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Stochastic Seismic Response Analysis of Base-Isolated High-rise Buildings

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Abstract

Stochastic seismic responses of a base-isolated high-rise building subjected to non-stationary random seismic earthquake ground motion are investigated by combining the pseudo excitation method and equivalent linearization method, for which the superstructure is modeled as a multi-degree-of-freedom system considering the shear-flexural effect, the hysteretic restoring forces of the isolators are described by the Bouc-Wen differential equation model which has a good performance on the transition from elastic responses to plastic responses. With the linearization for such a Wen's model, a first order differential equation will be obtained, which will couple with the governing equations of the isolated structure and so make up the closed-form expressions for base-isolated high-rise buildings, thus the stochastic seismic responses of the simplified systems will be obtained conveniently. Consequently, the solution of the stochastic seismic responses of a base-isolated structure considering hysteretic nonlinear behavior is transformed to a deterministic step-by-step integration problem. The precision of the pseudo excitation method is verified by Monte-Carlo simulation, the results for a 17-storey frame with height:width ratio=5.1 show that the isolation technique greatly reduces the inter-storey drifts and absolute accelerations of the superstructure for high-rise buildings and that the responses are substantially underestimated if the flexural effect is neglected.

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Keywords: non-stationary; random excitation; pseudo-excitation method; equivalent linearization method; base-isolated high-rise buildings

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1. INTRODUCTION

Seismic isolation has emerged as one of the most promising techniques for reducing destructive effects on structures caused by strong earthquakes. Because of the uncertainty of earthquake, the seismic stochastic analysis is a powerful tool in earthquake engineering, and attracted interests of many people in the past decades. Reference (Jangid and Datta 1995) studied the influence of the eccentricity of a superstructure on the responses of a base-isolated structure, the stochastic responses were obtained through solving the Lyapunov equation, for which the computational efficiency quickly decreases when the number of degrees of freedom of the structure increase. Reference (Wang and Lin 2001) studied the random responses of shear-type multi-degree of freedom (MDOF) hysteretic system by utilizing pseudo excitation method (PEM) and equivalent linearization method (ELM). Reference (Du et al. 2006) extended this method to analyze the responses of base-isolated structures under stationary random excitation; however, it was found that the flexural effect cannot be neglected when the ratio of height to width of the superstructure exceeds 4. In this paper, the responses of a base-isolated high-rise building due to seismic non-stationary random excitations are investigated, in which the superstructure is simplified as a MDOF system considering the shear-flexural effect with the hysteric restoring forces of the isolators described by the Bouc-Wen differential equation model. The main objectives of this paper are to: (i) present a method to analyze the dynamic responses of base-isolated high-rise buildings under seismic non-stationary random excitations; (ii) verify the validity of the isolation technology to such buildings; (iii) investigate the influence of the flexural effect to the responses of such structures.

2. Equivalent stiffness coefficients of the simplified MDOF system

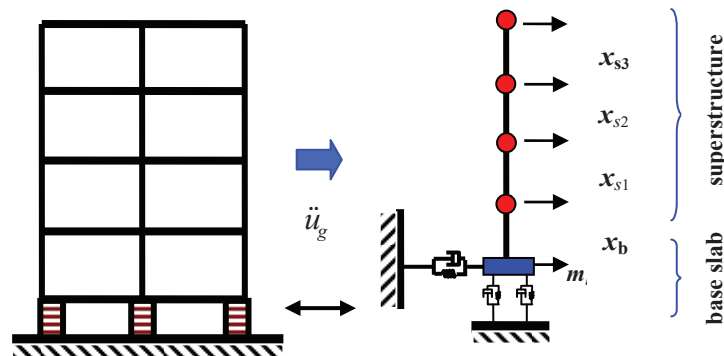


Fig.1: Mathematical model of a base-isolated high-rise building

Fig.1 shows the general elevation of a base-isolated high-rise building consisting of n_0 stories, the superstructure of the isolated system will be modeled as a MDOF system considering shear-flexural effects. The vertical stiffness of the isolators is assumed to be rigid. The equilibrium equation of the frame can be expressed as:

