Application of High-order Panel Method in Static Aeroelastic Analysis of Aircraft

Yaokun Wang\textsuperscript{a*}, Zhiqiang Wan\textsuperscript{a}, Chao Yang\textsuperscript{a}

\textsuperscript{a}School of Aeronautic Science and Engineering, Beijing University of Aeronautics and Astronautics, Beijing 100191, China

Abstract

A static aeroelastic analysis method based on the high-order panel method and modal method has been recommended through two examples of a high-aspect-ratio wing and a low-aspect-ratio wing. The subsonic aerodynamic forces of a high-aspect-ratio wing and the subsonic-supersonic aerodynamic forces of a low-aspect-ratio wing are investigated in this study. The influences of elastic structural deformation on aerodynamic forces are studied with an emphasis on the aerodynamic coefficients, wing root loads, structural deformation and pressure distribution of different sections, and these results are compared with the results from low-order panel method and the results based on experimental aerodynamic forces. It is concluded that aerodynamic forces can be accurately calculated with the high-order panel method. The method presented in this study is feasible, credible and efficient. Comprehensive static aeroelastic characteristics can be provided by the method for initial phases of aircraft design.

© 2011 Published by Elsevier Ltd. Selection and/or peer-review under responsibility of Kunming University of Science and Technology Open access under CC BY-NC-ND license.

Keywords: static aeroelasticity; flight loads; high-order panel method; modal method; experimental aerodynamic forces

1. Introduction

It is not appropriate in modern aircraft design to ignore the influences of the static structural deformation on flight loads, static stability, controlling derivatives and other flight performance. Therefore, static aeroelastic analysis is very important which considering deformation effect. Accurate predictions of static aeroelastic characteristics, which are especially important in the early design stage of aircraft, can be used to avoid revising the structural design repeatedly in the late phase of aircraft design.

* Corresponding author. Tel.: +86-10-8231-6034.
E-mail address: yaokun54@yahoo.com.cn.
To obtain accurate static aeroelastic analysis results, accurate aerodynamic distribution and precise structural stiffness data are critical.

It has become more common that the static aeroelastic analysis is based on linear aerodynamic forces calculated with the low-order panel method in the early design stage of aircraft. This analysis method is highly efficient and easy to use, but the analysis results are rough. And the method of using CFD aerodynamic forces or experimental aerodynamic forces is unrealistic because of great time consumption. Therefore, it is necessary to develop an analysis method for aerodynamic forces which is highly efficient and is easy to use and precise, and thus develop fast static aeroelastic analysis.

The high-order panel method can achieve a higher accuracy than the low-order panel method. The aerodynamic model is based on three-dimensional solid geometry and can be constructed with CAD software. Therefore, complex aircraft configurations can be modeled. With the high-order panel method, the aerodynamic load distribution of the leading and trailing edges of the wing can approximate the actual pressure distribution well, and the flow field can be accurately predicted. As a result, this method can provide precise aerodynamic forces in the early design stage of aircraft. Meanwhile, it is a good choice to combine the high-order panel method with the modal method for conducting aeroelastic analysis in the early phase of aircraft design.

The method for static aeroelastic analysis based on the high-order panel method and modal method has some application abroad, however, its domestic application is relatively less. Literature [11] proposed the method but only applied to subsonic, low aspect ratio wing calculations. In this paper, the application of the high-order method is expended through a subsonic aerodynamic analysis case of a high-aspect wing and sub/supersonic aerodynamic analysis case of a low-aspect wing, meanwhile the static aeroelasticity has been analyzed based on the high-order method and the modal method. In comparison with the results from the low-order method and CFD, the feasibility and reliability is verified.

