Research problems

The purpose of the research problems section is the presentation of unsolved problems in discrete mathematics. Older problems are acceptable if they are not as widely known as they deserve. Problems should be submitted using the format as they appear in the journal and sent to

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Readers wishing to make comments dealing with technical matters about a problem that has appeared should write to the correspondent for that particular problem. Comments of a general nature about previous problems should be sent to Professor Alspach.


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The following conjectures of Faigle and Sands [1] concerning finite distributive lattices \( \mathcal{D} \) of width \( w(\mathcal{D}) \) are still open:

(i) There is a universal constant \( c \) such that

\[
|\mathcal{D}| \geq c \cdot w(\mathcal{D}) \cdot \sqrt{\log w(\mathcal{D})}.
\]

(ii) The Boolean algebra \( \mathcal{B}_n \) is the unique distributive lattice of minimal size with width \( \binom{n}{n/2} \).

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Remark. Kahn and Saks [2] have proved
\[
\lim_{|\mathcal{D}| \to \infty} \frac{w(\mathcal{D})}{|\mathcal{D}|} = 0.
\]

References


Problem 137. Posed by H.-D.O.F. Gronau,

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Let \( X \) be a finite set, \( |X| = n \), and let \( \mathcal{F} \subseteq 2^X \) be a family of subsets of \( X \) such that for every pair \( Y, Z \in \mathcal{F} \) it holds that either \( Y \) and \( Z \) are complementary or \( Y \cap Z \neq \emptyset \) as well as \( Y \cup Z \neq X \). In [1] we proved
\[
2^{n-2} \leq \max|\mathcal{F}| \leq 2^{n-2} + \left( \binom{n}{[n/2]} - 1 \right),
\]
i.e.
\[
\max|\mathcal{F}| = (1 + o(1))2^{n-2}.
\]

Here is my 10-year-old conjecture.

Conjecture.
\[
\max|\mathcal{F}| = \begin{cases} \binom{n}{n/2} & \text{for } n = 2, 4, 6, 8, \\ 2^{n-2} & \text{otherwise}. \end{cases}
\]

Reference


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Let \( P \) be a partially ordered set (poset) and let \( H(P) \) denote the Hasse diagram of \( P \) (as a graph). If \( L \) is a linear extension of \( P \) then the matching \( M(P, L) = \{ e_1, e_2, \ldots, e_l \} \) in \( H(P) \), where \( e_i = \{ p_i, q_i \} \), is called sequential with respect to \( L \) if \( p_1 < p_2 < \cdots < p_l < q_1 < q_2 < \cdots < q_l \) in \( L \). It is known [1], that \( pn(P, L) \), the page number of \( P \) with respect to \( L \) is bounded from below by \( M^*(P, L) \), the cardinality of a maximum-size sequential matching (with respect to \( L \)).

Problem 138. Given \( P, L \) of \( P \), and integer \( k \). Is it true that \( M^*(P, L) \geq k \)?

Problem 139. Given \( P \) and integer \( m \), is it true that \( P \) has a linear extension \( L \) such that \( M^*(P, L) \leq m \)?

It is conjectured that Problem 138 can be answered in polynomial time whereas Problem 139 is NP-complete.

It is known that \( M^*(P, L) \) is not a tight bound for \( pn(P, L) \), and in general for \( pn(P) \).

Problem 140. Provide a family of posets for which this bound can be arbitrarily bad.

Reference