$$\mathbf{K}_0 \mathbf{U}_0 = \mathbf{F}_0 \quad (1)$$

The equilibrium equation of the simplified MDOF can be expressed as:

$$\mathbf{K}_s \mathbf{U}_s = \mathbf{F}_s \quad (2)$$

in which \mathbf{U}_s and \mathbf{F}_s can be evaluated from \mathbf{U}_0 and \mathbf{F}_0 , respectively, \mathbf{K}_s is a $n_0 \times n_0$ symmetric matrix with $n_0(n_0+1)/2$ unknown coefficients, which will be evaluated in terms of \mathbf{U}_s and \mathbf{F}_s (Sun et al. 1995).

3. Governing equations of motion for the isolation system

The governing equation of the motion for the superstructure subjected to seismic ground acceleration \ddot{u}_g can be expressed as (Naeim and Kelly 1999):

$$\mathbf{M}_s(\ddot{\mathbf{x}}_s + \mathbf{r}_s\ddot{u}_g + \mathbf{r}_s\ddot{x}_b) + \mathbf{C}_s\dot{\mathbf{x}}_s + \mathbf{K}_s\mathbf{x}_s = 0 \quad (3)$$

where \mathbf{M}_s , \mathbf{C}_s and \mathbf{K}_s represent the $n_0 \times n_0$ mass, damping and stiffness matrices of the superstructure, respectively; \mathbf{x}_s is the n_0 -vector of displacements relative to the base slab, and x_b is the relative displacement of the base slab to the ground; $\mathbf{r}_s = \{1, 1, \dots, 1\}^T$ is the n_0 -dimensional influence coefficient vector. Assuming the superstructure will keep elastic during the earthquake, thus mode superposition scheme can be employed,

$$\mathbf{x}_s = \sum_{i=1}^N q_i \phi_i = \Phi \mathbf{q} \quad (4)$$

where N is the number of the modes contributed to the responses, ϕ_i is the i th shape vector of size $n_0 \times 1$; q_i is modal coordinate; Φ is made up of ϕ_i , and satisfies

$$\Phi^T \mathbf{M}_s \Phi = \mathbf{I} \quad (5)$$

in which \mathbf{I} is an identity matrix, thus Eq. (3) can be replaced by the following equation

$$\bar{\mathbf{M}}_s \ddot{\mathbf{q}} + \bar{\mathbf{C}}_s \dot{\mathbf{q}} + \bar{\mathbf{K}}_s \mathbf{q} = -\mathbf{L}(\ddot{u}_g + \ddot{x}_b) \quad (6)$$

where

$$\begin{aligned} \bar{\mathbf{M}}_s &= \Phi^T \mathbf{M}_s \Phi = \mathbf{I}, \quad \bar{\mathbf{C}}_s = \Phi^T \mathbf{C}_s \Phi = \text{diag}(2\zeta\omega_1, 2\zeta\omega_2, \dots, 2\zeta\omega_N), \\ \bar{\mathbf{K}}_s &= \Phi^T \mathbf{K}_s \Phi = \text{diag}(\omega_1^2, \omega_2^2, \dots, \omega_N^2), \quad \mathbf{L} = \Phi^T \mathbf{M}_s \mathbf{r}_s \end{aligned} \quad (7)$$

