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## Thermodynamics of charged Brans–Dicke AdS black holes

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### ABSTRACT

It is well known that in four dimensions, black hole solution of the Brans–Dicke–Maxwell equations is just the Reissner–Nordstrom solution with a constant scalar field. However, in  $n \geq 4$  dimensions, the solution is not yet the  $(n + 1)$ -dimensional Reissner–Nordstrom solution and the scalar field is not a constant in general. In this Letter, by applying a conformal transformation to the dilaton gravity theory, we derive a class of black hole solutions in  $(n + 1)$ -dimensional ( $n \geq 4$ ) Brans–Dicke–Maxwell theory in the background of anti-de Sitter universe. We obtain the conserved and thermodynamic quantities through the use of the Euclidean action method. We find a Smarr-type formula and perform a stability analysis in the canonical ensemble. We find that the solution is thermally stable for small  $\alpha$ , while for large  $\alpha$  the system has an unstable phase, where  $\alpha$  is a coupling constant between the scalar and matter field.

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### 1. Introduction

The pioneering study on scalar–tensor theories was done by Brans and Dicke several decades ago who sought to incorporate Mach's principle into gravity [1]. According to Brans–Dicke (BD) theory the phenomenon of inertia arises from accelerations with respect to the general mass distribution of the universe. This theory can be regarded as an economic modification of general relativity which accommodates both Mach's principle and Dirac's large number hypothesis as new ingredients. There has been a renewed interest in studying BD theory ever since it has been disclosed that BD theory appears naturally in the low energy limit of superstring theory. In string theory, gravity becomes scalar–tensor in nature. The low-energy effective action of the string theory leads to the Einstein gravity, coupled non-minimally to a scalar dilaton field. Besides, recent observations show that at the present epoch, our Universe expands with acceleration instead of deceleration along the scheme of standard Friedmann models and since general relativity could not describe such Universe correctly, cosmologists have attended to alternative theories of gravity such as BD theory. Due to highly nonlinear character of BD theory, a desirable pre-requisite for studying strong field situation is to have

knowledge of exact explicit solutions of the field equations. Since black holes are very important both in classical and quantum gravity, many authors have investigated various aspects of them in BD theory [2]. It turned out that the dynamic scalar field in the BD theory plays an important role in the process of collapse and critical phenomenon. The first four-dimensional black hole solutions of BD theory was obtained by Brans in four classes [3]. It has been shown that among these four classes of the static spherically symmetric solutions of the vacuum BD theory of gravity only two are really independent, and only one of them is permitted for all values of  $\omega$ . It has been proved that in four dimensions, the stationary and vacuum BD solution is just the Kerr solution with constant scalar field everywhere [4]. It has also been shown that the charged black hole solution in four-dimensional Brans–Dicke–Maxwell (BDM) theory is just the Reissner–Nordstrom solution with a constant scalar field, however, in higher dimensions, one obtains the black hole solutions with a nontrivial scalar field [5]. This is because the stress–energy tensor of Maxwell field is not traceless in the higher dimensions and the action of Maxwell field is not invariant under conformal transformations. Accordingly, the Maxwell field can be regarded as the source of the scalar field in the BD theory [5]. The properties of charged black hole solutions in dilaton gravity [6–9] and BD theory [10] have been explored by many authors. However, these solutions [6–10] are neither asymptotically flat nor (anti-)de Sitter [(A)dS]. Recently, the dilaton potential leading to (A)dS-like solutions of dilaton gravity has been found [11–13]. It was shown that the cosmological constant is coupled to the dilaton in a very nontrivial way. Other studies on black

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hole solutions in BD theory have been carried out in [14–16]. In this Letter, we would like to construct black hole solutions of BDM theory in the background of (A)dS spaces in all higher dimensions for an arbitrary value of coupling constant and investigate their properties. We also want to perform a stability analysis and investigate the effect of the scalar field on the thermal stability of the solutions.

The outline of this Letter is as follows: In Section 2, we present the basic equations and the conformal transformation between the action of the dilaton gravity theory and the BD theory. In Section 3, we construct black hole solutions in BDM theory and investigate their properties. In Section 4, we study the thermodynamical properties of the solutions and calculate the conserved and thermodynamic quantities of BD black holes. We also investigate the effect of the scalar field on the thermal stability of the solutions in this section. The last section is devoted to summary and discussion.

