GEOSCIENCE FRONTIERS 2(2) (2011) 223–235

available at www.sciencedirect.com
China University of Geosciences (Beijing)
GEOSCIENCE FRONTIERS
journal homepage: www.elsevier.com/locate/gsf

ORIGINAL ARTICLE

On predicting mantle mushroom plumes

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Received 18 December 2010; accepted 21 February 2011
Available online 20 April 2011

KEYWORDS
Mantle mushroom plumes;
Sizes;
Critical times;
Transient Reynolds number

Abstract This study investigates the mechanism of formation of convection plumes of mushroom shape in sub-solidus mantle and their prediction. The seismic-tomographic images of columnar structures of several hundreds kilometers in diameter have been reported by several researchers, while the much cherished mushroom-shaped plume heads could only be found in computational geodynamics (CGD) models and simple small-scale laboratory analogue simulations. Our theory of transient instability shows that the formation of convection plumes is preceded by the onset of convection caused by unsteady-state heat conduction at the boundaries, from which filamentous plumes first appear. The plumes generated at the Core Mantle Boundary (CMB) and lithosphere rising and falling through the mantle have been predicted simply with our theory for various heat fluxes and viscosities, which still remain uncertain amongst geoscientists. The sizes of mushroom plumes in the sub-solidus mantle caused by heat fluxes of 20 and 120 mW/m^2 at the CMB are found to be 1842 km and 1173 km with critical times over 825 Myr and 334 Myr respectively. They are comparable to some large continental flood basalt provinces, and they number between 17 and 41. The thickness of the thermal boundary layers at the CMB from which convection plumes evolved are found to be 652 km and 415 km for 20 and 120 mW/m^2 respectively.

Top cooling may produce plunging plumes of diameter of 585 km and at least 195 Myr old. The number of cold plumes is estimated to be 569, which has not been observed by seismic tomography

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Peer-review under responsibility of China University of Geosciences (Beijing).
doi:10.1016/j.gsf.2011.03.001

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or as cold spots. The cold plunging plumes may overwhelm and entrap some of the hot rising plumes from CMB, so that together they may settle in the transition zone.

1. Introduction

Morgan (1971) first postulated that "hotspots are manifestations of convection in the lower mantle", and he estimated 20 deep mantle plumes with "thunderhead pattern of flow". He did not provide any theoretical equations for their predictions, nor any details of the plumes, such as their sizes and ages. The plume hypothesis is further "strengthened" and built on heuristic deductions and inference from physico-chemical models based on "speculative" laboratory geochemistry, largely imprecise seismic tomography and rather poor resolution and computational geodynamic (CGD) simulations with "realistic" mantle conditions. These claims of plumes and their interactions with their surroundings were difficult to verify, as the physical conditions of the deep mantle are extreme and are not measurable as yet. The continuing debate of the mantle plume hypothesis remained to be resolved by the emergence of the elusive mantle plumes, whilst Luguet et al. (2008) and Meibom (2008) had also shown that the so-called chemical or isotopic signatures of the "mantle plumes" may be flawed as the chemistry of Earth's mantle is too heterogeneous. Truly there is no direct evidence of mantle plumes (van der Hilst and de Hoop, 2006) of any age ascending from the deep mantle, much of the argument about interactions of the crust of the earth and plume heads and tails are imaginative deductions. Kerr (2006) has summarized the various skepticisms and doubts of various geologists over the validity of seismic tomography in representing hot mantle plumes, as there are weaknesses in the sophisticated wave analyses such as the limited number of monitoring stations, inadequate resolutions and data. More important, the sources of seismic waves from hypocenter of earthquakes reaching the receivers may not be well defined and clearly distinguishable. So far there has been no success in the detection of whole mushroom plumes complete with heads and tails much cherished by the supporters of the plume hypothesis (Campbell and Davies, 2006 Montelli et al., 2006; Campbell, 2007).