2. Methodology

2.1. Aerodynamic forces analysis of high-order panel method

The small disturbance potential equation is solved in the subsonic state as follows[11].

\[
(1 - M_\infty^2) \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0
\]  
(1)

where \( M_\infty \) is the freestream Mach number, and \( \phi \) is the velocity potential. Supersonic equation for small disturbance potential flow is similar, namely:

\[
(M_\infty^2 - 1) \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0
\]  
(2)

The compressible direction of the flow is inherently along the x-axis of the aerodynamic coordinate system, as shown in Figure 1.
The high-order lifting surface method is not required to pre-fix the force point location. Rather, the determination of the force point and control point locations in a high-order lifting surface is a result of substantial numerical simulations on a large collection of wing platforms and flow conditions[10]. In the three-dimensional aerodynamic model based on the aircraft configuration, a constant source singularity is distributed on each panel, and a doublet singularity is located at each grid point. In addition, these doublet singularities are also linearly distributed over the panels that surround the grid. This type of linear doublet distribution, which is shown in Figure 2, is called an elementary singularity distribution. At each panel, two boundary conditions are imposed to solve for the source and doublet strength: the Neumann boundary condition \( \frac{\partial \phi}{\partial n} = -V_n \cdot \vec{n} \) and the Dirichlet boundary condition \( \phi_L = 0 \). In addition, the zero-force condition \( \frac{\partial \phi}{\partial x} = 0 \) is imposed on the wake to satisfy the wake condition. The pressure coefficient, aerodynamic derivatives and so on can be obtained by solving the small disturbance potential equation based on these boundary conditions[10].

### 2.2. Aerodynamic forces analysis of low-order panel method

The low-order panel method based on the linear aerodynamic potential theory spends less time, and can be better combined with the analytical method of structure finite element, and applied to solve sub/supersonic problems. The method is widely used in engineering analysis of aeroelastic, but the accuracy is limited.

### 2.3. Aerodynamic forces analysis based on the experimental aerodynamics

The method based on extensive test aerodynamic forces considers nonlinear wind-tunnel aerodynamic forces as rigid aerodynamic forces, the non-linear variation of parameters such as aerodynamic forces vs. angle of attack is included. Based on abundant experimental data, the method has a high calculation precision and a rapid analysis, and has already been put into engineering practice.

### 2.4. Modal method

On the assumption that the elastic accelerations are negligible compared to the rigid-body maneuver accelerations, the static aeroelastic equation can be written as two equations: one for the rigid body and the other for the elastic motion:

\[
M_{RR} \ddot{\xi}_R = F_{AR}
\]
Eq. (3) can be solved to obtain the aerodynamic trim parameters, including the angle of attack, sideslip angle, control surface deflections, and rolling rates, to trim the aircraft to a maneuver prescribed by the rigid-body accelerations. Elastic effects are included in the generalized force vector. Eq. (4) can be solved to obtain the elastic deformations pertaining to the aerodynamic trim parameters and flight conditions, and wind-tunnel data can be imported for more accurate analysis.

2.5. Method for the static aeroelastic response analysis

The aerodynamic forces on a rigid aircraft are a function of the trim variables, i.e.,

\[ F_{AR} = F_{AR}(\alpha, \beta, p, q, r, \delta, ...) \]  

(5)

The incremental elastic forces can be related to the structural deformation with the aerodynamic influence coefficient matrix as follows:

\[ F_{AE} = q_e G^T A G u = q_e A u \]  

(6)

According to the above discussion, the equation used to analyze the static aeroelastic responses can be written as follows:

\[ K u + M \phi_R \ddot{u}_R = F_{AR} + F_{AE} \]  

(7)

3. Aerodynamic models of flexible wings

A high-aspect-ratio wing and a low-aspect-ratio wing are investigated in this study. The aerodynamic models of the high-aspect-ratio wing and the low-aspect-ratio wing are shown in Figures 2 (a) and (b), respectively.

4. Analysis and comparison of aerodynamic forces

4.1. Aerodynamic forces Analysis of the high-aspect-wing

The aerodynamic forces of the high-aspect-ratio wing in a fixed state are studied based on the high-order panel method, the state is Ma 0.4, attack angle 8°. The pressure distribution at 45% spanwise location by the high-order panel method, low-order panel method and experimental aerodynamic forces are shown in Figure 3.
Figure 3 shows that the results are reasonably well correlated with the wind-tunnel test data, even at the wing leading and trailing edges of the wing, and the error is acceptable for the early phase of aircraft design. Compared with the low-order panel method, the high-order panel method conforms with experimental results better, especially in the trailing edge position.