## 2. Field equations and conformal transformations

The action of the  $(n + 1)$ -dimensional Brans–Dicke–Maxwell theory with one scalar field  $\Phi$  and a self-interacting potential  $V(\Phi)$  can be written as

$$I_G = -\frac{1}{16\pi} \int_{\mathcal{M}} d^{n+1}x \sqrt{-g} \times \left( \Phi R - \frac{\omega}{\Phi} (\nabla\Phi)^2 - V(\Phi) - F_{\mu\nu} F^{\mu\nu} \right) - \frac{1}{8\pi} \int_{\partial\mathcal{M}} d^n x \sqrt{-h} \Phi K, \quad (1)$$

where  $R$  is the scalar curvature,  $V(\Phi)$  is a potential for the scalar field  $\Phi$ ,  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  is the electromagnetic field tensor, and  $A_\mu$  is the electromagnetic potential. The factor  $\omega$  is the coupling constant. The last term in Eq. (1) is the Gibbons–Hawking boundary term which is chosen such that the variational principle is well-defined. The manifold  $\mathcal{M}$  has metric  $g_{\mu\nu}$  and covariant derivative  $\nabla_\mu$ .  $K$  is the trace of the extrinsic curvature  $K^{ab}$  of the boundary  $\partial\mathcal{M}$  of the manifold  $\mathcal{M}$ , with induced metric  $h_{ab}$ . The equations of motion can be obtained by varying the action (1) with respect to the gravitational field  $g_{\mu\nu}$ , the scalar field  $\Phi$  and the gauge field  $A_\mu$  which yields the following field equations

$$G_{\mu\nu} = \frac{\omega}{\Phi^2} \left( \nabla_\mu \Phi \nabla_\nu \Phi - \frac{1}{2} g_{\mu\nu} (\nabla\Phi)^2 \right) - \frac{V(\Phi)}{2\Phi} g_{\mu\nu} + \frac{1}{\Phi} (\nabla_\mu \nabla_\nu \Phi - g_{\mu\nu} \nabla^2 \Phi) + \frac{2}{\Phi} \left( F_{\mu\lambda} F_\nu{}^\lambda - \frac{1}{4} F_{\rho\sigma} F^{\rho\sigma} g_{\mu\nu} \right), \quad (2)$$

$$\nabla^2 \Phi = -\frac{n-3}{2(n-1)\omega + 2n} F^2 + \frac{1}{2(n-1)\omega + 2n} \times \left( (n-1)\Phi \frac{dV(\Phi)}{d\Phi} - (n+1)V(\Phi) \right), \quad (3)$$

$$\nabla_\mu F^{\mu\nu} = 0, \quad (4)$$

where  $G_{\mu\nu}$  and  $\nabla$  are, respectively, the Einstein tensor and covariant differentiation in the spacetime metric  $g_{\mu\nu}$ . It is apparent that the right-hand side of Eq. (2) includes the second derivatives of the scalar field, so it is hard to solve the field equations (2)–(4) directly. We can remove this difficulty by a conformal transformation. Indeed, the BDM theory (1) can be transformed into the Einstein–Maxwell theory with a minimally coupled scalar dilaton field,  $\bar{\Phi}$ , via the conformal transformation [5]

$$\bar{g}_{\mu\nu} = \Omega^{-2} g_{\mu\nu}, \quad (5)$$

with

$$\Omega^{-2} = \Phi^{\frac{2}{n-1}}, \quad (6)$$

and

$$\alpha = \frac{n-3}{\sqrt{4(n-1)\omega + 4n}}, \quad \bar{\Phi} = \frac{n-3}{4\alpha} \ln \Phi. \quad (7)$$

Using this conformal transformation, the action (1) transforms to

$$\bar{I}_G = -\frac{1}{16\pi} \int_{\mathcal{M}} d^{n+1}x \sqrt{-\bar{g}} \times \left( \bar{R} - \frac{4}{n-1} (\bar{\nabla}\bar{\Phi})^2 - \bar{V}(\bar{\Phi}) - e^{-\frac{4\alpha\bar{\Phi}}{n-1}} \bar{F}_{\mu\nu} \bar{F}^{\mu\nu} \right) - \frac{1}{8\pi} \int_{\partial\mathcal{M}} d^n x \sqrt{-\bar{h}} \bar{K}, \quad (8)$$

where  $\bar{R}$  and  $\bar{\nabla}$  are, respectively, the Ricci scalar and covariant differentiation in the spacetime metric  $\bar{g}_{\mu\nu}$ , and  $\bar{V}(\bar{\Phi})$  is