The governing equation of motion of base slab can be expressed as

$$m_b(\ddot{x}_b + \ddot{u}_g) + c_b\dot{x}_b + \alpha_0 k_u x_b + (1 - \alpha_0)k_u z + \mathbf{r}_s^T \mathbf{M}_s (\ddot{\mathbf{x}}_s + \mathbf{r}_s\ddot{x}_b + \mathbf{r}_s\ddot{u}_g) = 0 \quad (8)$$

where α_0 represents post to pre-yielding stiffness ratio; m_b , c_b and k_u represent the mass, the damping and the pre-yielding stiffness of the base slab, respectively, and c_b is given by

$$c_b = 2\zeta_b \sqrt{k_d M_{sum}}, \quad k_d = \alpha_0 k_u, \quad M_{sum} = \mathbf{r}_s^T \mathbf{M}_s \mathbf{r}_s + m_b \quad (9)$$

where M_{sum} represents total mass of the superstructure and base slab, z is a hysteric component, a function of the time history of x_b , z is related to x_b through the following first-order non-linear differential equation.

$$\dot{z} = A\dot{x}_b - \frac{1}{D_y} (\gamma |\dot{x}_b| |z|^{n-1} + \beta \dot{x}_b |z|^n) \quad (10)$$

in which γ and β control the shape of the hysteric loop, A and D_y (yielding displacement) control the restoring force amplitude, and n controls the smoothness of the transition from elastic responses to the plastic responses. With ELM method, the above equation is given by (Wen 1980):

$$\dot{z} + c_e \dot{x}_b + k_e z = 0 \quad (11)$$

with,

$$c_e = \frac{1}{D_y} \sqrt{\frac{2}{\pi}} \left(\gamma \frac{E[\dot{x}_b z]}{\sigma_{\dot{x}_b}} + \beta \sigma_z \right) - A, \quad k_e = \frac{1}{D_y} \sqrt{\frac{2}{\pi}} \left(\gamma \sigma_{\dot{x}_b} + \beta \frac{E[\dot{x}_b z]}{\sigma_z} \right) \tag{12}$$

Eqs. (6) and (8) can be compacted as

$$\bar{\mathbf{M}}\ddot{\mathbf{y}} + \bar{\mathbf{C}}\dot{\mathbf{y}} + \bar{\mathbf{K}}\mathbf{y} + \begin{Bmatrix} (1-\alpha_0)k_u \\ \mathbf{0} \end{Bmatrix} z = -\bar{\mathbf{M}}\mathbf{r}\ddot{u}_g \tag{13}$$

where

$$\bar{\mathbf{M}} = \begin{bmatrix} M_{sum} & \mathbf{L}^T \\ \mathbf{L} & \bar{\mathbf{M}}_s \end{bmatrix}, \quad \bar{\mathbf{C}} = \begin{bmatrix} c_b & \mathbf{0} \\ \mathbf{0} & \bar{\mathbf{C}}_s \end{bmatrix}, \quad \bar{\mathbf{K}} = \begin{bmatrix} \alpha_0 k_u & \mathbf{0} \\ \mathbf{0} & \bar{\mathbf{K}}_s \end{bmatrix}, \tag{14}$$

$$\mathbf{y} = \{x_b, \mathbf{q}^T\}^T, \quad \mathbf{r} = \{1, 0, 0, \dots, 0\}^T$$

In which \mathbf{r} is vector of order $N+1$.

A state variable \mathbf{V} of size $(2N+3) \times 1$ will be introduced as,

$$\mathbf{V} = \{\mathbf{y}^T, \dot{\mathbf{y}}^T, z\}^T = \{x_b, \mathbf{q}^T, \dot{x}_b, \dot{\mathbf{q}}^T, z\}^T \tag{15}$$

Thus Eqs. (11) and (13) can be replaced by the following first-order differential equations

$$\dot{\mathbf{V}} = \mathbf{H}\mathbf{V} - \mathbf{f}(t) = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} \\ -\bar{\mathbf{M}}^{-1}\bar{\mathbf{K}} & -\bar{\mathbf{M}}^{-1}\bar{\mathbf{C}} & \mathbf{a} \\ \mathbf{0} & \mathbf{b} & -\mathbf{k}_e \end{bmatrix} \mathbf{V} - \begin{Bmatrix} \mathbf{0} \\ \mathbf{r} \\ \mathbf{0} \end{Bmatrix} \ddot{u}_g \tag{15}$$

with \mathbf{a}, \mathbf{b} of order $N+1$

$$\mathbf{a} = -\bar{\mathbf{M}}^{-1} \{(1-\alpha_0)k_u, 0, 0, \dots, 0\}^T, \quad \mathbf{b} = \{-c_e, 0, 0, \dots, 0\}^T \tag{16}$$