Generally, any fluid beset with an adverse density gradient is inherently unstable, for which Rayleigh (1916) had provided the basic criterion of onset of convection in a thin layer of fluid. His theory showed that the ratio of the thickness of the fluid layer d to the size or wavelength λ of the convection cell is given by $d/\lambda = \tilde{a}_c/2\pi$, where $\tilde{a}_c$ is the critical dimensionless wavenumber. Sparrow et al. (1964) showed that the value of $\tilde{a}_c$ is between about 2 and π, depending on the boundary conditions, hence, d/λ is between 0.3 and 0.5. However, convection in deep fluids will generate plumes that may traverse the depth of the fluid over several wavelengths or the diameters of the plumes before attaining their final sizes, coalescence or detachment, as have been observed in thermal experiments of Foster (1969) and Sparrow et al. (1970) and CFD simulations of Tan (1999) and CGD simulations of Olson et al. (1987), Kellogg and King (1997), Davies (1995) and Brunet and Yuen (2000). Experiments of Foster (1969), Sparrow et al. (1970) and Davenport and King (1972) showed that $d/\lambda \geq 5$ for plumes to be developed naturally to their critical sizes without any distortion. Therefore, the deep mantle of 2900 km will probably sustain a plume of diameter of about 2900/5 = 580 km, which is within the range of 200 km and 800 km plume diameters reported by Montelli et al. (2006), and it is only 50% of the size of plume of 1200 km postulated by Campbell and Davies (2006) recently.

Thermal plumes in deep mantle have been simulated by computational geodynamics (CGD) models with influence of various parameters, such as viscosity, composition and phase changes, which are based on abruptly increased surface temperatures of 200–2000 °C (Olson et al., 1987; Ji and Nataf, 1998; Kiefer and Kellogg, 1998; Montague et al., 1998; Goes et al., 2004; Ke and Solomatov, 2004; Davies, 2005). CGD simulations showed the formation of unstable thermal boundary layer up to the evolution of plumes, plume detachment and dissipation of heat in the bulk fluid (see review by Schubert et al. (2001) and Davies (2005)) identical to observations of Foster (1969) and Sparrow et al. (1970). Moreover, several numerical studies such as those of Tackley et al. (1994), Li and Romanowicz (1996), Larsen and Yuen (1997), Kiefer and Kellogg (1998), and Montague et al. (1998) have also shown that simultaneous formation of plumes generated by top cooling and bottom heating can co-exist in the mantle. They also indicated massive disruptive movement of down-welling of subduction of the oceanic lithosphere and plunging plumes which may well thwart the development of rising plumes or sweep away the protuberances of emerging plumes. Strangely there are no clear reports of cold plunging plumes in seismic-tomography studies or in field observation; instead they often report descending cold subducted plates.

Notwithstanding the strong criticism of the plume hypothesis, there seems to be "compelling" circumstantial evidence of a thermal structure below Iceland and various hot-spots. Seismic studies of Iceland plumes by Wolfe et al. (1997) revealed a narrow plume structure of diameters of 300 km at 410 km, Shen et al. (1998) 400 km at 660 km depth, while Bijwaard and Spakman (1999) reported 500 km near the D" layer at 2700 km. However, detailed reviews of some of these studies by Ritsema et al. (1999), Anderson (2000, 2001), Foulger et al. (2001), Foulger (2002), Ritsema and Allen (2003), and Zhao (2004, 2007, 2009) revealed little support of the idealized mushroom plumes beneath Iceland, instead they look more like blobs or patches of hot materials weakly resembling detached mushroom plume. Recently Montelli et al. (2006) claimed to have detected by tomography eleven plumes of diameters of 200 km–800 km at 2800 km depth, although they did not detect large mushroom plume heads and thin plume tails. Their P- and S-wave anomalies yielded very different sizes of plumes at the CMB and the mid-mantle, and some differences are as large as four folds (Table 1). The large discrepancies have cast doubt on the accuracy and validity of the tomographic analyses, especially the lower mantle and the transition zone are known to be highly heterogeneous which may cause high variation in density (Karato, 2010). The conflicting results of tomography analyses and the changing geochemistry seem not to dampen the support of the plume hypothesis.

Table 1 Plumes with large anomalous diameters from P- and S-waves (Montelli et al., 2006).

