



ELSEVIER

Available online at www.sciencedirect.com

SCIENCE @ DIRECT®

Journal of Computational and Applied Mathematics 182 (2005) 252–269

JOURNAL OF
COMPUTATIONAL AND
APPLIED MATHEMATICSwww.elsevier.com/locate/cam

Parametric completely generalized mixed implicit quasi-variational inclusions involving h -maximal monotone mappings[☆]

Xie Ping Ding*

College of Mathematics and Software Science, Sichuan Normal University, Chengdu, Sichuan 610066, PR China

Received 24 January 2004

Abstract

A new class of parametric completely generalized mixed implicit quasi-variational inclusions involving h -maximal monotone mappings is introduced. By applying resolvent operator technique of h -maximal monotone mapping and the property of fixed point set of set-valued contractive mappings, the behavior and sensitivity analysis of the solution set of the parametric completely generalized mixed implicit quasi-variational inclusions involving h -maximal monotone mappings are studied. The continuity and Lipschitz continuity of the solution set with respect to the parameter are proved under suitable assumptions. Our approach and results are new and improve, unify and extend previous many known results in this field.

© 2004 Elsevier B.V. All rights reserved.

MSC: 49J40; 47H05; 47J25; 47J20; 49J53

Keywords: Parametric completely generalized mixed implicit quasi-variational inclusion; h -maximal monotone mapping; Resolvent operator; Sensitivity analysis

1. Introduction

Variational inequality theory has become very effective and powerful tool for studying a wide range of problems arising in differential equations, mechanics, contact problems in elasticity, optimization

[☆] This project was supported by the NSF of Sichuan Education Department of China (2003A081) and SZD0406.

* Tel.: +86 028 4760 929; fax: +86 028 4790 994.

E-mail address: dingxp@sicru.edu.cn (X.P. Ding).

and control problems, management science, operations research, general equilibrium problems in economics and transportation, unilateral, obstacle, moving, etc., for example, see [3,5,21,22,33]. A useful and important generalization of variational inequalities is generalized mixed quasi-variational inclusions.

Hassouni and Moudafi [23] used the resolvent operator technique to study a class of mixed type variational inequalities with single-valued mappings which was called variational inclusions. Since then, Adly [1], Ding [6–14], Ding and Lou [16,18], Ding and Park [19], Huang [24,25,27], Huang and Deng [26], Fang and Huang [20], Kazmi [28], Noor [34,36,37], Noor et al. [41], Uko [45] have obtained some important extensions and generalizations of the results in [23] from various different directions. By studying an elastoplasticity problem, Panagiotopoulos and Stavroulakis [42] and Noor and Al-Said [39] considered a new class of generalized nonlinear variational inequality problems, which is a variant and generalization of the problem proposed by Verma [46] and Verma and Base [47].

In recent years, much attention has been devoted to develop general methods for the sensitivity analysis of solution set of various variational inequalities and variational inclusions. From the mathematical and engineering points of view, sensitivity properties of various variational inequalities can provide new insight concerning the problem being studied and can stimulate ideas for solving problems. The sensitivity analysis of solution set for variational inequalities have been studied extensively by many authors using quite different methods. By using the projection technique, Dafermos [4], Mukherjee and Verma [31], Noor [35] and Yen [48] dealt with the sensitivity analysis for variational inequalities with single-valued mappings. By using the implicit function approach that makes use of so-called normal mappings, Robinson [44] dealt with the sensitivity analysis of solutions for variational inequalities in finite-dimensional spaces. By using resolvent operator technique, Adly [1], Noor and Noor [40,38], and Agarwal et al. [2] study the sensitivity analysis for quasi-variational inclusions with single-valued mappings. Recently, by using projection technique and the property of fixed point set of set-valued contractive mappings, Ding and Lou [17], Liu et al. [30], and Ding [15] study the behavior and sensitivity analysis of solution set for generalized quasi-variational inequalities and generalized mixed quasi-variational inclusions with set-valued mappings respectively.

Inspired and motivated by recent research works in this field, in this paper, we introduce a new class of parametric completely generalized mixed implicit quasi-variational inclusions involving h -maximal monotone mappings which includes the most of (parametric) generalized quasi-variational inequalities and (parametric) generalized quasi-variational inclusions in 5bib41 bib42 bib44 bib45 bib46 bib47 [1,2,4,6–20, 22–28,30,31, 33–42,44–48] as very special cases. By using resolvent operator technique and the property of fixed point set of set-valued contractive mappings, the behavior and sensitivity analysis of solution set for the parametric completely generalized mixed implicit quasi-variational inclusion are studied. The continuity and Lipschitz continuity of solution set of the parametric completely generalized mixed implicit quasi-variational inclusions are proved under suitable conditions. As special cases, some known results in this fields are also discussed. Our results improve, unify and generalize many known results mentioned above.

2. Preliminaries

Let H be a real Hilbert space with norm $\|\cdot\|$ and inner product $\langle \cdot, \cdot \rangle$. Let 2^H and $C(H)$ denote the family of all subsets of H and the family of all nonempty compact subsets of H , respectively. $\tilde{H}(\cdot, \cdot)$ denotes the Hausdorff metric on $C(H)$. In the following, let us recall some concepts.

Definition 2.1. Let $h : H \rightarrow H$ be a single-valued mapping. h is said to be

(i) monotone if

$$\langle h(x) - h(y), x - y \rangle \geq 0 \quad \forall x, y \in H;$$

(ii) strictly monotone if h is monotone and

$$\langle h(x) - h(y), x - y \rangle = 0 \quad \text{if and only if } x = y;$$

(iii) strongly monotone if there exists a constant $r > 0$ such that

$$\langle h(x) - h(y), x - y \rangle \geq r \|x - y\|^2 \quad \forall x, y \in H.$$

(iv) L_h -Lipschitz continuous if there exists a constant $L_h > 0$ such that

$$\|h(x) - h(y)\| \leq L_h \|x - y\| \quad \forall x, y \in H.$$

Definition 2.2. Let $M : H \rightarrow 2^H$ be a set-valued mapping. M is said to be

(i) monotone if

$$\langle u - v, x - y \rangle \geq 0 \quad \forall x, y \in H, \quad u \in M(x), \quad v \in M(y);$$

(ii) maximal monotone if M is monotone and $(I + \rho M)(H) = H$ for all $\rho > 0$, where I is the identity mapping on H .

The following concept was introduced in [20].

Definition 2.3. Let $h : H \rightarrow H$ be a single-valued mapping and $M : H \rightarrow 2^H$ be a set-valued mapping. M is said to be h -maximal monotone (which is called h -monotone in [20]), if M is monotone and $(h + \rho M)(H) = H$ for all $\rho > 0$.

Remark 2.1. It is clear that if $h = I$, the identity mapping, the concept of I -maximal monotone mapping coincides with that of maximal monotone mapping. Example 2.1 in [20] shows that a maximal monotone mapping may not be h -maximal monotone for some h . Theorem 2.1 in [20] shows that if h is strictly monotone and M is h -maximal monotone, then the operator $R_{M,\rho}^h = (h + \rho M)^{-1} : H \rightarrow H$ is a single-valued mapping and is called the resolvent operator of the h -maximal mapping M .

Lemma 2.1 (Fang and Huang [20]). Let $h : H \rightarrow H$ be a strongly monotone mapping with constant $r > 0$ and M is an h -maximal monotone mapping, then the resolvent operator $R_{M,\rho}^h$ of M is Lipschitz continuous with constant $1/r$, i.e.,

$$\|R_{M,\rho}^h(u) - R_{M,\rho}^h(v)\| \leq \frac{1}{r} \|u - v\| \quad \forall u, v \in H.$$

Let Ω be a nonempty open subset of H in which the parameter λ takes values. Let $N : H \times H \times H \times \Omega \rightarrow H$, $W : H \times H \times \Omega \rightarrow H$, $m, i, j : H \times \Omega \rightarrow H$ and $h : H \rightarrow H$ be single-valued mappings. Let $A, B, C, D, E, F, G : H \times \Omega \rightarrow C(H)$ be set-valued mappings. Let $M : H \times H \times \Omega \rightarrow 2^H$ be a

set-valued mapping such that for each given $(f, \lambda) \in H \times \Omega$, $M(\cdot, f, \lambda) : H \rightarrow 2^H$ is a h -maximal monotone mapping with $(G(H, \lambda) - m(H, \lambda)) \cap \text{dom } M(\cdot, f, \lambda) \neq \emptyset$. Throughout this paper, unless otherwise stated, we will consider the following parametric completely generalized mixed implicit quasi-variational inclusion problem (PCGMIQVIP):

$$\begin{aligned} &\text{for each } (\lambda, w) \in \Omega \times H, \quad \text{find } x = x(\lambda) \in H, \quad a = a(x, \lambda) \in A(i(x, \lambda), \lambda), \\ &b = b(x, \lambda) \in B(x, \lambda), \quad c = c(x, \lambda) \in C(x, \lambda), \quad d = d(x, \lambda) \in D(x, \lambda), \quad e = e(x, \lambda) \in E(x, \lambda), \\ &f = f(x, \lambda) \in F(x, \lambda), \quad g(x, \lambda) \in G(x, \lambda) \quad \text{such that} \\ &w \in W(j(e, \lambda), a, \lambda) - N(b, c, d, \lambda) + M(g - m(x, \lambda), f, \lambda). \end{aligned} \quad (2.1)$$

2.1. Special cases

(I) If $w = 0$, $W \equiv 0$, $N(b, c, d, \lambda) = -\tilde{N}(b, c, \lambda)$ for all $b, c, d \in H$ and $\lambda \in \Omega$, and for each $(f, \lambda) \in H \times \Omega$, $M(\cdot, f, \lambda)$ is a maximal monotone mapping (i.e., h is the identity mapping on H), then the PCGMIQVIP (2.1) collapses to the following parametric generalized nonlinear implicit quasi-variational inclusion problem:

$$\begin{aligned} &\text{for each } \lambda \in \Omega, \quad \text{find } x = x(\lambda) \in H, \quad b = b(x, \lambda) \in B(x, \lambda), \quad c = c(x, \lambda) \in C(x, \lambda), \\ &f = f(x, \lambda) \in F(x, \lambda), \quad g = g(x, \lambda) \in G(x, \lambda) \quad \text{such that} \\ &0 \in \tilde{N}(b, c, \lambda) + M(g - m(x, \lambda), f, \lambda). \end{aligned} \quad (2.2)$$

The Problem (2.2) was introduced and studied in [15].