4. Non-stationary stochastic analysis by PEM

4.1. Model of earthquake excitation

In the present study, the power spectral density function (PSDF) of the earthquake excitation, $\ddot{u}_f(\omega)$, is considered as that suggested by Clough and Penzien,

$$S_{\ddot{u}_f}(\omega) = S_0 \frac{1 + 4\xi_g^2 (\omega / \omega_g)^2}{\left(1 - (\omega / \omega_g)^2\right)^2 + 4\xi_g^2 (\omega / \omega_g)^2} \frac{(\omega / \omega_f)^4}{\left(1 - (\omega / \omega_f)^2\right)^2 + 4\xi_f^2 (\omega / \omega_f)^2} \tag{17}$$

where S_0 is the constant PSDF of input white-noise; ω_g and ξ_g generally represent the pre-dominant frequency and damping ratio of the soil strata, respectively; ω_f and ξ_f are the ground filter parameters. Thus the earthquake excitation can be expressed by

$$S_{\ddot{u}_g}(\omega, t) = |g(t)|^2 S_{\ddot{u}_f}(\omega) \tag{18}$$

in which $g(t)$ is the deterministic modulating function, and can be given by

$$g(t) = \begin{cases} (t/t_1)^2 & 0 < t \leq t_1 \\ 1 & t_1 < t \leq t_2 \\ \exp[c(t-t_2)] & t > t_2 \end{cases} \quad (19)$$

where t_1 and t_2 denotes the time for start and end of the strong motion duration, respectively; c describes the intensity attenuation of the input PSDF.

4.2. PEM computational procedure

(1) The pseudo excitation at the instant t_j is constituted as

$$\ddot{u}_g(\omega, t_j) = \sqrt{S_{\ddot{u}_g}(\omega, t_j)} \exp(i\omega t_j) \quad (20)$$

c_e and k_e will be given an initial value at the instant $t=0$.

(2) Substituting $\ddot{u}_g(\omega, t_j)$ into Eq.(16), the corresponding non-stationary random vibration responses analysis is transformed into ordinary direct dynamic analysis, high-precision integration of mixed type precise integration method (PIM) is adopted due to its high computational efficiency (Lin et al. 2005), thus, $\mathbf{\bar{V}}(\omega_i, t_j)$ can be evaluated at a series of frequency points $\omega_i (i=1, 2, 3, \dots, N_0)$.

$$\begin{aligned} S_{\dot{x}_b z}(\omega_i, t_j) &= \dot{x}_b^*(\omega_i, t_j) z(\omega_i, t_j), S_{zz}(\omega_i, t_j) = z^*(\omega_i, t_j) z(\omega_i, t_j), \\ S_{\dot{x}_b \dot{x}_b}(\omega_i, t_j) &= \dot{x}_b^*(\omega_i, t_j) \dot{x}_b(\omega_i, t_j) \end{aligned} \quad (21)$$

$$\begin{aligned} E[\dot{x}_b z] &= 2 \int_0^{+\infty} S_{\dot{x}_b z} d\omega = \sum_{i=1}^{N_0} \dot{x}_b^*(\omega_i, t_j) z(\omega_i, t_j) \delta_\omega \\ \sigma_z^2 &= 2 \int_0^{+\infty} S_{zz} d\omega = \sum_{i=1}^{N_0} z^*(\omega_i, t_j) z(\omega_i, t_j) \delta_\omega, \end{aligned} \quad (22)$$

$$\sigma_{\dot{x}_b}^2 = 2 \int_0^{+\infty} S_{\dot{x}_b \dot{x}_b} d\omega = \sum_{i=1}^{N_0} \dot{x}_b^*(\omega_i, t_j) \dot{x}_b(\omega_i, t_j) \delta_\omega$$

where $\#^*$ denotes the complex conjugate of $\#$. When the corresponding responses become convergent, replace t_j by t_{j+1} , and repeat steps (1) and (2) for the next time step. Eqs. (12), (16), (17), (22) and (23) make up the closed-form expressions of the isolated system.