Name	P-wave anomaly		S-wave anomaly	
	Depth (km)	Diameter (km)	Depth (km)	Diameter (km)
Azores	2800	600	2800	200
Canary	2800	800	2800	200
Easter	2800	800	2800	400
Iceland	2800	800	2800	400
Kerguelen	2800	800	2800	400
Samoa	2800	400	2800	800
Bowie	650	200	1450	600
Eifel	300	200	650	400
Juan de Fuca	1000	200	1000	400

Ironically, while seismic tomography can detect the purported vertical thermal structures of several thousands of kilometers standing unmolested in a perfect quiescent mantle, seemingly quite free of the influence of subducting slabs or plunging columnar plumes, yet it has failed to detect even a single plume head thought to be more than 1000 km in diameter. The plume heads, if they exist at all, are actually thin folding vortex sheets, so that seismic waves may pass through them undetected without any time delay. Thermal plumes are generated by an adverse density gradient in a fluid layer heated at the bottom or cooled at the top, and they are often mistaken for full solid spheres or blobs, which are produced from two liquids of different density and viscosity (Olson and Singer (1985), Bercovici and Kelly (1997)). There has been no serious attempt to compare the sizes of the plumes generated by CGD simulations with those detected by tomography, so that confirmation may be reached on the plausibility of formation of such plumes under the same mantle conditions.

Morgan's (1971) claim of 20 deep mantle mushroom plumes was based on his survey of hotspots assumed to be indicative of the number of mantle plumes; the hotspots were largely oceanic. Malamud and Turcotte (1999) conducted steady-state heat balance for the whole earth and deduced 5200 mantle plumes. However, Courtillot et al. (2003) could identify with five strict criteria only seven out of 49 hotspots that may be of deep origin, while twenty may be attributed to the transition zone. A quick examination of their data shows that about two thirds of hotspots and plumes are found in the Southern hemisphere. The recent seismic-tomographic analyses of Montelli et al. (2004) first indicated eighteen plumes and later (Montelli et al., 2006) confirmed thirty five mantle plumes, while the computational modeling of Zhong (2005, 2006) suggested "tens of plumes". Eleven of the plumes with diameters between 200 and 800 km detected by Montelli et al. (2006) were of deep mantle origin, all of which lie within 30° N and S of the equator. Their methods could resolve only minimum visible diameters of 600 km at the CMB, and only three of the ten deep plumes have diameter greater than 600 km. The analyses of Oliver and Ghent (2000) showed bimodal distribution of plumes mainly within 30° N and S of the equator, which constitutes 30% of the earth's surface area, due to the centrifugal and differential rotational forces of the earth, and the fact that the polar regions do not receive enough sunshine. This rather constrained distribution is also observed in Venus (Crumpler et al., 1993) and the butterfly cluster of sun spots along the equator of the sun. It will be shown later that the number of plumes can be estimated easily if the size of plumes can be predicted.

Quite unbeknown to the mantle plume community (see review of Schubert et al. (2001), Davies (2005), Ritter and Christensen (2007)), Tan and Thorpe (1999a and b) first proposed the possibility of predicting the onset times of instability and sizes of large mushroom plumes in deep mantle and stellar bodies with equations derived from their transient instability theory. They showed that the size of the mushroom plume is proportional to (heat flux)$^{-1/4}$ and for bottom heating with an insulating interface, and is proportional to (temperature change)$^{-1/3}$ for a conducting boundary. The phenomenological theory and equations of Tan (1994), Tan and Thorpe (1996, 1999a and b) will be employed to predict the occurrence of mantle plumes in rocky mantle under various boundary conditions. The mantle has been considered a subsolidus that allows creeping flow of mantle rock, hence the conventional no-slip boundary condition at the CMB corresponding to a solid wall will be reduced to a free surface. Moreover, the deep mantle is supported by molten iron at the bottom and an insulating atmospheric layer at the top, both of which have negligible viscosities compared to that of the mantle rock. Thus, the mantle rock is confined between two free surfaces. This special case of a fluid layer bounded by two free and conducting surfaces has been treated in Rayleigh's (1916) seminal treatise. Apparently, his theory has been verified by the experiments of Goldstein and Graham (1969) by confining a viscous silicone oil layer between a thin layer of helium at the top and mercury at the bottom. The objective of this paper is to provide the fundamental principles and equations for the prediction and assessment of plumes generated by unstable-steady heat conduction both by bottom heating and top cooling. It will be shown that the sizes and types of plumes can be predicted easily and simply with known values of step change in temperature and heat flux, and the physical properties of the mantle.