(II) If $N(b, c, d, \lambda) = -\tilde{N}(b, c, d, \lambda)$, $j(x, \lambda) = x$, $i(x, \lambda) = x$, $G(x, \lambda) = \{g(x, \lambda)\}$ and $W(e, a, \lambda) = -\tilde{W}(e, a, \lambda)$ for all $b, c, d, x, e, a \in H$ and $\lambda \in \Omega$, and for each $(f, \lambda) \in H \times \Omega$, $M(\cdot, f, \lambda)$ is a maximal monotone mapping, then the PCGMIQVIP (2.1) reduces to the following parametric completely generalized nonlinear implicit quasi-variational inclusion problem:

$$\begin{aligned} &\text{for each } (w, \lambda) \in H \times \Omega, \quad \text{find } x = x(\lambda) \in H, \quad a = a(x, \lambda) \in A(x, \lambda), \quad b = b(x, \lambda) \in B(x, \lambda), \\ &c = c(x, \lambda) \in C(x, \lambda), \quad d = d(x, \lambda) \in D(x, \lambda), \quad e = e(x, \lambda) \in E(x, \lambda), \quad f = f(x, \lambda) \in F(x, \lambda), \\ &\text{such that } w \in \tilde{N}(b, c, d, \lambda) - \tilde{W}(e, a, \lambda) + M((g - m)(x, \lambda), f, \lambda). \end{aligned} \quad (2.3)$$

The parametric problem (2.3) is new. When $\tilde{N}(b, c, d, \lambda) = \tilde{N}(b, c, \lambda)$ for all $b, c, d \in H$ and $\lambda \in \Omega$, the parametric problem (2.3) has been introduced and studied in [30].

(III) If $i(x, \lambda) = g(x, \lambda)$ for all $(x, \lambda) \in H \times \Omega$, and for each $(f, \lambda) \in H \times \Omega$, $M(\cdot, f, \lambda)$ is a maximal monotone mapping, then the PCGMIQVIP (2.1) reduces to the following parametric problem:

$$\begin{aligned} &\text{for each } (\lambda, -w) \in \Omega \times H, \quad \text{find } x = x(\lambda) \in H, \quad b = b(x, \lambda) \in B(x, \lambda), \\ &c = c(x, \lambda) \in C(x, \lambda), \quad d = d(x, \lambda) \in D(x, \lambda), \quad e = e(x, \lambda) \in E(x, \lambda), \quad f = f(x, \lambda) \in F(x, \lambda), \\ &g = g(x, \lambda) \in G(x, \lambda), \quad a = a(x, \lambda) \in A(g, \lambda) \quad \text{such that} \\ &w \in W(j(e, \lambda), a, \lambda) - N(b, c, d, \lambda) + M(g - m(x, \lambda), f, \lambda). \end{aligned} \quad (2.4)$$

(IV) Let $\varphi : H \times H \times \Omega \rightarrow \mathbb{R} \cup \{+\infty\}$ be such that for each fixed $(f, \lambda) \in H \times \Omega$, $\varphi(\cdot, f, \lambda)$ is a proper convex lower semicontinuous functional satisfying $(G(H, \lambda) - m(H, \lambda)) \cap \text{dom}(\partial\varphi(\cdot, f, \lambda)) \neq \emptyset$ where $\partial\varphi(\cdot, f, \lambda)$ is the subdifferential of $\varphi(\cdot, f, \lambda)$. In [43], $\partial\varphi(\cdot, f, \lambda) : H \rightarrow 2^H$ is a maximal monotone mapping. Let $M(\cdot, f, \lambda) = \partial\varphi(\cdot, f, \lambda)$, $\forall (f, \lambda) \in H \times \Omega$. For given $(f, \lambda) \in H \times \Omega$, by the definition

of the subdifferential of $\varphi(\cdot, f, \lambda)$, it is easy to see that the PCGMIVIP (2.1) reduces to the following parametric problem:

$$\begin{aligned} &\text{for each fixed } (w, \lambda) \in H \times \Omega, \quad \text{find } x = x(\lambda) \in H, \quad a = a(x, \lambda) \in A(i(x, \lambda), \lambda), \\ &b = b(x, \lambda) \in B(x, \lambda), \quad c = c(x, \lambda) \in C(x, \lambda), \quad d = d(x, \lambda) \in D(x, \lambda), \quad e = e(x, \lambda) \in E(x, \lambda), \\ &f = f(x, \lambda) \in F(x, \lambda), \quad g = g(x, \lambda) \in G(x, \lambda) \quad \text{such that} \\ &\langle W(j(e, \lambda), a, \lambda) - N(b, c, d, \lambda) - w, y - (g - m(x, \lambda)) \rangle \\ &\geq \varphi(g - m(x, \lambda), f, \lambda) - \varphi(y, f, \lambda), \quad \forall y \in H. \end{aligned} \quad (2.5)$$

(V) If $W(j(e, \lambda), a, \lambda) = j(e, \lambda) + a$, for all $a, e \in H$ and $\lambda \in \Omega$, $G = g = i : H \times \Omega \rightarrow H$ is a single-valued mapping and $m = 0$, then the parametric problem (2.5) reduces to the following parametric problem:

$$\begin{aligned} &\text{for each } (w, \lambda) \in H \times \Omega, \quad \text{find } x = x(\lambda) \in H, \quad a = a(x, \lambda) \in A(g(x, \lambda), \lambda), \\ &b = b(x, \lambda) \in B(x, \lambda), \quad c = c(x, \lambda) \in C(x, \lambda), \quad d = d(x, \lambda) \in D(x, \lambda), \quad e = e(x, \lambda) \in E(x, \lambda) \\ &f = f(x, \lambda) \in F(x, \lambda) \quad \text{such that } \langle j(e, \lambda) + a - N(b, c, d, \lambda) - w, y - g(x, \lambda) \rangle \\ &\geq \varphi((g(x, \lambda), \lambda), f, \lambda) - \varphi(y, f, \lambda), \quad \forall y \in H. \end{aligned} \quad (2.6)$$

Let $\psi : H \times H \times \Omega \rightarrow \mathbb{R}$ be a real function satisfying

- (I) $\psi(x, y, \lambda)$ is linear in first argument,
- (II) $\psi(x, y, \lambda)$ is bounded, i.e., there exists a constant $v > 0$ such that

$$\psi(x, y, \lambda) \leq v \|x\| \|y\| \quad \forall x, y \in H, \quad \lambda \in \Omega,$$

- (III) for all $x, y, z \in H$ and $\lambda \in \Omega$,

$$\psi(x, y, \lambda) - \psi(x, z, \lambda) \leq \psi(x, y - z, \lambda).$$

Lemma 2.2. Let $\psi : H \times H \times \Omega \rightarrow \mathbb{R}$ be a real function satisfying conditions (I)–(III). Then for each $(y, \lambda) \in H \times \Omega$ there exists a unique point $j(y, \lambda) \in H \times \Omega$ such that

$$\psi(x, y, \lambda) = \langle j(y, \lambda), x \rangle \quad \forall (x, \lambda) \in H \times \Omega,$$

and the mapping $y \mapsto j(y, \lambda)$ is uniformly Lipschitz continuous with respect to $\lambda \in \Omega$ with constant $v > 0$.

Proof. For each fixed $\lambda \in \Omega$, by conditions (I) and (II) on ψ , we have

$$|\psi(x, y, \lambda)| \leq v \|x\| \|y\| \quad \forall x, y \in H$$

and hence $\psi(x, 0, \lambda) = \psi(0, y, \lambda) = 0$ and for each $x, y \in H$ and $\lambda \in \Omega$, and $x \mapsto \psi(x, y, \lambda)$ is continuous. By conditions (II) and (III) on ψ , we have

$$|\psi(x, y, \lambda) - \psi(x, z, \lambda)| \leq v \|x\| \|y - z\| \quad \forall x, y, z \in H \text{ and } \lambda \in \Omega$$

and so for each $(x, \lambda) \in H \times \Omega$, $y \mapsto \psi(x, y, \lambda)$ is also continuous. Hence for each given $(y, \lambda) \in H \times \Omega$, $x \mapsto \psi(x, y, \lambda)$ is a continuous linear functional on H . By the Riesz representation theorem, there is a unique point $j(y, \lambda) \in H$ such that

$$\psi(x, y, \lambda) = \langle j(y, \lambda), x \rangle \quad \forall x \in H$$

and for all $(y, z, \lambda) \in H \times H \times \Omega$

$$\begin{aligned}\|j(y, \lambda) - j(z, \lambda)\| &= \sup_{\|x\| \leq 1} |\langle j(y, \lambda) - j(z, \lambda), x \rangle| \\ &= \sup_{\|x\| \leq 1} |\psi(x, y, \lambda) - \psi(x, z, \lambda)| \leq \sup_{\|x\| \leq 1} |\psi(x, y - z, \lambda)| \\ &\leq \sup_{\|x\| \leq 1} v\|x\| \|y - z\| \leq v\|y - z\| \quad \forall y, z \in H.\end{aligned}$$