5. Example results

The stochastic responses of a 17-storey two-span frame structure under non-stationary random excitation is evaluated, with the storey-height $h=3.6\text{m}$, span length $d=6\text{m}$, cross section of reinforced concrete columns and beams, $0.8\text{m} \times 0.8\text{m}$ and $0.6\text{m} \times 0.75\text{m}$, respectively. The mass of base slab is $60 \times 10^3\text{kg}$, the storey mass is $45 \times 10^3\text{kg}$ for each of storeys 1-5 and $30 \times 10^3\text{kg}$ for each of storeys 6-17. The damping ratio of the superstructure and isolators is 0.05 and 0.20, respectively. The design period $T_d = 4\text{s}$, $\alpha_0 = 0.1$, $D_y = 0.01\text{m}$, and $S_0 = 0.02\text{m}^2/\text{s}^3$, parameters of ground filter and deterministic modulating function are listed in Table 1.

The root mean square (RMS) displacement responses of the base slab evaluated by PEM are compared with the Monte-Carlo simulation in Fig.2. Obviously, the agreement is very good, thus the accuracy of PEM is verified. Fig.3 shows the influences of the stiffness of the superstructure to the peak RMS absolute acceleration. Shear-flexural type and shear type stiffness evaluated by D-value method of the superstructure is employed in case1 and case2, respectively. The peak RMS absolute acceleration of the superstructure in the two cases is shown in Fig. 3, the differences of RMS enlarger with the height of the superstructure increasing. The RMS of the storey drift and absolute acceleration of base-isolated and base-fixed structures are compared in Fig.4. Apparently, isolation technology greatly decreases the responses of the superstructure. The probability of the third storey (the easiest storey to go into plastic) remaining elastic is 75.6%, which agrees with the assumption previously, and the probability of the LRB not being damaged by uplift is 70.8%.

Table 1: parameters of ground filter and deterministic modulating function

$\omega_g(\text{rad/s})$	ξ_g	$\omega_f(\text{rad/s})$	ξ_f	$t_1(\text{s})$	$t_2(\text{s})$	c
15.7	0.6	1.57	0.6	6	12	-0.18

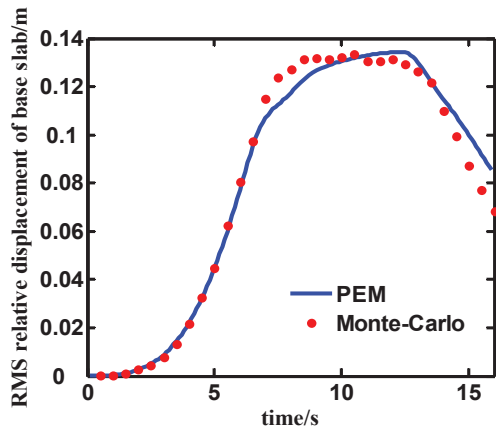


Fig. 2: Comparison of RMS relative displacement of base slab evaluated by PEM and Monte-Carlo.