2. Transient instability theory and the generation of mushroom plumes

It is assumed that the core mantle boundary is heated uniformly so that plumes may form uniformly in the mantle after the emergence of the unstable thermal boundary layer. Equations of critical time of onset of convection and critical size of mushroom plume derived by Tan and Thorpe (1996, 1999a and b) are modified for various boundary conditions with their corresponding critical Rayleigh numbers and critical dimensionless wavenumbers. The thermal boundary layer above the CMB and below the asthenosphere will be bounded by a moving boundary within the bulk mantle that may be conducting and semi-free surface, since the flow of fluid will always generate shear between fluid layers (Tan and Thorpe, 1996). However, the pure free-surface boundary conditions will be employed for simplicity. Both cases of bottom heating and top cooling with a conducting and an insulating interface will be treated in this study. The effect of internal heating is essentially to prolong the onset of convection and the reduction of the number of plumes (Roberts, 1967; Bercovici et al., 1989; Weienstein and Olson, 1991; Davies and Davies, 2009). However, the radiogenic heating in the barren lower mantle is negligible, 2.15×10^{-9} W/m^3 (Schubert and Spohn, 1981). Indeed the resultant rise of temperature for 1 Gyr is only about $\kappa t\rho H/k$ = 12 °C, which is negligible compared to the range of temperature contrast of 200–2000 °C reported in literature.

The time of onset of convection and the size of the mushroom plume caused by unsteady-state heat conduction may be predicted by a transient Rayleigh number defined as $Ra = gaz^4(\partial T/\partial Z)_t/VK$, which tracks the instantaneous local hydrostatic equilibrium till the onset of convection. The local temperature gradient may be determined from the spatio-temporally varying temperature profile for the boundary condition under consideration (Tan and Thorpe, 1992, 1999a and b).

2.1. Fixed surface temperature (FST)

When the surface of a fluid is heated (or cooled) by a step change in temperature instantaneously to a fixed surface temperature T_s, the temperature profile in the bulk fluid of temperature T_0 initially is predicted by penetration theory as $(T - T_s)/(T - T_0) = erf(z/2\sqrt{\kappa t})$ (Carslaw and Jaeger, 1973): the FST boundary corresponds to $Bi = \infty$ and a conducting interface. It is envisaged that the convection for $Bi = \infty$ will commence in a fluid layer bounded by two free surfaces since the mantle will assume the sub-solidus state similar to Rayleigh's (1916) seminal treatise that results in a critical Rayleigh number of 657.5 and a critical dimensionless wavenumber of $\tilde{a}_c = \pi/\sqrt{2} = 2.22$.

The maximum transient Rayleigh number at any instant can be found by differentiation of the transient Rayleigh number and setting it to zero to give the position of $z_{max} = 2\sqrt{2\kappa t} = 2.83\sqrt{\kappa t}$ and a maximum Rayleigh number (Tan and Thorpe, 1999a):

$$Ra_{max} = \frac{4.89 g\alpha\sqrt{\kappa t_c^3}(T_0 - T_s)}{\nu} \quad (1)$$

or in the conventional form $Ra_{max} = g\alpha(1.7\sqrt{\kappa t_c})^3 \Delta T_c/\nu\kappa$ with an "effective" thermal depth of $\delta_e = 1.70\sqrt{\kappa t_c}$. The critical time for Ra_c of 657.5 is calculated from Eq. (1) as:

$$t_c = \left[\frac{225.1\nu}{g\alpha\sqrt{\kappa}\Delta T_c}\right]^{\frac{2}{3}} = 37.0\left[\frac{\nu}{g\alpha\sqrt{\kappa}\Delta T_c}\right]^{\frac{2}{3}} \quad (2)$$

The critical time is independent of the depth of the liquid and depends on the fixed ΔT_c and fluid properties. The full cycle of plume formation includes the conduction phase, the evolution of thin filamentous to mushroom plume, and its detachment from the thermal boundary layer. Studies of Davaille and Jaupart (1993) for convection in liquid of low and high contrast viscosities, Sparrow et al. (1970) and Foster (1965) for water, and Foster (1971) for fluid of infinite Prandtl number, all showed that the periodicity of the plumes is approximately 1.4 times the onset time.