$$\bar{V}(\bar{\Phi}) = \Phi^{-\frac{n+1}{n-1}} V(\Phi). \quad (9)$$

This action is just the action of the  $(n + 1)$ -dimensional Einstein–Maxwell–dilaton gravity, where  $\bar{\Phi}$  is the dilaton field and  $\bar{V}(\bar{\Phi})$  is a potential for  $\bar{\Phi}$ .  $\alpha$  is an arbitrary constant governing the strength of the coupling between the dilaton and the Maxwell field. Varying action (8), we obtain the equations of motion

$$\bar{R}_{\mu\nu} = \frac{4}{n-1} \left( \bar{\nabla}_\mu \bar{\Phi} \bar{\nabla}_\nu \bar{\Phi} + \frac{1}{4} \bar{V}(\bar{\Phi}) \bar{g}_{\mu\nu} \right) + 2e^{-\frac{4\alpha\bar{\Phi}}{n-1}} \left( \bar{F}_{\mu\lambda} \bar{F}_\nu{}^\lambda - \frac{1}{2(n-1)} \bar{F}_{\rho\sigma} \bar{F}^{\rho\sigma} \bar{g}_{\mu\nu} \right), \quad (10)$$

$$\bar{\nabla}^2 \bar{\Phi} = \frac{n-1}{8} \frac{\partial \bar{V}}{\partial \bar{\Phi}} - \frac{\alpha}{2} e^{-\frac{4\alpha\bar{\Phi}}{n-1}} \bar{F}_{\rho\sigma} \bar{F}^{\rho\sigma}, \quad (11)$$

$$\bar{\nabla}_\mu \left( e^{-\frac{4\alpha\bar{\Phi}}{n-1}} \bar{F}^{\mu\nu} \right) = 0. \quad (12)$$

Comparing Eqs. (2)–(4) with Eqs. (10)–(12), we find that if  $(\bar{g}_{\mu\nu}, \bar{F}_{\mu\nu}, \bar{\Phi})$  is the solution of Eqs. (10)–(12) with potential  $\bar{V}(\bar{\Phi})$ , then

$$[g_{\mu\nu}, F_{\mu\nu}, \Phi] = \left[ \exp\left(\frac{-8\alpha\bar{\Phi}}{(n-1)(n-3)}\right) \bar{g}_{\mu\nu}, \bar{F}_{\mu\nu}, \exp\left(\frac{4\alpha\bar{\Phi}}{n-3}\right) \right], \quad (13)$$

is the solution of Eqs. (2)–(4) with potential  $V(\Phi)$ .

## 3. Brans–Dicke black holes in AdS spaces

Asymptotically (A)dS-like solutions of the dilaton field equations (10)–(12) have been constructed in [11–13,17–19]. Here we would like to obtain black hole solutions of the Brans–Dicke field equations (2)–(4) in the background of AdS universe. Our strategy for constructing these solutions is applying the conformal transformation (13) to black hole solutions of Eqs. (10)–(12) in the dilaton gravity theory. The dilaton potential leading to (A)dS-like solutions of dilaton gravity has been found recently in [12]. For an arbitrary value of  $\alpha$  in (A)dS spaces the form of the dilaton potential  $\bar{V}(\bar{\Phi})$  in  $n + 1$  dimensions is chosen as

$$\begin{aligned} \bar{V}(\bar{\Phi}) &= \frac{\Lambda(n-1)}{3(n-2+\alpha^2)^2} \\ &\times \left\{ -\alpha^2[(n+1)^2 - (n+1)\alpha^2 - 6(n+1)] \right. \\ &+ \alpha^2 + 9 \left. \right\} e^{\frac{-4(n-2)\bar{\Phi}}{(n-1)\alpha}} + (n-2)^2(n-\alpha^2)e^{\frac{4\alpha\bar{\Phi}}{n-1}} \\ &+ 4\alpha^2(n-1)(n-2)e^{\frac{-2\bar{\Phi}(n-2-\alpha^2)}{(n-1)\alpha}}. \end{aligned} \quad (14)$$