This shows that the mapping $y \mapsto j(y, \lambda)$ is uniformly Lipschitz continuous with respect to $\lambda \in \Omega$ with constant $v > 0$. \square

(VII) If $\psi : H \times H \times \Omega \rightarrow \mathbb{R}$ satisfies conditions (I)–(III) and $j(\cdot, \lambda) : H \times \Omega \rightarrow H$ is the mapping defined by ψ in Lemma 2.2, then the parametric problem (2.6) is equivalent to the following parametric completely generalized mixed quasi-variational inequality problem:

$$\begin{aligned}\text{For each } (w, \lambda) \in H \times \Omega, \quad &\text{find } x = x(\lambda) \in H, \quad a = a(x, \lambda) \in A(g(x, \lambda), \lambda), \\ &b = b(x, \lambda) \in B(x, \lambda), \\ &c = c(x, \lambda) \in C(x, \lambda), \quad d = d(x, \lambda) \in D(x, \lambda), \quad e = e(x, \lambda) \in E(x, \lambda) \\ &f = f(x, \lambda) \in F(x, \lambda), \quad \text{such that} \\ &\langle a - N(b, c, d, \lambda) - w, y - g(x, \lambda) \rangle + \psi(y, e, \lambda) - \psi(g(x, \lambda), e, \lambda) \\ &\geq \varphi(g(x, \lambda), f, \lambda) - \varphi(y, f, \lambda) \quad \forall y \in H.\end{aligned}\tag{2.7}$$

(VIII) If $K : H \times \Omega \rightarrow 2^H$ is a set-valued mapping such that for each $(x, \lambda) \in H \times \Omega$, $K(x, \lambda)$ is a closed convex subset of H and for each fixed $(f, \lambda) \in H \times \Omega$, $\varphi(\cdot, f, \lambda) = I_{K(f, \lambda)}(\cdot)$ is the indicator function of $K(f, \lambda)$,

$$I_{K(f, \lambda)}(x) = \begin{cases} 0 & \text{if } x \in K(f, \lambda), \\ +\infty & \text{otherwise,} \end{cases}$$

then parametric problem (2.7) reduces to the following parametric generalized strongly nonlinear implicit quasi-variational inequality problem:

$$\begin{aligned}\text{For each } (w, \lambda) \in H \times \Omega, \quad &\text{find } x = x(\lambda) \in H, \quad a = a(x, \lambda) \in A(g(x, \lambda), \lambda), \\ &b = b(x, \lambda) \in B(x, \lambda), \quad c = c(x, \lambda) \in C(x, \lambda), \quad d = d(x, \lambda) \in D(x, \lambda), \quad e = e(x, \lambda) \in E(x, \lambda), \\ &f = f(x, \lambda) \in F(x, \lambda), \quad \text{such that } g(x, \lambda) \in K(f, \lambda) \text{ and} \\ &\langle a - N(b, c, d, \lambda) - w, y - g(x, \lambda) \rangle + \psi(e, y, \lambda) - \psi(e, g(x, \lambda)) \geq 0 \quad \forall y \in K(f, \lambda).\end{aligned}\tag{2.8}$$

The nonparametric types of Problems (2.7) and (2.8) were introduced and studied in [14].

In brief, for appropriate and suitable choices of $N, W, A, B, C, D, E, F, G, h, i, j, m$ and M , it is easy to see that the PCGMQVIP (2.1) includes a number of (parametric) quasi-variational inclusions (parametric) generalized quasi-variational inclusions (parametric) quasi-variational inequalities (parametric) generalized implicit quasi-variational inequalities studied by many authors as special cases, for example see [1,2,4,6–20, 22–28,30,31, 33–42,44–48] and the references therein.

Definition 2.4. A mapping $m : H \times \Omega \rightarrow H$ is called L_m -Lipschitz continuous in first argument, if there exists a constant $L_m > 0$ such that

$$\|m(x, \lambda) - m(y, \lambda)\| \leq L_m \|x - y\| \quad \forall x, y \in H, \lambda \in \Omega.$$

Definition 2.5. A set-valued mapping $A : H \times \Omega \rightarrow C(H)$ is said to be L_A -Lipschitz continuous in first argument, if there exists a constant $L_A > 0$ such that

$$\tilde{H}(A(x, \lambda), A(y, \lambda)) \leq L_A \|x - y\| \quad \forall x, y \in H, \lambda \in \Omega.$$

Definition 2.6. Let $B, C : H \times \Omega \rightarrow C(H)$ be set-valued mappings and $N : H \times H \times H \times \Omega \rightarrow H$ be a single-valued mapping.

(i) N is said to be γ -relaxed Lipschitz continuous in first argument with respect to B , if there exists a constant $\gamma > 0$ such that

$$\langle N(u, c, d, \lambda) - N(v, c, d, \lambda), x - y \rangle \leq -\gamma \|x - y\|^2 \quad \forall x, y, c, d \in H, u \in B(x, \lambda), \\ v \in B(y, \lambda), \lambda \in \Omega.$$

(ii) N is said to be σ -generalized pseudo-contractive in second argument with respect to C , if there exists a constant $\sigma > 0$ such that

$$\langle N(b, u, d, \lambda) - N(b, v, d, \lambda), x - y \rangle \leq \sigma \|x - y\|^2 \quad \forall x, y, b, d \in H, u \in C(x, \lambda), \\ v \in C(y, \lambda), \lambda \in \Omega.$$

(iii) N is said to be Lipschitz continuous in the first argument, if there exists a constant $L_{(N,1)} > 0$ such that

$$\|N(x, c, d, \lambda) - N(y, c, d, \lambda)\| \leq L_{(N,1)} \|x - y\| \quad \forall x, y, c, d \in H, \lambda \in \Omega.$$

In a similar way, one can define the Lipschitz continuity of N in the second and third argument, respectively.

Now, for each fixed $\lambda \in \Omega$, the solution set $S(\lambda)$ of the PCGMIQVIP (2.1) is denoted as

$$S(\lambda) = \{x = x(\lambda) \in H : \exists a = a(x, \lambda) \in A(i(x, \lambda), \lambda), b = b(x, \lambda) \in B(x, \lambda), \\ c = c(x, \lambda) \in C(x, \lambda), d = d(x, \lambda) \in D(x, \lambda), e = e(x, \lambda) \in E(x, \lambda), \\ f = f(x, \lambda) \in F(x, \lambda), \\ g = g(x, \lambda) \in G(x, \lambda) \text{ such that } w \in W(j(e, \lambda), a, \lambda) - N(b, c, d, \lambda) \\ + M(g - m(x, \lambda), f, \lambda)\}.$$

The main aim of this paper is to study the behavior and sensitivity analysis of the solution set $S(\lambda)$, and the conditions on these mappings $A, B, C, D, E, G, W, N, M, h, i, j, m$ under which the solution set $S(\lambda)$ of the PCGMIQVIP (2.1) is nonempty and continuous or Lipschitz continuous with respect to the parameter $\lambda \in \Omega$.

3. Sensitivity analysis of solution set

We first transfer the PCGMIQVIP (2.1) into a fixed point problem.

Theorem 3.1. For each $(w, \lambda) \in H \times \Omega$, (x, a, b, c, d, e, f, g) is a solution of the PCGMIQVIP (2.1) if and only if $x = x(\lambda) \in H$, $a = a(x, \lambda) \in A(i(x, \lambda), \lambda)$, $b = b(x, \lambda) \in B(x, \lambda)$, $c = c(x, \lambda) \in C(x, \lambda)$, $d = d(x, \lambda) \in D(x, \lambda)$, $e = e(x, \lambda) \in E(x, \lambda)$, $f = f(x, \lambda) \in F(x, \lambda)$ and $g = g(x, \lambda) \in G(x, \lambda)$ satisfy

$$g = m(x, \lambda) + R_{M(\cdot, f, \lambda), \rho}^h [h(g - m(x, \lambda)) - \rho W(j(e, \lambda), a, \lambda) + \rho N((b, c, d, \lambda) + \rho w)], \quad (3.1)$$

where $\rho > 0$ is a constant.

Proof. For each $(w, \lambda) \in H \times \Omega$, suppose that (x, a, b, c, d, e, f, g) is a solution of the PCGMIQVIP (2.1), then $x = x(\lambda) \in H$, $a = a(x, \lambda) \in A(i(x, \lambda), \lambda)$, $b = b(x, \lambda) \in B(x, \lambda)$, $c = c(x, \lambda) \in C(x, \lambda)$, $d = d(x, \lambda) \in D(x, \lambda)$, $e = e(x, \lambda) \in E(x, \lambda)$, $f = f(x, \lambda) \in F(x, \lambda)$ and $g = g(x, \lambda) \in G(x, \lambda)$ satisfy

$$w \in W(j(e, \lambda), a, \lambda) - N(b, c, d, \lambda) + M(g - m(x, \lambda), f, \lambda). \quad (3.2)$$

Relation (3.2) holds if and only if

$$\begin{aligned} h(g - m(x, \lambda)) + \rho[N(b, c, d, \lambda) - W(j(e, \lambda), a, \lambda) + w] \\ \in (h + \rho M(\cdot, f, \lambda))(g - m(x, \lambda)), \end{aligned} \quad (3.3)$$

where $\rho > 0$ is a constant. Since for each $(f, \lambda) \in H \times \Omega$, $M(\cdot, f, \lambda)$ is h -maximal monotone, by the definition of the resolvent operator $R_{M(\cdot, f, \lambda), \rho}^h$ of $M(\cdot, f, \lambda)$, relation (3.3) holds if and only if

$$g = m(x, \lambda) + R_{M(\cdot, f, \lambda), \rho}^h [h((g - m(x, \lambda)) + \rho N(b, c, d, \lambda) - \rho W(j(e, \lambda), a, \lambda) + \rho w)].$$

This completes the proof. \square

If $N(b, c, d, \lambda) = -\tilde{N}(b, c, d, \lambda)$, $j(x, \lambda) = x$, $i(x, \lambda) = x$, $G(x, \lambda) = \{g(x, \lambda)\}$ and $W(e, a, \lambda) = -\tilde{W}(e, a, \lambda)$ for all $b, c, d, x, e, a \in H$ and $\lambda \in \Omega$, and for each $(f, \lambda) \in H \times \Omega$, $M(\cdot, f, \lambda)$ is a maximal monotone mapping in Theorem 3.1, then we obtain the following result.