The critical wavelength for a convection plume with a critical dimensionless wavenumber of $\tilde{a}_c = \pi/\sqrt{2} = 2.22$ (Rayleigh, 1916) can be predicted from $\lambda_c = 2\pi z_{max}/\tilde{a}_c$ as:

$$\lambda_c = 8.0\sqrt{\kappa t_c} \text{ or } 41.0\left(\frac{\nu\kappa}{g\alpha\Delta T_c}\right)^{1/3} \quad (3)$$

which shows that the size of the plume is inversely proportional to $\Delta T_c^{1/3}$. Large ΔT_c will result in small plume, which may rise faster and dissipate heat more effectively deep into the bulk mantle.

2.2. Constant heat flux or insulating boundary

The interface of the CMB is considered to be rather insulating as the transient Biot number as defined by Pearson (1958) and Tan and Thorpe (1996) $Bi = (dq''/dT_s)_T/(dq''/dT_s)_B = \sqrt{(k\rho c_p)_T/(k\rho c_p)_B}$ is about 0.27, which is close to an insulating boundary with a constant heat flux (CHF) when $Bi = 0$; while the air covering the earth will ensure the surface of the earth is insulating. Its corresponding temperature profile in a semi-infinite fluid is given by $T_0 - T = 2q^o\sqrt{\kappa t}(ierfc(z/2\sqrt{\kappa t}))/k$, and its constant heat flux is $q^o = k\Delta T_s\sqrt{\pi/4\kappa t}$. The earth is shielded by an insulating atmospheric layer of air that ensures low heat loss to the outer space. The low constant heat flux of the CHF model is in great contrast to the prediction of infinite heat flux in the initial phase of an abruptly increased fixed surface temperature (FST) model given by $\overline{q}_t = 2k\Delta T_c/\sqrt{\pi\kappa t}$. For long heat conduction time in the geologic time scale of million years, the average heat flux predicted by the (FST) model will approach a low and approximately constant value. Thus, imposing an instantaneous temperature of 1000 °C over 100 Myr will yield an average heat flux of $\overline{q}_t = 0.11k/\sqrt{\kappa}$, which will be in the same order of magnitude as and near to that of CHF model, $q^o = 0.089k/\sqrt{\kappa}$, when the unit of time is in year.

The maximum transient Rayleigh number can be found (Tan and Thorpe, 1996) at $z_{max} = 2.56\sqrt{\kappa t_c}$ as

$$Ra_{max} = \frac{3.02 g\alpha q^o \kappa t_c^2}{k\nu} \quad (4)$$

which suggests that the transient Rayleigh number is independent of conductivity, since $\kappa = k/\rho c_p$. The transient Rayleigh number can also be expressed in the conventional form with substitution of $q^o = k\Delta T_s\sqrt{\pi/4\kappa t_c}$ when the surface-temperature change ΔT_s is known,

$$Ra_{max} = \frac{2.676 g\alpha(\kappa t_c)^{3/2}\Delta T_s}{\kappa\nu} \quad (5)$$

For a layer of liquid heated at the bottom with a constant heat flux, then Ra_{max} = 474 (estimated from results of Rayleigh (1916) and Sparrow et al. (1964)), the critical time at the onset of convection can be estimated as follow:

$$t_c = 12.5\left(\frac{k\nu}{g\alpha q^o\kappa}\right)^{1/2} \text{ or } 31.6\left(\frac{\nu}{g\alpha\sqrt{\kappa}\Delta T_s}\right)^{2/3} \quad (6)$$

which shows that the critical time is solely controlled by $q^{o-1/2}$, since the physical properties are constant.

The critical wavelength for a convection plume with a critical dimensionless wavenumber of $\tilde{a}_c = 1.84$ (estimated from Sparrow et al. (1964)) can be predicted from $\lambda_c = 2\pi z_{max}/\tilde{a}_c$ as:

$$\lambda_c = 8.75\sqrt{\kappa t_c} \text{ or } 31.0\left(\frac{\kappa k\nu}{g\alpha q^{\circ}}\right)^{1/4} \quad (7)$$

which shows that the size of the plume is inversely proportional to $q^{o-1/4}$, and high heat flux will produce small plumes. If the temperature change is known, then the size of plume may be found from $\lambda_c = 49.1(\nu\kappa/(g\alpha\Delta T_c))^{1/3}$.

