,0.1]^T$. The SNR was set to 30 dB. For the CMA part of CMA+SDD and CMA, they all have 21 taps with a center-spike initialization and the DD part of CMA+SDD has 21 taps with all zeros initialization. μ_c and μ_d were set to 0.001 and 0.007 respectively for CMA+SDD. μ_c was set to 0.001 for CMA. The results of this simulation are shown in figure 3-6.

The learning curves presented with CMA and CMA+SDD in terms of the convergence rate and MSE are depicted in figure 3. It is clear that CMA+SDD has the lower steady-state MSE and the faster convergence rate than CMA. The equalizers input and output signal constellations are shown in 4, 5 and 6 respectively. From the figures, it is clear that the CMA+SDD outputs are clearer than CMA outputs.
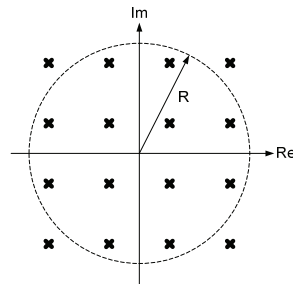

Fig. 2. Zero-error contours of CMA's error function

5. Conclusion

In this paper, we propose a new concurrent blind equalization combined constant modulus algorithm with decision directed algorithm. The proposed algorithm has "initially closed eye" ability and of CMA, and faster convergence rate and lower steady state mean square error abilities of DD algorithm.

Simulation studies have confirmed that CMA+SDD is more suitable for mitigating the effect of ISI in the communication systems.
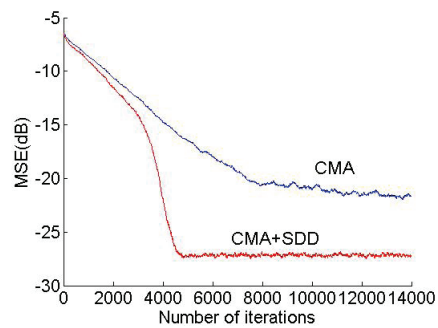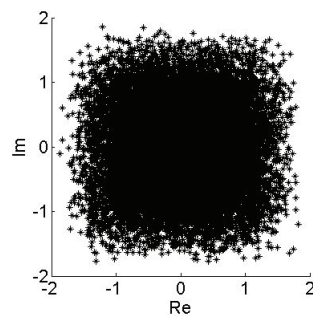

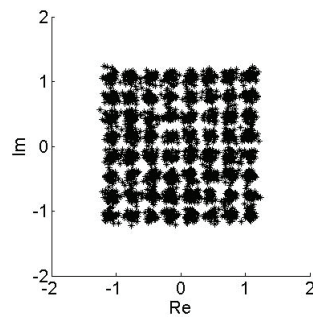Fig. 3. Learning curves of algorithms

Fig. 4. Equalizer input

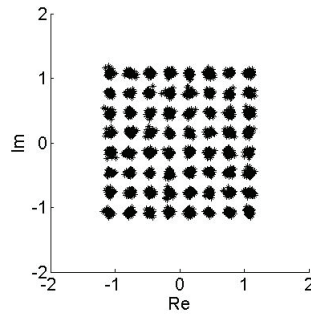Fig. 5. CMA output after convergence

Fig. 6. CMA+SDD output after convergence

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