Here  $\Lambda$  is the cosmological constant. It is clear the cosmological constant is coupled to the dilaton in a very nontrivial way. This type of dilaton potential can be obtained when a higher-dimensional theory is compactified to four dimensions, including various supergravity models [20]. In the absence of the dilaton field the action (8) reduces to the action of Einstein–Maxwell gravity with cosmological constant. Asymptotically AdS black hole solutions of the field equations (10)–(12) have been obtained in [12] and we review it briefly here. Assuming the  $(n+1)$ -dimensional metric has the following form

$$d\bar{s}^2 = -f(r)dt^2 + \frac{dr^2}{g(r)} + r^2 R^2(r) d\Omega_{n-1}^2, \quad (15)$$

where  $d\Omega_{n-1}^2$  denotes the metric of an unit  $(n-1)$ -sphere and  $f(r)$ ,  $g(r)$  and  $R(r)$  are functions of  $r$  which should be determined. First of all, the Maxwell equations (12) can be integrated immediately, where all the components of  $\bar{F}_{\mu\nu}$  are zero except  $\bar{F}_{tr}$ :

$$\bar{F}_{tr} = \sqrt{\frac{f(r)}{g(r)}} \frac{qe^{\frac{4\alpha\bar{\Phi}}{n-1}}}{(rR)^{n-1}}, \quad (16)$$

where  $q$ , an integration constant, is the charge parameter of the black hole. According to the Gauss theorem, the electric charge is

$$Q = \frac{1}{4\pi} \int_{r \rightarrow \infty} \bar{F}_{tr} \sqrt{-\bar{g}} d^{n-1}x = \frac{\Omega_{n-1}}{4\pi} q, \quad (17)$$

where  $\Omega_{n-1}$  is the volume of the unit  $(n-1)$ -sphere. Notice that  $Q$  is invariant under the conformal transformation (13). Using metric (15) and the Maxwell field (16), one can show that the system of equations (10)–(11) have solutions of the form [12]

$$\begin{aligned} f(r) &= \left[ 1 - \left( \frac{r_+}{r} \right)^{n-2} \right] \left[ 1 - \left( \frac{r_-}{r} \right)^{n-2} \right]^{1-\gamma(n-2)} \\ &\quad - \frac{1}{3} \Lambda r^2 \left[ 1 - \left( \frac{r_-}{r} \right)^{n-2} \right]^\gamma, \end{aligned} \quad (18)$$

$$\begin{aligned} g(r) &= \left\{ \left[ 1 - \left( \frac{r_+}{r} \right)^{n-2} \right] \left[ 1 - \left( \frac{r_-}{r} \right)^{n-2} \right]^{1-\gamma(n-2)} \right. \\ &\quad \left. - \frac{1}{3} \Lambda r^2 \left[ 1 - \left( \frac{r_-}{r} \right)^{n-2} \right]^\gamma \right\} \left[ 1 - \left( \frac{r_-}{r} \right)^{n-2} \right]^{\gamma(n-3)}, \end{aligned} \quad (19)$$

$$\bar{\Phi}(r) = \frac{n-1}{4} \sqrt{\gamma(2+2\gamma-n\gamma)} \ln \left[ 1 - \left( \frac{r_-}{r} \right)^{n-2} \right], \quad (20)$$

$$R(r) = \left[ 1 - \left( \frac{r_-}{r} \right)^{n-2} \right]^{\gamma/2}. \quad (21)$$

Here  $r_+$  and  $r_-$  are, respectively, the event horizon and Cauchy horizon of the black hole, and the constant  $\gamma$  is

$$\gamma = \frac{2\alpha^2}{(n-2)(n-2+\alpha^2)}. \quad (22)$$

The charge parameter  $q$  is related to  $r_+$  and  $r_-$  by

$$q^2 = \frac{(n-1)(n-2)^2}{2(n-2+\alpha^2)} r_+^{n-2} r_-^{n-2}. \quad (23)$$

The quasilocal mass the dilaton AdS black hole can be calculated through the use of the subtraction method of Brown and York [21]. Such a procedure causes the resulting physical quantities to depend on the choice of reference background. According to this formalism if we write the metric of spherically symmetric spacetime in the form

$$ds^2 = -W(r)dt^2 + \frac{dr^2}{V(r)} + r^2 d\Omega_{n-1}^2, \quad (24)$$

and the matter action contains no derivatives of the metric, then the quasilocal mass is given by [22]

$$\mathcal{M} = \frac{n-1}{2} r^{n-2} W^{1/2}(r) (V_0^{1/2}(r) - V^{1/2}(r)). \quad (25)$$