For the case of top-cooling by an insulating atmosphere with a critical Rayleigh number of about 335 (estimated from Sparrow et al. (1964), Eq. (6)) may be rewritten as $t_c = 10.5(k\nu/(g\alpha q^o\kappa))^{1/2}$, which is only 16% lower than Eq. (6). The corresponding estimated dimensionless wavenumber of $\tilde{a}_c = 1.7$ (Sparrow et al. (1964)) will result in a wavelength of $\lambda_c = 9.5\sqrt{\kappa t_c}$ or $\lambda_c = 30.8(\kappa k\nu/(g\alpha q^o))^{1/4}$, which is almost identical to Eq. (7).

Griffiths and Campbell (1990) have derived an empirical equation for predicting the diameter of a rising plume D_p at any depth based on a constant buoyancy B, which is not applicable to the transient stage of development of the plumes, but it may be used with caution for estimating the size of the detached plume at various depths, $D_p = (B\nu/g\alpha\Delta T)^{1/5}k^{2/5}z^{3/5}$.

The rise velocity of the plumes may be estimated with the modified equation of Olson and Corcos (1980) with zero horizontal shear, $v_p = 1.65(\kappa/\delta_e)Ra^{2/3}$ originally derived for the fluid velocity of convection rolls confined between two plates. If the value of the critical Rayleigh number is 657.5 and $\delta_e = 1.70\sqrt{\kappa t_c}$, then the rise velocity is $v_p = 74\sqrt{\kappa/t_c}$. Foster (1965) has shown that the onset of convection can occur with a minimum amplification of velocity by a factor of 20 and a maximum amplification of 100 times, while Horton and Rogers (1945) measured a 100 fold increase of velocity of fluid in porous media, and Tan et al. (2009) reported thirty folds increase in velocity. If the rise velocity is assumed to be twenty times the diffusion velocity, then the rise velocity of plume may be given as $v_p = 20(z_{max}/t_c)$ or $v_p = 57\sqrt{\kappa/t_c}$. An average rise velocity may be assumed, thus

$$v_p = 66\sqrt{\frac{\kappa}{t_c}} \tag{8}$$

The seafloor spreading velocities worldwide have been estimated to be between 0.01 and 0.05 m/yr, which would mean a critical time of between 2.83 Gyr and 113 Myr respectively for a thermal diffusivity of 2.05×10^{-6} m^2/s. The long critical time of 2.83 Gyr will yield a plume diameter of 3750 km, which is larger than the depth of the D" layer, and it will be quite unlikely for it to occur.

3. Prediction of mushroom plumes in lower mantle

The prediction of the sizes of mushroom plumes and the onset of convection are largely dependent on the boundary condition, the temperature change at the interface and the heat flux that drives the convection in the mantle. The earth has been cooled very rapidly in its initial phase of formation until the formation and stabilization of the atmospheric boundary layer, which insulates the earth and inhibits the flow of heat to the outer space. Williams (1998) has considered a temperature contrast of 2000 K for mantle plumes and Gubbins (2001) has assumed a temperature drop of 1500 K across a 250-km boundary to estimate a heat flux at the outer core to be about 25 mW/m^2. However, the average basal heating flux of 35 mW/m^2 at the upper mantle, excluding the radiogenic heating of 37 mW/m^2, by steady-state analysis (Malamud and Turcotte, 1999) will entail a CMB heat flux of $35 \times (6370/3485)^2 = 117$ mW/m^2; although the latter could well be over 200 mW/m^2 because heat transfer in the lower mantle is still in an unsteady-state. It is assumed that the extremely hot core will continue to heat the core mantle boundary until it reaches the present temperature of about 4000 ± 600 K, which is about 2000 °C above the average mantle temperature of $\sim 2000 \pm 250$ K (Schubert et al. (2001), pp. 191 and 205). If the CMB has been abruptly raised to the present temperature by a step change in temperature, then one can predict the onset times and sizes of the mushroom plumes for a range of temperature differences as shown in Table 2.