4.2. Subsonic aerodynamic forces analysis of low aspect ratio wing

The aerodynamic forces of the low-aspect-ratio wing in a fixed state are studied based on the high-order panel method. The state are Ma0.6, attack angle 6°, and Ma 1.45, attack angle 6°, respectively. The pressure distribution at 58% spanwise location by the high-order panel method, low-order panel method and experimental aerodynamic forces are shown in Figure 4.

It is clearly that in subsonic state, the results from the high-level panel method are reasonably well correlated with the wind-tunnel test data and the error of the results from the low-level panel method is correspondingly large.

As for supersonic state, there are some differences between the results from the high-order panel method and wind-tunnel test, but the overall trend still maintain a good consistency. Curve can be drawn from the comparison. Low-order panel method results in a big error, which has some limitations in the calculation of supersonic aerodynamic forces.
5. Static aeroelastic analysis

5.1. static aeroelastic analysis for high-aspect-ratio wing

The high-order panel method combined with the modal method can solve the static aeroelastic problem. The first ten elastic structural modes of the high-aspect-ratio wing are used in the static aeroelastic analysis. The wing tip deformation for Mach 0.4 and an attack angle of 4° is calculated based on aerodynamic forces of high-order panel method, and it is compared with the results from experimental aerodynamic forces and aerodynamic forces of low-order panel method. The experimental aerodynamic forces is set as 100%, as shown in Figure 5 and Table 1.

![Aerodynamic model for showing structural deformation](image)

**Table.1 contrast of wing tip deformation**

<table>
<thead>
<tr>
<th>Aerodynamic Method</th>
<th>Wing tip deformation/%</th>
<th>relative error/%</th>
</tr>
</thead>
<tbody>
<tr>
<td>High-order panel method</td>
<td>94.44</td>
<td>-5.56</td>
</tr>
<tr>
<td>Low-order panel method</td>
<td>113.36</td>
<td>13.36</td>
</tr>
</tbody>
</table>

The results show that wing-tip deformation calculated from high-order panel method is consistency with the results of experimental aerodynamic forces. It illustrates that there is little difference in aerodynamic pressure distribution over the wing surface for the aerodynamic forces based on high-order panel method and experimental aerodynamic method.

The load distribution along the span is the important contents in static aeroelastic characteristics of high-aspect-ratio wing. The lift coefficient vs. the angle of attack for Mach 0.4 of the wing is presented in this study. The variation of shear force, bending moment and torsion moment along the span for Mach 0.4 and an attack angle of 4° is investigated, the influence of elastic structural deformation is put into consideration. The result is compared with the results of experimental aerodynamic forces and the low-order panel method, as shown in Figure 6 to Figure 7.

![Lift coefficient vs angle of attack](image)

Figure 6. Lift coefficient vs angle of attack (elastic, Ma0.4)
The result is non-dimensional, and the benchmark is that the shear force is divided by \(mg\), the torsion moment by \(mgc\), and the bending moment by \(mgl\), where \(m\) is the mass of the wing, \(g\) is acceleration of gravity, \(l\) is the reference span of the wing, and \(c\) is the reference chord.

The results show that the coefficient varies linearly with attack angle in elastic state, the difference between the lift coefficient calculated based on the high-order panel method and wind-tunnel data is small. However, the difference between the lift coefficient calculated based on the high-order panel method and experimental aerodynamic forces is a little large.

The variation of shear force, bending moment and torsion moment along the span for an attack angle of 4° show that both the result calculated based on the high-order panel method and the low-order panel method are well correlated with the one of wind-tunnel test data, and meet the load distribution trend of the high-aspect-ratio wing.