Here  $V_0(r)$  is an arbitrary function which determines the zero of the energy for a background spacetime and  $r$  is the radius of the spacelike hypersurface boundary. When the spacetime is asymptotically (A)dS, the Arnowitt–Deser–Misner (ADM) mass  $M$  is the  $\mathcal{M}$  determined in (25) in the limit  $r \rightarrow \infty$ . If no cosmological horizon is present, the large  $r$  limit of (25), is used to determine the mass. If a cosmological horizon is present one cannot take the large  $r$  limit to identify the quasilocal mass. However, one can still identify the small mass parameter in the solution [21]. For the solution under consideration, there is no cosmological horizon and if we transform the metric (15) in the form (24) by using the transformation

$$r^2 R^2(r) \rightarrow r^2, \quad (26)$$

then we obtain the mass of the dilaton black hole as

$$\bar{M} = \frac{\Omega_{n-1}}{16\pi} (n-1) \left[ r_+^{n-2} + \frac{n-2-\alpha^2}{n-2+\alpha^2} r_-^{n-2} \right]. \quad (27)$$

In the absence of the dilaton field ( $\alpha = 0$ ) this expression for the mass reduces to the mass of the  $(n+1)$ -dimensional Reissner–Nordstrom–AdS black holes. It is worth noting that our result for ADM mass coincides with ones found in [23] for asymptotically flat dilaton black holes. Next we calculate the entropy of the dilatonic black hole. Black hole entropy typically satisfies the so-called area law of the entropy, which states that the entropy of the black hole is a quarter of the event horizon area [24]. This near-universal law applies to almost all kinds of black holes, including dilaton black holes, in Einstein gravity [25]. It is easy to show that the entropy of the black hole is

$$\bar{S} = \frac{\Omega_{n-1} r_+^{n-1}}{4} \left[ 1 - \left( \frac{r_-}{r_+} \right)^{n-2} \right]^{\gamma(n-1)/2}. \quad (28)$$

The Hawking temperature of the black hole on the outer horizon  $r_+$ , in dilaton gravity, can be calculated using the relation

$$\bar{T}_+ = \left( \frac{f'}{4\pi\sqrt{f/g}} \right)_{r=r_+}, \quad (29)$$

where a prime denotes derivative with respect to  $r$ . It is a matter of calculation to show that

$$\bar{T}_+ = \frac{(n-2)}{4\pi r_+} \left[ 1 - \left( \frac{r_-}{r_+} \right)^{n-2} \right]^{1-\gamma(n-1)/2}. \quad (30)$$

It is apparent that the metric corresponding to (18)–(21) is asymptotically (A)dS. Using the conformal transformation (13), the  $(n+1)$ -dimensional black hole solutions of BDM theory in the background of AdS universe can be obtained as

$$ds^2 = -U(r)dt^2 + \frac{dr^2}{V(r)} + r^2 H^2(r) d\Omega_{n-1}^2, \quad (31)$$

where  $U(r)$ ,  $V(r)$ ,  $H(r)$  and  $\Phi(r)$  are

$$U(r) = \left[ 1 - \left( \frac{r_-}{r} \right)^{n-2} \right]^{\frac{-2(n-2)\gamma}{n-3}} f(r), \quad (32)$$

$$V(r) = \left[ 1 - \left( \frac{r_-}{r} \right)^{n-2} \right]^{\frac{2(n-2)\gamma}{n-3}} g(r), \quad (33)$$

$$H(r) = \left[ 1 - \left( \frac{r_-}{r} \right)^{n-2} \right]^{\frac{-(n-1)\gamma}{2(n-3)}}, \quad (34)$$

$$\Phi(r) = \left[ 1 - \left( \frac{r_-}{r} \right)^{n-2} \right]^{\frac{(n-1)(n-2)\gamma}{(n-3)}}. \quad (35)$$

Applying the conformal transformation (13), the electromagnetic field in BDM theory can be written as

$$F_{tr} = \bar{F}_{tr} = \frac{q}{r^{n-1}}, \quad (36)$$

while the scalar potential in BDM theory becomes

$$V(\Phi) = \frac{\Lambda(n-1)}{3(n-2+\alpha^2)^2} \left\{ 4\alpha^2(n-1)(n-2)\Phi^{\frac{(3n-1)\alpha^2-(n-2)(n-3)}{2\alpha^2(n-1)}} \right. \\ \left. + (n-2)^2(n-\alpha^2)\Phi^2 \right. \\ \left. + \alpha^2[n\alpha^2 - (n-2)^2]\Phi^{\frac{(n+1)\alpha^2-(n-2)(n-3)}{\alpha^2(n-1)}} \right\}. \quad (37)$$