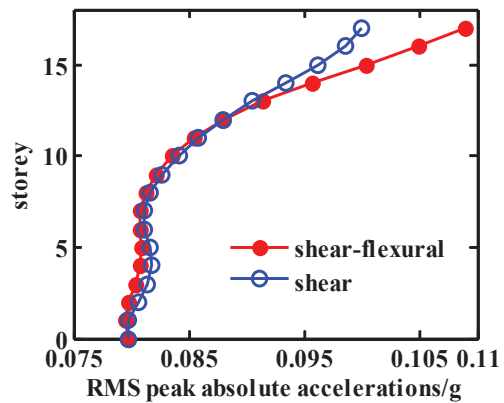
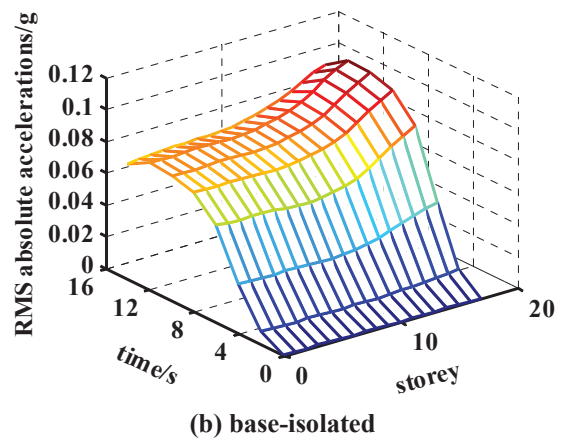
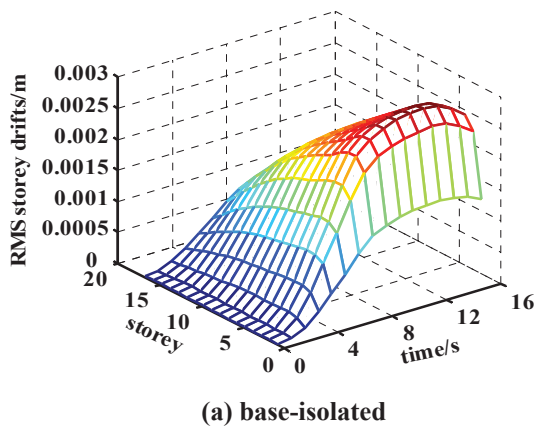


Fig. 3: Comparison of the absolute storey accelerations of the superstructure given by shear-flexural model with those given by the shear-type MDOF model.



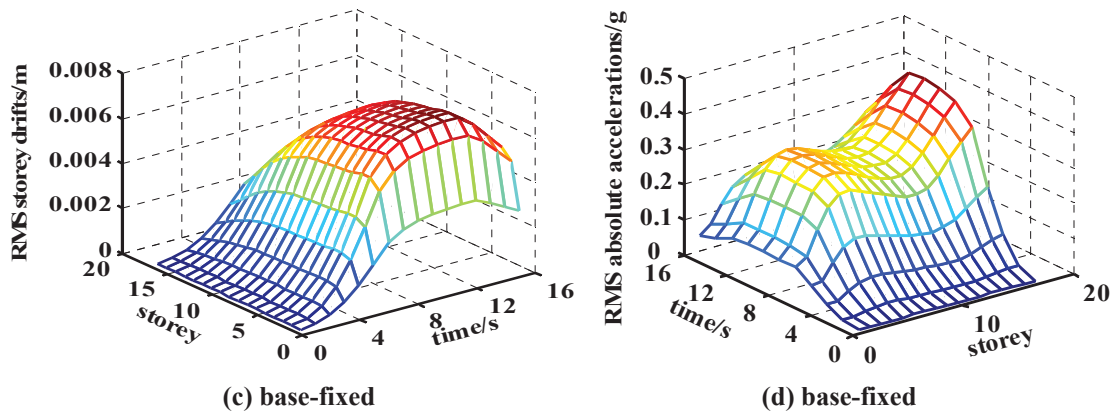


Fig. 4: Comparison of RMS storey drifts and absolute storey accelerations of the base-isolated and base-fixed structures.

6. CONCLUSIONS

Combining PEM and ELM, a method for analyzing the stochastic responses of base-isolated high-rise buildings is proposed; it's proved that isolation technology has great potential in decreasing the inter-storey drifts and absolute accelerations of the superstructure even for high-rise buildings, and the absolute accelerations of such buildings may be underestimated if the flexural effect is neglected.

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