Theorem 3.2. For each $(w, \lambda) \in H \times \Omega$, (x, a, b, c, d, e, f) is a solution of the parametric problem (2.3) if and only if $x = x(\lambda) \in H$, $a = a(x, \lambda) \in A(x, \lambda)$, $b = b(x, \lambda) \in B(x, \lambda)$, $c = c(x, \lambda) \in C(x, \lambda)$, $d = d(x, \lambda) \in D(x, \lambda)$, $e = e(x, \lambda) \in E(x, \lambda)$, and $f = f(x, \lambda) \in F(x, \lambda)$ such that

$$g(x, \lambda) = m(x, \lambda) + J_{\rho}^{M(\cdot, f, \lambda)} [(g - m)(x, \lambda) + \rho(\tilde{W}(e, a, \lambda) - \tilde{N}(b, c, d, \lambda) + w)], \quad (3.4)$$

where $J_{\rho}^{M(\cdot, f, \lambda)} = (I + \rho M(\cdot, f, \lambda))^{-1}$ is the resolvent operator of $M(\cdot, f, \lambda)$ and $\rho > 0$ is a constant.

Proof. For each $(w, \lambda) \in H \times \Omega$, suppose that (x, a, b, c, d, e, f) is a solution of the parametric problem (2.3), then $x = x(\lambda) \in H$, $a = a(x, \lambda) \in A(x, \lambda)$, $b = b(x, \lambda) \in B(x, \lambda)$, $c = c(x, \lambda) \in C(x, \lambda)$, $d = d(x, \lambda) \in D(x, \lambda)$, $e = e(x, \lambda) \in E(x, \lambda)$, $f = f(x, \lambda) \in F(x, \lambda)$ satisfy

$$w \in \tilde{N}(b, c, d, \lambda) - \tilde{W}(e, a, \lambda) + M((g - m)(x, \lambda), f, \lambda). \quad (3.5)$$

Relation (3.5) holds if and only if

$$g(x, \lambda) - m(x, \lambda) + \rho[\tilde{W}(e, a, \lambda) - \tilde{N}(b, c, d, \lambda) + w] \in (I + \rho M(\cdot, f, \lambda))(g - m)(x, \lambda). \quad (3.6)$$

Since for each $(f, \lambda) \in H \times \Omega$, $M(\cdot, f, \lambda)$ is maximal monotone, by the definition of the resolvent operator $J_\rho^{M(\cdot, f, \lambda)}$ of $M(\cdot, f, \lambda)$, relation (3.6) holds if and only if

$$g(x, \lambda) = m(x, \lambda) + J_\rho^{M(\cdot, f, \lambda)}[(g - m)(x, \lambda) + \rho(\tilde{W}(e, a, \lambda) - \tilde{N}(b, c, d, \lambda) + w)]. \quad \square$$

Theorem 3.3. Let $A, B, C, D, E, F, G : H \times \Omega \rightarrow C(H)$ be set-valued mappings such that A, B, C, D, E, F and G are Lipschitz continuous in first argument with constants $L_A, L_B, L_C, L_D, L_E, L_F$ and L_G , respectively, and G be δ -strongly monotone in first argument. Let $N : H \times H \times H \times \Omega \rightarrow H$ be γ -relaxed Lipschitz continuous in first argument with respect to B and σ -pseudo-contractive in second argument with respect to C . $N(\cdot, \cdot, \cdot, \cdot)$ be Lipschitz continuous in the first, second and third arguments with constants $L_{(N,1)}, L_{(N,2)}$ and $L_{(N,3)}$, respectively. Let $W : H \times H \times \Omega \rightarrow H$ be Lipschitz continuous in first and second arguments with constants $L_{(W,1)}$ and $L_{(W,2)}$, respectively. Let $m, i, j : H \times \Omega \rightarrow H$ be Lipschitz continuous in first argument with constants L_m, L_i and L_j , respectively, and h be r -strongly monotone and L_h -Lipschitz continuous. Let $M : H \times H \times \Omega \rightarrow 2^H$ be such that for each fixed $(f, \lambda) \in H \times \Omega$, $M(\cdot, f, \lambda) : H \rightarrow 2^H$ is a h -maximal monotone mapping satisfying $(G(H, \lambda) - m(H, \lambda)) \cap \text{dom } M(\cdot, f, \lambda) \neq \emptyset$. Suppose that for any $(x, y, z, \lambda) \in H \times H \times H \times \Omega$,

$$\|R_{M(\cdot, x, \lambda), \rho}^h(z) - R_{M(\cdot, y, \lambda), \rho}^h(z)\| \leq \mu \|x - y\| \quad (3.7)$$

and there exists a constant $\rho > 0$ such that

$$\begin{aligned} k &= \left(1 + \frac{1}{r}\right) \left(\sqrt{1 - 2\delta + L_G^2} + L_m\right) + \frac{L_G + L_m}{r} \sqrt{1 - 2r + L_h^2} + \mu L_F < 1, \\ p &= L_{(N,1)} L_B + L_{(N,2)} L_C > L_{(N,3)} L_D + L_{(W,1)} L_j L_E + L_{(W,2)} L_A L_i = q, \\ \gamma &> \sigma + r q (1 - k) + \sqrt{(p^2 - q^2)(1 - r^2(1 - k)^2)}, \\ \left| \rho - \frac{\gamma - \sigma - r q (1 - k)}{p^2 - q^2} \right| &< \frac{\sqrt{[\gamma - \sigma - r q (1 - k)]^2 - (p^2 - q^2)(1 - r^2(1 - k)^2)}}{p^2 - q^2}. \end{aligned} \quad (3.8)$$

Then for each $\lambda \in \Omega$, the solution set $S(\lambda)$ of the PCGMIQVIP (2.1) is nonempty and closed.

Proof. (1) Define a set-valued mapping $Q : H \times \Omega \rightarrow 2^H$ by

$$\begin{aligned} Q(x, \lambda) &= \bigcup_{a \in A(i(x, \lambda), \lambda), b \in B(x, \lambda), c \in C(x, \lambda), d \in D(x, \lambda), e \in E(x, \lambda), f \in F(x, \lambda), g \in G(x, \lambda))} [x - (g - m(x, \lambda)) \\ &\quad + R_{M(\cdot, f, \lambda), \rho}^h(h(g - m(x, \lambda)) - \rho W(j(e, \lambda), a, \lambda) + \rho N(b, c, d, \lambda) + \rho w)] \\ &\quad \forall (x, \lambda) \in H \times \Omega. \end{aligned}$$

Let $(x, \lambda) \in H \times \Omega$ be an arbitrary element. Since A, B, C, D, E, F, G are compact valued, for any sequences $\{a_n\} \subset A(i(x, \lambda), \lambda)$, $\{b_n\} \subset B(x, \lambda)$, $\{c_n\} \subset C(x, \lambda)$, $\{d_n\} \subset D(x, \lambda)$, $\{e_n\} \subset E(x, \lambda)$, $\{f_n\} \subset F(x, \lambda)$, $\{g_n\} \subset G(x, \lambda)$, there exist subsequences $\{a_{n_i}\}$, $\{b_{n_i}\}$, $\{c_{n_i}\}$, $\{d_{n_i}\}$, $\{e_{n_i}\}$, $\{f_{n_i}\}$, $\{g_{n_i}\}$ and elements $a \in A(i(x, \lambda), \lambda)$, $b \in B(x, \lambda)$, $c \in C(x, \lambda)$, $d \in D(x, \lambda)$, and $e \in E(x, \lambda)$, $f \in F(x, \lambda)$, $g \in G(x, \lambda)$ such that $a_{n_i} \rightarrow a$, $b_{n_i} \rightarrow b$, $c_{n_i} \rightarrow c$, $d_{n_i} \rightarrow d$, $e_{n_i} \rightarrow e$, $f_{n_i} \rightarrow f$ and $g_{n_i} \rightarrow g$ as $i \rightarrow \infty$. By (3.7), Lemma 2.1, the Lipschitz continuity of W in the first and second arguments, the