As one can see from Eq. (36), in the background of (A)dS universe, the scalar field in BD theory does not exert any direct influence on the matter field  $F_{tr}$ , however, the scalar field modifies the geometry of the spacetime as it participate in the field equations. This is in contrast to the solutions presented in [10,15]. The solutions in [10,15] are neither asymptotically flat nor (A)dS and the gauge field crucially depends on the scalar BD field. It is worth noting that the scalar field  $\Phi(r)$  and the electromagnetic field  $F_{tr}$  become zero as  $r \rightarrow \infty$ . It is also notable to mention that these solutions are valid for all values of  $\omega$ . When  $\omega \rightarrow \infty$  ( $\alpha = 0 = \gamma$ ), these solutions reduce to

$$U(r) = V(r) = \left[ 1 - \left( \frac{r_+}{r} \right)^{n-2} \right] \left[ 1 - \left( \frac{r_-}{r} \right)^{n-2} \right] - \frac{1}{3}\Lambda r^2, \quad (38)$$

which describes an  $(n+1)$ -dimensional asymptotically (A)dS Reissner–Nordstrom black hole.

#### 4. Thermodynamics of BD black holes

We now turn to the investigation of the thermodynamics of charged BD black holes we have just found. The Hawking temperature of BD black holes on the outer horizon  $r_+$  can be calculated using the relation

$$T_+ = \frac{\kappa}{2\pi} = \left( \frac{U'}{4\pi\sqrt{UV}} \right)_{r=r_+}, \quad (39)$$

where  $\kappa$  is the surface gravity. We obtain

$$T_+ = \frac{(n-2)}{4\pi r_+} \left[ 1 - \left( \frac{r_-}{r_+} \right)^{n-2} \right]^{1-\gamma(n-1)/2}. \quad (40)$$

If we compare Eq. (40) with the temperature obtained in the dilaton gravity theory in Eq. (30), we find that the temperature is invariant under the conformal transformation (13). This is due to the fact that the conformal parameter  $\Omega^2$  is regular at the horizon. Therefore, the Hawking temperature is an invariant quantity under conformal transformations only if the transformations are regular at event horizon.

The ADM mass  $M$ , the entropy  $S$  and the electric potential  $U$  of the BD black hole can be calculated through the use of the Euclidean action method [26,27]. In this approach, first the electric potential and the temperature are fixed on a boundary with a fixed radius  $r_+$ . The Euclidean action has two parts; bulk and surface. The first step to make the Euclidean action is to substitute  $t$  with  $i\tau$ . This makes the metric positive definite:

$$ds^2 = U(r) d\tau^2 + \frac{1}{V(r)} dr^2 + r^2 H^2(r) d\Omega_{n-1}^2. \quad (41)$$

There is a conical singularity at the horizon  $r = r_+$  in the Euclidean metric [27]. To eliminate it, the Euclidean time  $\tau$  is made periodic with period  $\beta$ , where  $\beta$  is the inverse of Hawking temperature. Now we obtain the Euclidean action of  $(n+1)$ -dimensional Brans–Dicke–Maxwell theory. The Euclidean action can be calculated analytically and continuously changing of action (1) to Euclidean time  $\tau$ , i.e.,

$$I_{GE} = -\frac{1}{16\pi} \int_{\mathcal{M}} d^{n+1}x \sqrt{g} \left( \Phi R - \frac{\omega}{\Phi} (\nabla\Phi)^2 - V(\Phi) - F_{\mu\nu} F^{\mu\nu} \right) \\ - \frac{1}{8\pi} \int_{\partial\mathcal{M}} d^n x \sqrt{h} \Phi (K - K_0), \quad (42)$$

where  $K_0$  is the trace of the extrinsic curvature of the vacuum metric background (here it is the  $(n+1)$ -dimensional AdS spacetime). This term must be added so that it can normalize the Euclidean action to zero in this spacetime [28]. Using metric (41), we find

$$R = -g^{-1/2} (g^{1/2} U' V / U)' - 2G_0^0, \quad (43)$$

$$K = -\frac{\sqrt{V}[rHU' + 2(n-1)(UH + rUH')]}{2rHU}, \quad (44)$$

where  $G_0^0$  is the (00) component of the Einstein tensor, and again the prime denotes derivative with respect to  $r$ . Inserting  $U(r)$ ,  $V(r)$  and  $H(r)$  from (32)–(34) with  $r_+ = 0 = r_-$  in Eq. (44) we obtain the extrinsic curvature for the metric background