### 5.2. Sub/supersonic static aeroelastic analysis for low-aspect-ratio wing

The static aeroelastic characteristics of the wing for Mach 0.6, an attack angle of 6°, and Mach 1.45, an attack angle of 6° is investigated, including the shear force, bending moment and torsion moment of the wing root. The result is compared with aerodynamic forces based on low-order panel method and experimental aerodynamic forces, set the experimental aerodynamic forces as 100%. As shown in Table 2 to Table 4.

#### Table 2. Contrast of shear force

<table>
<thead>
<tr>
<th>Mach</th>
<th>Aerodynamic Method</th>
<th>Shear force/%</th>
<th>Relative error/%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>High-order panel method</td>
<td>102.93</td>
<td>2.93</td>
</tr>
<tr>
<td></td>
<td>Low-order panel method</td>
<td>105.55</td>
<td>5.55</td>
</tr>
<tr>
<td>1.45</td>
<td>High-order panel method</td>
<td>100.13</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>Low-order panel method</td>
<td>97.4</td>
<td>-2.6</td>
</tr>
</tbody>
</table>

#### Table 3. Contrast of bending moment

<table>
<thead>
<tr>
<th>Mach</th>
<th>Aerodynamic Method</th>
<th>Bending moment/%</th>
<th>Relative error/%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>High-order panel method</td>
<td>99.13</td>
<td>-0.87</td>
</tr>
<tr>
<td></td>
<td>Low-order panel method</td>
<td>106.12</td>
<td>6.12</td>
</tr>
<tr>
<td>1.45</td>
<td>High-order panel method</td>
<td>102.81</td>
<td>2.81</td>
</tr>
<tr>
<td></td>
<td>Low-order panel method</td>
<td>91.94</td>
<td>-8.06</td>
</tr>
</tbody>
</table>
Table 4. Contrast of torsion moment

<table>
<thead>
<tr>
<th>Mach</th>
<th>Aerodynamic Method</th>
<th>Torsion moment/%</th>
<th>Relative error/%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>High-order panel method</td>
<td>107.84</td>
<td>7.84</td>
</tr>
<tr>
<td></td>
<td>Low-order panel method</td>
<td>97.6</td>
<td>2.4</td>
</tr>
<tr>
<td>1.45</td>
<td>High-order panel method</td>
<td>106.15</td>
<td>6.15</td>
</tr>
<tr>
<td></td>
<td>Low-order panel method</td>
<td>85.68</td>
<td>-14.32</td>
</tr>
</tbody>
</table>

According to the comparison result among shear force, bending moment and torsion moment at wing root for an attack angle of 6° and in the subsonic states, both the result calculated based on the high-order panel method and the low-order panel method are well correlated with wind-tunnel test data; in the supersonic states, the result based on the high-order panel method is still well correlated with wind-tunnel test data. However, great error arises when the low-order panel method is employed. In the supersonic states, low-order panel method has its limitations in static aeroelastic analysis.

For the cases of high-aspect-ratio and low-aspect-ratio wing in this study, it is efficient to analyze the static aeroelastic characteristics based on both the low-order panel method and high-order panel method. The difference is mainly the modeling of aerodynamic forces. Compared with model of flat aerodynamic forces, 3D aeroelastic forces model is more complex. Time consumption can also fulfill engineering research. Of course, high-order panel method is able to get a higher calculation precision than low-order panel method. In addition, the method is also applicable to the trim analysis of the elastic aircraft, although it is used in the analysis of the fixed states for the current research.

6. Conclusion

A method for the static aeroelastic analysis of aircraft based on the high-order panel method and modal method is presented in the study. By taking a high-aspect ratio wing and a low-aspect ratio wing as example, the application field of the method is extended.

By the comparison of the results from the three methods, it can be seen as below:

- The error of the results from Low-order panel method is large, especially in the supersonic case. It is only applicable to the concept design stage or the early design stage of aircraft.
- The high-level panel method for subsonic and supersonic aerodynamic calculations with its accuracy and effectiveness can be applied to the early detailed design phase of aircraft.
- The method using experimental aerodynamic forces based on extensive wind-tunnel test dates, has higher accuracy and can be used at the late detailed design phase of aircraft.

References