Lipschitz continuity of N in the first, second and third arguments, and the Lipschitz continuity of h and j in first argument, we have

$$\begin{aligned}
& \|R_{M(\cdot, f_{n_i}, \lambda), \rho}^h[h(g_{n_i} - m(x, \lambda)) - \rho W(j(e_{n_i}, \lambda), a_{n_i}, \lambda) + \rho N(b_{n_i}, c_{n_i}, d_{n_i}, \lambda) + \rho w] \\
& - R_{M(\cdot, f, \lambda), \rho}^h[h(g - m(x, \lambda)) - \rho W(j(e, \lambda), a, \lambda) + \rho N(b, c, d, \lambda) + \rho w]\| \\
& \leq \|R_{M(\cdot, f_{n_i}, \lambda), \rho}^h[h(g_{n_i} - m(x, \lambda)) - \rho W(j(e_{n_i}, \lambda), a_{n_i}, \lambda) + \rho N(b_{n_i}, c_{n_i}, d_{n_i}, \lambda) + \rho w] \\
& - R_{M(\cdot, f, \lambda), \rho}^h[h(g_{n_i} - m(x, \lambda)) - \rho W(j(e_{n_i}, \lambda), a_{n_i}, \lambda) + \rho N(b_{n_i}, c_{n_i}, d_{n_i}, \lambda) + \rho w]\| \\
& + \|R_{M(\cdot, f, \lambda), \rho}^h[h(g_{n_i} - m(x, \lambda)) - \rho W(j(e_{n_i}, \lambda), a_{n_i}, \lambda) + \rho N(b_{n_i}, c_{n_i}, d_{n_i}, \lambda) + \rho w] \\
& - R_{M(\cdot, f, \lambda), \rho}^h[h(g - m(x, \lambda)) - \rho W(j(e, \lambda), a, \lambda) + \rho N(b, c, d, \lambda) + \rho w]\| \\
& \leq \mu \|f_{n_i} - f\| + \frac{1}{r} [\|h(g_{n_i} - m(x, \lambda)) - h(g - m(x, \lambda))\| \\
& + \rho \|W(j(e_{n_i}, \lambda), a_{n_i}, \lambda) - W(j(e, \lambda), a_{n_i}, \lambda)\| \\
& + \rho \|W(j(e, \lambda), a_{n_i}, \lambda) - W(j(e, \lambda), a, \lambda)\| \\
& + \rho \|N(b_{n_i}, c_{n_i}, d_{n_i}, \lambda) - N(b, c_{n_i}, d_{n_i}, \lambda)\| + \rho \|N(b, c_{n_i}, d_{n_i}, \lambda) - N(b, c, d_{n_i}, \lambda)\| \\
& + \rho \|N(b, c, d_{n_i}, \lambda) - N(b, c, d, \lambda)\|] \\
& \leq \mu \|f_{n_i} - f\| + \frac{1}{r} [L_h \|g_{n_i} - g\| + \rho(L_{(W,1)} L_j \|e_{n_i} - e\| + L_{(W,2)} \|a_{n_i} - a\| \\
& + L_{(N,1)} \|b_{n_i} - b\| + L_{(N,2)} \|c_{n_i} - c\| + L_{(N,3)} \|d_{n_i} - d\|)] \rightarrow 0, \quad \text{as } i \rightarrow \infty.
\end{aligned}$$

It follows that for each $(x, \lambda) \in H \times \Omega$, $Q(x, \lambda)$ is closed.

Now for each fixed $\lambda \in \Omega$, we prove that $Q(x, \lambda)$ is a set-valued contractive mapping. For any $(x, y, \lambda) \in H \times H \times \Omega$ and any $u \in Q(x, \lambda)$, there exist $a_1 \in A(i(x, \lambda), \lambda)$, $b_1 \in B(x, \lambda)$, $c_1 \in C(x, \lambda)$, $d_1 \in D(x, \lambda)$, $e_1 \in E(x, \lambda)$, $f_1 \in F(x, \lambda)$ and $g_1 \in G(x, \lambda)$ such that

$$\begin{aligned}
u &= x - (g_1 - m(x, \lambda)) + R_{M(\cdot, f_1, \lambda), \rho}^h[h(g_1 - m(x, \lambda)) \\
& - \rho W(j(e_1, \lambda), a_1, \lambda) + \rho N(b_1, c_1, d_1, \lambda) + \rho w].
\end{aligned}$$

Note that $A(i(y, \lambda), \lambda)$, $B(y, \lambda)$, $C(y, \lambda)$, $D(y, \lambda)$, $E(y, \lambda)$, $F(y, \lambda)$, $G(y, \lambda) \in C(H)$, there exist $a_2 \in A(i(y, \lambda), \lambda)$, $b_2 \in B(y, \lambda)$, $c_2 \in C(y, \lambda)$, $d_2 \in D(y, \lambda)$, $e_2 \in E(y, \lambda)$, $f_2 \in F(y, \lambda)$ and $g_2 \in G(y, \lambda)$ such that

$$\begin{aligned}
\|a_1 - a_2\| &\leq H(A(i(x, \lambda), \lambda), A(i(y, \lambda), \lambda)), \\
\|b_1 - b_2\| &\leq H(B(x, \lambda), B(y, \lambda)), \\
\|c_1 - c_2\| &\leq H(C(x, \lambda), C(y, \lambda)), \\
\|d_1 - d_2\| &\leq H(D(x, \lambda), D(y, \lambda)), \\
\|e_1 - e_2\| &\leq H(E(x, \lambda), E(y, \lambda)), \\
\|f_1 - f_2\| &\leq H(F(x, \lambda), F(y, \lambda)), \\
\|g_1 - g_2\| &\leq H(G(x, \lambda), G(y, \lambda)).
\end{aligned} \tag{3.9}$$

Let

$$\begin{aligned}
v &= y - (g_2 - m(y, \lambda)) + R_{M(\cdot, f_2, \lambda), \rho}^h[h(g_2 - m(y, \lambda)) \\
& - \rho W(j(e_2, \lambda), a_2, \lambda) + \rho N(b_2, c_2, d_2, \lambda) + \rho w],
\end{aligned}$$

then we have $v \in Q(y, \lambda)$. It follows from (3.7) and Lemma 2.1 that

$$\begin{aligned}
 \|u - v\| &= \|x - (g_1 - m(x, \lambda)) + R_{M(\cdot, f_1, \lambda), \rho}^h[h(g_1 - m(x, \lambda)) \\
 &\quad - \rho W(j(e_1, \lambda), a_1, \lambda) + \rho N(b_1, c_1, d_1, \lambda) + \rho w] \\
 &\quad - [y - (g_2 - m(y, \lambda)) + R_{M(\cdot, f_2, \lambda), \rho}^h(h(g_2 - m(y, \lambda)) \\
 &\quad - \rho W(j(e_2, \lambda), a_2, \lambda) + \rho N(b_2, c_2, d_2, \lambda) + \rho w)]\| \\
 &\leq \|x - y - (g_1 - m(x, \lambda)) - (g_2 - m(y, \lambda))\| \\
 &\quad + \|R_{M(\cdot, f_1, \lambda), \rho}^h[h(g_1 - m(x, \lambda)) - \rho W(j(e_1, \lambda), a_1, \lambda) + \rho N(b_1, c_1, d_1, \lambda) + \rho w] \\
 &\quad - R_{M(\cdot, f_1, \lambda), \rho}^h[h(g_2 - m(y, \lambda)) - \rho W(j(e_2, \lambda), a_2, \lambda) + \rho N(b_2, c_2, d_2, \lambda) + \rho w]\| \\
 &\quad + \|R_{M(\cdot, f_1, \lambda), \rho}^h[h(g_2 - m(y, \lambda)) - \rho W(j(e_2, \lambda), a_2, \lambda) + \rho N(b_2, c_2, d_2, \lambda) + \rho w] \\
 &\quad - R_{M(\cdot, f_2, \lambda), \rho}^h[h(g_2 - m(y, \lambda)) - \rho W(j(e_2, \lambda), a_2, \lambda) + \rho N(b_2, c_2, d_2, \lambda) + \rho w]\| \\
 &\leq \|x - y - (g_1 - m(x, \lambda)) - (g_2 - m(y, \lambda))\| \\
 &\quad + \frac{1}{r} \|x - y - (h(g_1 - m(x, \lambda)) - h(g_2 - m(y, \lambda)))\| \\
 &\quad + \frac{1}{r} \|x - y + \rho(N(b_1, c_1, d_1, \lambda) - N(b_2, c_2, d_1, \lambda))\| \\
 &\quad + \frac{\rho}{r} \|N(b_2, c_2, d_1, \lambda) - N(b_2, c_2, d_2, \lambda)\| \\
 &\quad + \frac{\rho}{r} \|W(j(e_1, \lambda), a_1, \lambda) - W(j(e_2, \lambda), a_2, \lambda)\| + \mu \|f_1 - f_2\| \\
 &\leq \left(1 + \frac{1}{r}\right) [\|x - y - (g_1 - g_2)\| + \|m(x, \lambda) - m(y, \lambda)\|] \\
 &\quad + \frac{1}{r} \|g_1 - m(x, \lambda) - (g_2 - m(y, \lambda)) - (h(g_1 - m(x, \lambda)) - h(g_2 - m(y, \lambda)))\| \\
 &\quad + \frac{1}{r} \|x - y + \rho(N(b_1, c_1, d_1, \lambda) - N(b_2, c_2, d_1, \lambda))\| \\
 &\quad + \frac{\rho}{r} \|N(b_2, c_2, d_1, \lambda) - N(b_2, c_2, d_2, \lambda)\| \\
 &\quad + \frac{\rho}{r} \|W(j(e_1, \lambda), a_1, \lambda) - W(j(e_2, \lambda), a_2, \lambda)\| + \mu \|f_1 - f_2\|. \tag{3.10}
 \end{aligned}$$

Since G is δ -strongly monotone and L_G -Lipschitz continuous in first argument, we have

$$\|x - y - (g_1 - g_2)\|^2 = \|x - y\|^2 - \langle g_1 - g_2, x - y \rangle + \|g_1 - g_2\|^2 \leq (1 - 2\delta + L_G^2) \|x - y\|^2.$$