$$K_0 = \frac{1 - n(1 - \Lambda r^2/3)}{r\sqrt{1 - \Lambda r^2/3}}. \quad (45)$$

For  $\Lambda = 0$ , we get  $K_0 = -(n-1)/r$  which is the trace of the extrinsic curvature of the metric background for asymptotically flat spacetimes [27]. Substituting Eqs. (43)–(45) in action (42) and using Eqs. (32)–(37), after a long calculation, we obtain the Euclidean action as

$$I_{GE} = \beta \frac{\Omega_{n-1}}{16\pi} (n-1) \left[ r_+^{n-2} + \frac{n-2-\alpha^2}{n-2+\alpha^2} r_-^{n-2} \right] \\ - \frac{\Omega_{n-1} r_+^{n-1}}{4} \left[ 1 - \left( \frac{r_-}{r_+} \right)^{n-2} \right]^{\gamma(n-1)/2} \\ - \beta \frac{\Omega_{n-1} q^2}{4\pi(n-2)r_+^{n-2}}. \quad (46)$$

According to Refs. [28–30], the thermodynamical potential can be given by  $I_{GE}$ , we get

$$I_{GE} = \beta M - S - \beta U Q, \quad (47)$$

where  $M$  is the ADM mass,  $S$  and  $U$  are the entropy and the electric potential, respectively. Comparing Eq. (46) with Eq. (47), we find

$$M = \frac{\Omega_{n-1}}{16\pi} (n-1) \left[ r_+^{n-2} + \frac{n-2-\alpha^2}{n-2+\alpha^2} r_-^{n-2} \right], \quad (48)$$

$$S = \frac{\Omega_{n-1} r_+^{n-1}}{4} \left[ 1 - \left( \frac{r_-}{r_+} \right)^{n-2} \right]^{\gamma(n-1)/2}, \quad (49)$$

$$U = \frac{q}{(n-2)r_+^{n-2}}. \quad (50)$$

Comparing the conserved and thermodynamic quantities calculated in this section with those obtained in the previous section, we find that they are invariant under the conformal transformation (13). This is because the Euclidean action is invariant under the conformal transformation (up to a surface term associated with the scalar field). It is worth emphasizing that in BD theory, where we have the additional gravitational scalar degree of freedom, the entropy of the black hole does not follow the area law [31]. This is due to the fact that the black hole entropy comes from the boundary term in the Euclidean action formalism. Nevertheless, the entropy remains unchanged under the conformal transformations. The advantage of the Euclidean action method is that, in principle, we can find all the thermodynamic quantities because in this method the characteristic thermodynamic function, i.e., the thermodynamical potential, is found.

Then, we consider the first law of thermodynamics for the black hole. In order to check the first law, we obtain the mass  $M$  as a function of extensive quantities  $S$  and  $Q$ . Using the expression for the charge, the mass and the entropy given in Eqs. (17), (48) and (49), we can obtain a Smarr-type formula as

$$M(S, Q) = \frac{\Omega_{n-1}}{16\pi} (n-1) \left[ Z + \frac{32Q^2(n-2-\alpha^2)\pi^2}{Z(n-1)(n-2)^2} \right], \quad (51)$$

where  $Z = r_+^{n-2}$  is the positive real root of the following equation:

$$Z^{\frac{n-1}{n-2}} \left[ 1 - \frac{32Q^2(n-2+\alpha^2)\pi^2}{Z^2(n-1)(n-2)^2} \right]^{\gamma(n-1)/2} - 4S = 0. \quad (52)$$

One may then regard the parameters  $S$  and  $Q$  as a complete set of extensive parameters for the mass  $M(S, Q)$  and define the intensive parameters conjugate to  $S$  and  $Q$ . These quantities are the temperature and the electric potential

$$T = \left( \frac{\partial M}{\partial S} \right)_Q = \left( \frac{\partial M}{\partial Z} \right)_Q \left( \frac{\partial Z}{\partial S} \right)_Q, \quad (53)$$

$$U = \left( \frac{\partial M}{\partial Q} \right)_S = \left( \frac{\partial M}{\partial Q} \right)_Z + \left( \frac{\partial M}{\partial Z} \right)_Q \left( \frac{\partial Z}{\partial Q} \right)_S. \quad (54)$$