It follows that

$$\|x - y - (g_1 - g_2)\| \leq \sqrt{1 - 2\delta + L_G^2} \|x - y\|. \tag{3.11}$$

By the Lipschitz continuity of m in first argument, we have

$$\|m(x, \lambda) - m(y, \lambda)\| \leq L_m \|x - y\|. \tag{3.12}$$

Since h is r -strongly monotone and L_h -Lipschitz continuous, we have

$$\begin{aligned} & \|g_1 - m(x, \lambda) - (g_2 - m(y, \lambda)) - (h(g_1 - m(x, \lambda)) - h(g_2 - m(y, \lambda)))\|^2 \\ &= \|g_1 - g_2 - (m(x, \lambda) - m(y, \lambda))\|^2 + \|h(g_1 - m(x, \lambda)) - h(g_2 - m(y, \lambda))\|^2 \\ &\quad - 2\langle h(g_1 - m(x, \lambda)) - h(g_2 - m(y, \lambda)), g_1 - m(x, \lambda) - (g_2 - m(y, \lambda)) \rangle \\ &\leq (1 - 2r + L_h^2) \|g_1 - g_2 - (m(x, \lambda) - m(y, \lambda))\|^2 \\ &\leq (1 - 2r + L_h^2) (\|g_1 - g_2\|^2) + 2\langle g_1 - g_2, m(y, \lambda) - m(x, \lambda) \rangle + \|m(x, \lambda) - m(y, \lambda)\|^2 \\ &\leq (1 - 2r + L_h^2) (L_G^2 + 2L_G L_m + L_m^2) \|x - y\|^2. \end{aligned}$$

It follows that

$$\begin{aligned} & \| (g_1 - m(x, \lambda)) - (g_2 - m(y, \lambda)) - (h(g_1 - m(x, \lambda)) - h(g_2 - m(y, \lambda))) \| \\ & \leq (L_G + L_m) \sqrt{(1 - 2r + L_h^2)} \|x - y\|. \end{aligned} \quad (3.13)$$

Since B and C are Lipschitz continuous in first argument, $N(\cdot, \cdot, \cdot)$ is $L_{(N,1)}$ -Lipschitz continuous and γ -relaxed Lipschitz continuous with respect to B in first argument, and $N(\cdot, \cdot, \cdot)$ is $L_{(N,2)}$ -Lipschitz continuous and σ -generalized pseudo-contractive with respect to C in second argument, we have

$$\begin{aligned} & \|x - y + \rho(N(b_1, c_1, d_1, \lambda) - N(b_2, c_2, d_1, \lambda))\|^2 \\ &= \|x - y\|^2 + 2\rho \langle N(b_1, c_1, d_1, \lambda) - N(b_2, c_1, d_1, \lambda), x - y \rangle \\ &\quad + 2\rho \langle N(b_2, c_1, d_1, \lambda) - N(b_2, c_2, d_1, \lambda), x - y \rangle + \rho^2 [\|N(b_1, c_1, d_1, \lambda) - N(b_2, c_1, d_1, \lambda)\| \\ &\quad + \|N(b_2, c_1, d_1, \lambda) - N(b_2, c_2, d_1, \lambda)\|]^2 \\ &\leq \|x - y\|^2 - 2\rho\gamma \|x - y\|^2 + 2\rho\sigma \|x - y\|^2 + \rho^2 (L_{(N,1)} L_B + L_{(N,2)} L_C)^2 \|x - y\|^2 \\ &\leq (1 - 2\rho(\gamma - \sigma)) + \rho^2 \beta^2 (L_{(N,1)} L_B + L_{(N,2)} L_C)^2 \|x - y\|^2. \end{aligned}$$

It follows that

$$\begin{aligned} & \|x - y + \rho(N(b_1, c_1, d_1, \lambda) - N(b_2, c_2, d_1, \lambda))\| \\ & \leq \sqrt{1 - 2\rho(\gamma - \sigma) + \rho^2 (L_{(N,1)} L_B + L_{(N,2)} L_C)^2} \|x - y\|. \end{aligned} \quad (3.14)$$

Since B is Lipschitz continuous in first argument and N is Lipschitz continuous in third argument, we have

$$\|N(b_2, c_2, d_1, \lambda) - N(b_2, c_2, d_2, \lambda)\| \leq L_{(N,3)} L_D \|x - y\|. \quad (3.15)$$

By the Lipschitz continuity of W, j, A, E and i in first argument, we have

$$\begin{aligned} & \|W(j(e_1, \lambda), a_1, \lambda) - W(j(e_2, \lambda), a_2, \lambda)\| \\ & \leq \|W(j(e_1, \lambda), a_1, \lambda) - W(j(e_2, \lambda), a_1, \lambda)\| + \|W(j(e_2, \lambda), a_1, \lambda) - W(j(e_2, \lambda), a_2, \lambda)\| \\ & \leq (L_{(W,1)} L_j L_E + L_{(W,2)} L_A L_i) \|x - y\|. \end{aligned} \quad (3.16)$$

Since F is Lipschitz continuous in first argument, we have

$$\|f_1 - f_2\| \leq H(F(x, \lambda), F(y, \lambda)) \leq L_F \|x - y\|. \quad (3.17)$$

By (3.10)–(3.17), we obtain

$$\begin{aligned} \|u - v\| &\leq \left[\left(1 + \frac{1}{r}\right) (\sqrt{1 - 2\delta + L_G^2} + L_m) + \frac{L_G + L_m}{r} \sqrt{1 - 2r + L_h^2} \right. \\ &\quad + \frac{1}{r} \sqrt{1 - 2\rho(\gamma - \sigma) + \rho^2(L_{(N,1)}L_B + L_{(N,2)}L_C)^2} \\ &\quad \left. + \frac{\rho}{r} (L_{(N,3)}L_D + L_{(W,1)}L_JL_E + L_{(W,2)}L_AL_i) + \mu L_F \right] \|x - y\| \\ &= (k + t(\rho)) \|x - y\| = \theta \|x - y\|, \end{aligned} \quad (3.18)$$

where

$$\begin{aligned} k &= \left(1 + \frac{1}{r}\right) (\sqrt{1 - 2\delta + L_G^2} + L_m) + \frac{L_G + L_m}{r} \sqrt{1 - 2r + L_m^2} + \mu L_F, \\ t(\rho) &= \frac{1}{r} [\sqrt{1 - 2\rho(\gamma - \sigma) + \rho^2(L_{(N,1)}L_B + L_{(N,2)}L_C)^2} + \rho(L_{(N,3)}L_D \\ &\quad + L_{(W,1)}L_JL_E + L_{(W,2)}L_AL_i)], \end{aligned}$$

and $\theta = k + t(\rho)$. It follows from condition (3.8) that $\theta < 1$. Hence we have

$$d(u, Q(y, \lambda)) = \inf_{v \in Q(y, \lambda)} \|u - v\| \leq \theta \|x - y\|.$$

Since $u \in F(x, \lambda)$ is arbitrary, we obtain

$$\sup_{u \in Q(x, \lambda)} d(u, Q(y, \lambda)) \leq \theta \|x - y\|.$$

By using same argument, we can prove

$$\sup_{v \in Q(y, \lambda)} d(Q(x, \lambda), v) \leq \theta \|x - y\|.$$

By the definition of the Hausdorff metric \tilde{H} on $C(H)$, we obtain that for all $(x, y, \lambda) \in H \times H \times \Omega$,

$$\tilde{H}(Q(x, \lambda), Q(y, \lambda)) \leq \theta \|x - y\|,$$

i.e., $Q(x, \lambda)$ is a set-valued contractive mapping which is uniform with respect to $\lambda \in \Omega$. By a fixed point theorem of Nadler [32], for each $\lambda \in \Omega$, $Q(x, \lambda)$ has a fixed point $x = x(\lambda) \in H$, i.e., $x = x(\lambda) \in Q(x, \lambda)$. By the definition of Q , there exist $a = a(x, \lambda) \in A(i(x, \lambda), \lambda)$, $b = b(x, \lambda) \in B(x, \lambda)$, $c = c(x, \lambda) \in C(x, \lambda)$, $d = d(x, \lambda) \in D(x, \lambda)$, $e = e(x, \lambda) \in E(x, \lambda)$, $f = f(x, \lambda) \in F(x, \lambda)$, $g = g(x, \lambda) \in G(x, \lambda)$ such that

$$g = m(x, \lambda) + R_{M(\cdot, f, \lambda), \rho}^h [h(g - m(x, \lambda)) - \rho W(j(e, \lambda), a, \lambda) + \rho N(b, c, d, \lambda) + \rho w].$$

By Theorem 3.1, $x(\lambda) \in S(\lambda)$ is a solution of the PCGMIQVIP (2.1) and so $S(\lambda)$ is nonempty for each $\lambda \in \Omega$. \square

Theorem 3.4. Let $A, B, C, D, E, F, G, W, N, M, h, m, i, j$ and Ω be as in Theorem 3.3. Further assume

(i) for any $x \in H$, $A(x, \lambda), B(x, \lambda), C(x, \lambda), D(x, \lambda), E(x, \lambda), F(x, \lambda), G(x, \lambda), h(x, \lambda), m(x, \lambda), i(x, \lambda), j(x, \lambda)$, are Lipschitz continuous (or continuous) in second arguments with Lipschitz constants $\ell_A, \ell_B, \ell_C, \ell_D, \ell_E, \ell_F, \ell_G, \ell_h, \ell_m, \ell_i$ and ℓ_j respectively,

(ii) for any $b, c, d, f, t \in H, \lambda \mapsto N(b, c, d, \lambda), \lambda \mapsto W(a, c, \lambda)$, and $\lambda \mapsto R_{M(\cdot, f, \lambda), \rho}^h(t)$ are Lipschitz continuous (or continuous) with Lipschitz constants ℓ_N, ℓ_W and ℓ_R , respectively,

(iii) Conditions (3.7) and (3.8) in Theorem 3.3 are satisfied.

Then solution set $S(\lambda)$ of the PCGMIQVIP (2.1) is a Lipschitz continuous (or continuous) mapping from Ω to H .

Proof. For each $\lambda, \bar{\lambda} \in \Omega$, by Theorem 3.3, $S(\lambda)$ and $S(\bar{\lambda})$ are both nonempty closed subsets of H . By the proof of Theorem 3.3, $Q(x, \lambda)$ and $Q(x, \bar{\lambda})$ are both set-valued contractive mappings with same contractive constant $\theta \in (0, 1)$. By Lemma 2.1 of Lim [29], we obtain

$$\tilde{H}(S(\lambda), S(\bar{\lambda})) \leq \frac{1}{1 - \theta} \sup_{x \in H} \tilde{H}(Q(x, \lambda), Q(x, \bar{\lambda})). \quad (3.19)$$

For any $u \in Q(x, \lambda)$, there exists $a = a(x, \lambda) \in A(i(x, \lambda), \lambda), b = b(x, \lambda) \in B(x, \lambda), c = c(x, \lambda) \in C(x, \lambda), d = d(x, \lambda) \in D(x, \lambda), e(x, \lambda) \in E(x, \lambda), f = f(x, \lambda) \in F(x, \lambda)$, and $g = g(x, \lambda) \in G(x, \lambda)$ such that

$$u = x - (g - m(x, \lambda)) + R_{M(\cdot, f, \lambda), \rho}^h[h(g - m(x, \lambda), \lambda) - \rho W(j(e, \lambda), a, \lambda) + \rho N(b, c, d, \lambda) + \rho w].$$

It is easy to see that there exist $\bar{a} = a(x, \bar{\lambda}) \in A(i(x, \bar{\lambda}), \bar{\lambda}), \bar{b} = b(x, \bar{\lambda}) \in B(x, \bar{\lambda}), \bar{c} = c(x, \bar{\lambda}) \in C(x, \bar{\lambda}), \bar{d} = d(x, \bar{\lambda}) \in D(x, \bar{\lambda}), \bar{e} = e(x, \bar{\lambda}) \in E(x, \bar{\lambda}), \bar{f} = f(x, \bar{\lambda}) \in F(x, \bar{\lambda})$ and $\bar{g} = g(x, \bar{\lambda}) \in G(x, \bar{\lambda})$ such that

$$\begin{aligned} \|a - \bar{a}\| &\leq \tilde{H}(A(i(x, \lambda), \lambda), A(i(x, \bar{\lambda}), \bar{\lambda})), \\ \|b - \bar{b}\| &\leq \tilde{H}(B(x, \lambda), B(x, \bar{\lambda})), \quad \|c - \bar{c}\| \leq \tilde{H}(C(x, \lambda), C(x, \bar{\lambda})), \\ \|d - \bar{d}\| &\leq \tilde{H}(D(x, \lambda), D(x, \bar{\lambda})), \quad \|e - \bar{e}\| \leq \tilde{H}(E(x, \lambda), E(x, \bar{\lambda})), \\ \|f - \bar{f}\| &\leq \tilde{H}(F(x, \lambda), F(x, \bar{\lambda})), \quad \|g - \bar{g}\| \leq \tilde{H}(G(x, \lambda), G(x, \bar{\lambda})). \end{aligned}$$

Let

$$v = x - (\bar{g} - m(x, \bar{\lambda})) + R_{M(\cdot, \bar{f}, \bar{\lambda}), \rho}^h[h(\bar{g} - m(x, \bar{\lambda})) - \rho W(j(\bar{e}, \bar{\lambda}), \bar{a}, \bar{\lambda}) + \rho N(\bar{b}, \bar{c}, \bar{d}, \bar{\lambda}) + \rho w],$$

and

$$z = h(\bar{g} - m(x, \bar{\lambda})) - \rho W(j(\bar{e}, \bar{\lambda}), \bar{a}, \bar{\lambda}) + \rho N(\bar{b}, \bar{c}, \bar{d}, \bar{\lambda}) + \rho w.$$

Then $v \in Q(x, \bar{\lambda})$. It follows that

$$\begin{aligned}
 \|u - v\| &\leq \|g - \bar{g}\| + \|m(x, \lambda) - m(x, \bar{\lambda})\| \\
 &\quad + \|R_{M(\cdot, f, \lambda), \rho}^h[h(g - m(x, \lambda)) - \rho W(j(e, \lambda), a, \lambda) \\
 &\quad + \rho N(b, c, d, \lambda) + \rho w] - R_{M(\cdot, \bar{f}, \bar{\lambda}), \rho}^h(z)\| \\
 &\leq \|g - \bar{g}\| + \|m(x, \lambda) - m(x, \bar{\lambda})\| \\
 &\quad + \|R_{M(\cdot, f, \lambda), \rho}^h[h(g - m(x, \lambda)) - \rho W(j(e, \lambda), a, \lambda) \\
 &\quad + \rho N(b, c, d, \lambda) + \rho w] - R_{M(\cdot, f, \lambda), \rho}^h(z)\| \\
 &\quad + \|R_{M(\cdot, f, \lambda), \rho}^h(z) - R_{M(\cdot, \bar{f}, \bar{\lambda}), \rho}^h(z)\| + \|R_{M(\cdot, \bar{f}, \bar{\lambda}), \rho}^h(z) - R_{M(\cdot, \bar{f}, \bar{\lambda}), \rho}^h(z)\| \\
 &\leq \left(1 + \frac{L_h}{r}\right) (\|g - \bar{g}\| + \|m(x, \lambda) - m(x, \bar{\lambda})\|) \\
 &\quad + \frac{\rho}{r} \|W(j(e, \lambda), a, \lambda) - W(j(\bar{e}, \bar{\lambda}), \bar{a}, \bar{\lambda})\| \\
 &\quad + \frac{\rho}{r} \|N(b, c, d, \lambda) - N(\bar{b}, \bar{c}, \bar{d}, \bar{\lambda})\| + \mu \|f - \bar{f}\| + \ell_R \|\lambda - \bar{\lambda}\|. \tag{3.20}
 \end{aligned}$$

By Lipschitz continuity of G and m in second arguments, we have

$$\|g - \bar{g}\| \leq H(G(x, \lambda), G(x, \bar{\lambda})) \leq \ell_G \|\lambda - \bar{\lambda}\|, \tag{3.21}$$

$$\|m(x, \lambda) - m(x, \bar{\lambda})\| \leq \ell_m \|\lambda - \bar{\lambda}\|. \tag{3.22}$$

By the Lipschitz continuity of W in first, second, and third arguments, j and i in first and second arguments, and E and A in second argument, we have

$$\begin{aligned}
 &\|W(j(e, \lambda), a, \lambda) - W(j(\bar{e}, \bar{\lambda}), \bar{a}, \bar{\lambda})\| \\
 &\leq [L_{(W,1)}(L_j \ell_E + \ell_j) + L_{(W,2)}(L_A L_i \ell_E + L_A \ell_i + \ell_A) + \ell_W] \|\lambda - \bar{\lambda}\|. \tag{3.23}
 \end{aligned}$$

By Lipschitz continuity of $N(b, c, d, \lambda)$ in first, second, third, fourth arguments and the Lipschitz continuity of B, C, D in second argument, we have

$$\|N(b, c, d, \lambda) - N(\bar{b}, \bar{c}, \bar{d}, \bar{\lambda})\| \leq [L_{(N,1)} \ell_B + L_{(N,2)} \ell_C + L_{(N,3)} \ell_D + \ell_N] \|\lambda - \bar{\lambda}\|. \tag{3.24}$$

By Lipschitz continuity of F in the second argument, we have

$$\|f - \bar{f}\| \leq \tilde{H}(F(x, \lambda), F(x, \bar{\lambda})) \leq \ell_F \|\lambda - \bar{\lambda}\|. \tag{3.25}$$

follows from (3.20)–(3.25) that

$$\|u - v\| \leq M \|\lambda - \bar{\lambda}\|,$$

where

$$\begin{aligned}
 M &= \left(1 + \frac{L_h}{r}\right) (\ell_G + \ell_m) + \frac{\rho}{r} [L_{(W,1)}(L_j \ell_E + \ell_j) + L_{(W,2)}(L_A L_i \ell_E + L_A \ell_i + \ell_A) + \ell_W \\
 &\quad + L_{(N,1)} \ell_B + L_{(N,2)} \ell_C + L_{(N,3)} \ell_D + \ell_N] + \mu \ell_F + \ell_R.
 \end{aligned}$$

Hence we obtain

$$\sup_{u \in Q(x, \lambda)} d(u, F(x, \bar{\lambda})) \leq M \|\lambda - \bar{\lambda}\|.$$

By using similar argument as above, we can obtain

$$\sup_{v \in F(x, \bar{\lambda})} d(F(x, \lambda), v) \leq M \|\lambda - \bar{\lambda}\|.$$

It follows that

$$\tilde{H}(Q(x, \lambda), Q(x, \bar{\lambda})) \leq M \|\lambda - \bar{\lambda}\| \quad \forall (x, \lambda), (x, \bar{\lambda}) \in H \times \Omega.$$

By Lemma 2.2, we obtain

$$H(S(\lambda), S(\bar{\lambda})) \leq \frac{M}{1 - \theta} \|\lambda - \bar{\lambda}\|.$$

This proves that $S(\lambda)$ is Lipschitz continuous in $\lambda \in \Omega$. If, each mapping in conditions (i) and (ii) is assumed to be continuous in $\lambda \in \Omega$, then by similar argument as above, we can show that $S(\lambda)$ is also continuous in $\lambda \in \Omega$. \square

Remark 3.2. The PCGMIVIP (2.1) includes the parametric problems (2.2)–(2.8) and many parametric (generalized) quasi-variational inclusions and parametric (generalized) nonlinear implicit quasi-variational inequalities in [1,2,4,6–20, 22–28,30,31, 33–42,44–48] as special cases. Theorems 3.1–3.4 improve and generalize the corresponding known results in [1,2,4,6–20, 22–28,30,31, 33–42,44–48]. As special cases, we also can obtain the corresponding sensitivity analysis results of the parametric problems (2.2)–(2.8) and other parametric forms of the variational inclusions and the quasi-variational inequalities considered in [1,2,4,6–20, 22–28,30,31, 33–42,44–48].