Straightforward calculations show that the intensive quantities calculated by Eqs. (53) and (54) coincide with Eqs. (40) and (50). Thus, these thermodynamic quantities satisfy the first law of black hole thermodynamics,

$$dM = T dS + U dQ. \quad (55)$$

Finally, we study the thermal stability of the solutions in the canonical ensemble. The stability of a thermodynamic system with respect to small variations of the thermodynamic coordinates is usually performed by analyzing the behavior of the entropy  $S(M, Q)$  around the equilibrium. The local stability in any ensemble requires that  $S(M, Q)$  be a convex function of the extensive variables or its Legendre transformation must be a concave function of the intensive variables. The stability can also be studied by the behavior of the energy  $M(S, Q)$  which should be a convex function of its extensive variable. Thus, the local stability can in principle be carried out by finding the determinant of the Hessian matrix of  $M(S, Q)$  with respect to its extensive variables  $X_i$ ,  $\mathbf{H}_{X_i X_j}^M = [\partial^2 M / \partial X_i \partial X_j]$  [32,33]. In our case the mass  $M$  is

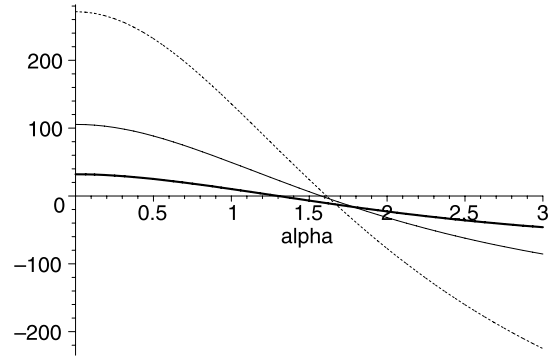


Fig. 1.  $(\partial^2 M / \partial S^2)_Q$  versus  $\alpha$  for  $r_- = 0.2$  and  $r_+ = 0.3$ ,  $n = 4$  (bold line),  $n = 5$  (continuous line), and  $n = 6$  (dashed line).

a function of entropy and charge. The number of thermodynamic variables depends on the ensemble that is used. In the canonical ensemble, the charge is a fixed parameter and therefore the positivity of the  $(\partial^2 M / \partial S^2)_Q$  is sufficient to ensure local stability. We have shown in Fig. 1 the behavior of  $(\partial^2 M / \partial S^2)_Q$  versus  $\alpha$  in various dimensions. This figure shows that for fixed value of the other parameters, the solution is thermally stable for small value of  $\alpha$  in any dimension, while it has an unstable phase for large values of  $\alpha$ . This shows that the scalar field makes the solution unstable.

## 5. Summary and discussion

The construction and analysis of the black hole solutions in anti-de Sitter (AdS) spaces is a subject of much recent interest. This interest is motivated by the correspondence between the gravitating fields in an AdS spacetime and conformal field theory on the boundary of the AdS spacetime. In this Letter, with an appropriate combination of three Liouville-type dilaton potentials and applying a conformal transformation to the dilatonic black hole solutions, we construct a class of  $(n+1)$ -dimensional ( $n \geq 4$ ) black hole solutions in BDM theory for arbitrary values of the coupling constant  $\omega$ . These solutions are asymptotically anti-de Sitter. We found the scalar potential leading to AdS-like solutions in BDM theory. The cosmological constant couples to the scalar field in a very nontrivial way, and the scalar potential has a complicated form (see Eq. (37)). This scalar potential plays a crucial role in the existence of these black holes, as the negative cosmological constant does in the Einstein–Maxwell theory. We found that the scalar field in BD theory does not exert any direct influence on the gauge field  $F_{tr}$ , however, the scalar field modifies the geometry of the spacetime as it participates in the field equations. We obtained the conserved and thermodynamic quantities through the use of the Euclidean action method, and verified that the conserved and thermodynamic quantities of the solutions satisfy the first law of black hole thermodynamics. We found that the conserved and thermodynamic quantities are invariant under the conformal transformation. We also analyzed the thermal stability of the solutions in the canonical ensemble by finding a Smarr-type formula and considering  $(\partial^2 M / \partial S^2)_Q$  for the charged BD black hole solutions in  $(n+1)$  dimensions. We found that there is no Hawking–Page phase transition in spite of charge of the BD black hole provided  $\alpha \leq \alpha_{\max}$ , while the solutions have an unstable phase for  $\alpha > \alpha_{\max}$ .

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