References

- [1] S. Adly, Perturbed algorithms and sensitivity analysis for a general class of variational inclusions, *J. Math. Anal. Appl.* 201 (1996) 609–630.
- [2] R.P. Agarwal, Y.J. Cho, N.J. Huang, Sensitivity analysis for strongly nonlinear quasi-variational inclusions, *Appl. Math. Lett.* 13 (6) (2000) 19–24.
- [3] C. Baiocchi, A. Capelo, *Variational and Quasi-variational Inequalities*, Wiley, New York, 1984.
- [4] S. Dafermos, Sensitivity analysis in variational inequalities, *Math. Oper. Res.* 13 (1988) 421–434.
- [5] M. Deluca, A. Maugeri, Quasi-variational inequalities and applications to the traffic equilibrium problem: discussion of a paradox, *J. Comput. Appl. Math.* 28 (1989) 163–171.
- [6] X.P. Ding, Perturbed proximal point algorithm for generalized quasivariational inclusions, *J. Math. Anal. Appl.* 210 (1997) 88–101.
- [7] X.P. Ding, On a class of generalized nonlinear implicit quasivariational inclusions, *Appl. Math. Mech.* 20 (10) (1999) 1087–1098.
- [8] X.P. Ding, On generalized mixed variational-like inequalities, *J. Sichuan Normal Univ.* 22 (5) (1999) 494–503.
- [9] X.P. Ding, Generalized implicit quasivariational inclusions with fuzzy set-valued mappings, *Comput. Math. Appl.* 38 (1) (1999) 71–79.
- [10] X.P. Ding, Algorithm of solutions for mixed implicit quasi-variational inequalities with fuzzy mappings, *Comput. Math. Appl.* 38 (5–6) (1999) 231–241.

- [11] X.P. Ding, Generalized quasi-variational-like inclusions with nonconvex functions, *Appl. Math. Comput.* 122 (2001) 267–282.
- [12] X.P. Ding, Perturbed Ishikawa type iterative algorithm for generalized quasivariational inclusions, *Appl. Math. Comput.* 141 (2003) 359–373.
- [13] X.P. Ding, Existence and algorithm of solutions for nonlinear mixed variational-like inequalities in Banach spaces, *J. Comput. Appl. Math.* 157 (2003) 419–434.
- [14] X.P. Ding, Existence and algorithm of solutions for generalized mixed implicit quasivariational inequalities, mixed implicit quasi-variational inclusions, *Appl. Math. Comput.* 148 (1) (2004) 47–66.
- [15] X.P. Ding, Sensitivity analysis for generalized nonlinear implicit quasi-variational inclusions, *Appl. Math. Lett.* 17 (2) (2004) 225–235.
- [16] X.P. Ding, C.L. Lou, Existence and algorithm for solving some generalized mixed variational inequalities, *Comput. Math. Appl.* 37 (3) (1999) 23–30.
- [17] X.P. Ding, C.L. Lou, On parametric generalized quasivariational inequalities, *J. Optim. Theory Appl.* 100 (1) (1999) 195–205.
- [18] X.P. Ding, C.L. Lou, Perturbed proximal point algorithms for general quasi-variational-like inclusions, *J. Comput. Appl. Math.* 113 (1–2) (2000) 153–165.
- [19] X.P. Ding, J.Y. Park, A new class of generalized nonlinear implicit quasivariational inclusions with fuzzy mappings, *J. Comput. Appl. Math.* 138 (2002) 243–257.
- [20] Y.P. Fang, N.J. Huang, H -monotone operator and resolvent operator technique for variational inclusions, *Appl. Math. Comput.* 145 (2003) 795–803.
- [21] F. Giannessi, A. Maugeri, *Variational Inequalities and Network Equilibrium Problems*, Plenum Press, New York, 1995.
- [22] P.T. Harker, J.S. Pang, Finite-dimensional variational inequality and nonlinear complementarity problems: a survey of theory, algorithms, and applications, *Math. Programming* 48 (1990) 161–220.
- [23] A. Hassouni, A. Moudafi, A perturbed algorithm for variational inclusions, *J. Math. Anal. Appl.* 185 (1994) 706–721.
- [24] N.J. Huang, Generalized nonlinear variational inclusions with noncompact set-valued mappings, *Appl. Math. Lett.* 9 (3) (1996) 25–29.
- [25] N.J. Huang, Mann and Ishikawa type perturbed iterative algorithms for nonlinear implicit quasi-variational inclusions, *Comput. Math. Appl.* 35 (10) (1998) 1–7.
- [26] N.J. Huang, C.X. Deng, Auxiliary principle and iterative algorithms for generalized set-valued strongly nonlinear mixed variational inequalities, *J. Math. Anal. Appl.* 256 (2001) 345–359.
- [27] N.J. Huang, A new completely general class of variational inclusions with noncompact set-valued mappings, *Comput. Math. Appl.* 35 (10) (1998) 9–14.
- [28] K.R. Kazmi, Mann and Ishikawa perturbed iterative algorithms for generalized quasivariational inclusions, *J. Math. Anal. Appl.* 209 (1997) 572–587.
- [29] T.C. Lim, On fixed point stability for set-valued contractive mappings with application to generalized differential equations, *J. Math. Anal. Appl.* 110 (1985) 436–441.
- [30] Z. Liu, L. Debnath, S.M. Kang, J.S. Ume, Sensitivity analysis for parametric completely generalized nonlinear implicit quasivariational inclusions, *J. Math. Anal. Appl.* 277 (1) (2003) 142–154.
- [31] R.N. Mukherjee, H.L. Verma, Sensitivity analysis of generalized variational inequalities, *J. Math. Anal. Appl.* 167 (1992) 299–304.
- [32] S.B. Nadler, Multivalued contraction mappings, *Pacific J. Math.* 30 (1969) 475–485.
- [33] A. Nagureay, *Network Economics: A Variational Inequality Approach*, Kluwer Academic Publishers, Boston, 1999.
- [34] M.A. Noor, An iterative scheme for a class of quasivariational inequalities, *J. Math. Anal. Appl.* 110 (1985) 462–468.
- [35] M.A. Noor, Sensitivity analysis for quasi-variational inequalities, *J. Optim. Theory Appl.* 95 (1997) 399–407.
- [36] M.A. Noor, Generalized set-valued variational inclusions and resolvent equations, *J. Math. Anal. Appl.* 228 (1998) 206–220.
- [37] M.A. Noor, Algorithms for general monotone mixed variational inequalities, *J. Math. Anal. Appl.* 229 (1999) 330–343.
- [38] M.A. Noor, Sensitivity analysis framework for general quasi-variational inclusions, *Comput. Math. Appl.* 44 (2002) 1175–1181.
- [39] M.A. Noor, E.A. Al-Said, Iterative methods for generalized nonlinear variational inequalities, *Comput. Math. Appl.* 33 (8) (1997) 1–11.
- [40] M.A. Noor, K.I. Noor, Sensitivity analysis for quasi-variational inclusions, *J. Math. Anal. Appl.* 236 (1999) 290–299.

- [41] M.A. Noor, K.I. Noor, T.M. Rassias, Set-valued resolvent equations and mixed variational inequalities, *J. Math. Anal. Appl.* 220 (1998) 741–759.
- [42] P.D. Panagiotopoulos, G.E. Stavroulakis, New type of variational principles based on the notion of quasidifferentiability, *Acta Mech.* 94 (1992) 171–194.
- [43] D. Pascali, S. Sburlan, *Nonlinear Mappings of Monotone Type*, Sijthoff & Noordhoff, Romania, 1978.
- [44] S.M. Robinson, Sensitivity analysis for variational inequalities by normal-map technique, in: F. Giannessi, A. Maugeri (Eds.), *Variational Inequalities and Network Equilibrium Problems*, Plenum Press, New York, 1995, pp. 257–269.
- [45] L.U. Uko, Strongly nonlinear generalized equations, *J. Math. Anal. Appl.* 220 (1998) 65–76.
- [46] R.U. Verma, On generalized variational inequalities, *J. Math. Anal. Appl.* 213 (1997) 387–392.
- [47] R.U. Verma, I.P.L.R.D. Base, Iterative algorithms for variational inequalities and associated nonlinear equations involving relaxed Lipschitz operators, *Appl. Math. Lett.* 9 (4) (1996) 61–63.
- [48] N.D. Yen, Lipschitz continuity of solutions of variational inequalities with a parametric polyhedral constraint, *Math. Oper. Res.* 20 (1995) 695–708.