For step changes of temperatures between 335 °C and 1300 °C, the sizes of mushroom mantle plumes are predicted by Eq. (4) to be between 1842 km and 1173 km, which are bigger than the maximum 800 km detected by Montelli et al. (2006), Table 2. However, the resulting average heat flux for a ΔT_s of 1300 °C is similar to the upper bound of 120 mW/m^2. While the expected plume size of 800 km will entail a very high ΔT_s of 4000 °C and a high heat flux of 538 mW/m^2, which are very unlikely to be possible in the recent geologic past. The low excess temperature of 200 K assumed by Sleep (1990) would result in extremely large plume of 2190 km developed over a period of 1160 Myr, with an impossibly low heat flux of only 10 mW/m^2. The initial rapid cooling phase of the earth might have passed through a range of temperature change at the CMB, which may generate plumes of sizes as shown in Fig. 1.

Schubert et al. (2005) have suggested the existence of clusters of plumes of 700 km and 622 km diameter in Reunion and Hawaii

Table 2 Mushroom plumes generated by an FST boundary at the CMB for mantle viscosity of 10^{22} Pa·s.

ΔT_s (°C)	Plume size (km)	Number of plumes		Critical times (Myr)	Heat flux (mW/m^2)
		Whole earth	@ 30%		
335	1842	57	17	825	20
500	1612	75	22	632	34
1000	1280	119	35	398	85
1200	1200	134	40	353	108
1300	1173	141	41	334	120
1500	1118	155	47	304	145
2000	1016	188	56	251	213
2500	943	218	66	216	287
3000	888	247	74	191	367
3500	843	273	82	173	450
4000	807[a]	299	90	158	538

[a] The largest plumes detected by Montelli et al. (2006).

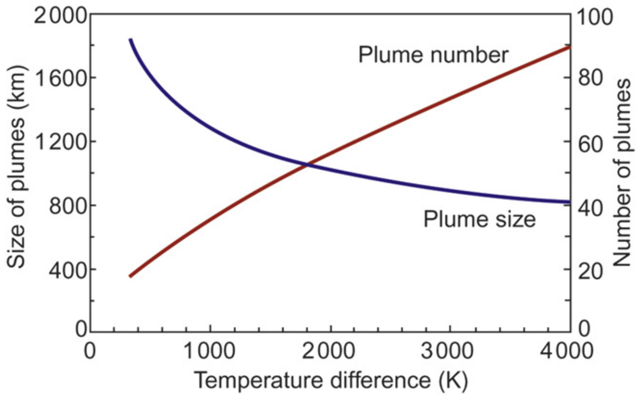

Figure 1 Sizes and number of mushroom plumes generated by FST boundary at the CMB.

respectively, based on volumetric flux and hydrodynamics, which although plausible, would be difficult to realize from temperature and heat flux considerations. Hill et al. (1992) had postulated plume heads of up to 3000 km, which would require a very low temperature rise of 75 K and a low heat flux of 3.0 mW/m^2 and extremely long onset time of about 2.3 Gyr, which is about half the age of the earth. These computations show that there exists a moderate temperature rise that will generate the reasonable plume size within a geological age without requiring excessive high heat flux.

The critical times of onset of convection for 335 °C and 1300 °C are found by Eq. (2) to be long, 825 and 334 Myr respectively. Actually the life span of plume is about 1.4 times the onset time. Therefore, the onset times of 825 and 334 Myr will yield a life span of plumes of about 1155 and 468 Myr, which are longer than the age of the oldest oceanic crust of 180 Myr where most hotspots are found, but they are still below the average age of continental crust of 2 Gyr. Ernst and Buchan (2003) have claimed the existence of fossil plumes formed as old as 2.5 Ga ago based on the geological record of some LIPs and giant dike swarms (GDS). Worsley et al. (1984, 1986) estimated a supercontinent cycle of about 400 Myr, while Yale and Carpenter (1998) estimated a periodicity of about 500 Myr for mantle and LIPs cycling rates since 3 Ga, although strong correlation occurred only since about 1.3 Ga. This periodicity would entail a temperature rise of 1200 K and a heat flux of 108 mW/m^2; although the size of the plume is rather large at 1200 km. This seems to suggest that mushroom plumes $\geq$ 800 km diameter much sought after by the plume advocates are unlikely to be found in the mantle, unless the temperature rise and heat flux at the CMB are higher than those just mentioned. The latter cannot be discounted easily as the mantle is deep and the atmospheric boundary layer is very insulting, except for the polar regions where they are not sufficiently insulated. Moreover, the history and record of formation of plumes is quite uncertain because plumes are primarily intermittent since the birth of the earth 4.5 Ga ago, and continental flood basalts may also be caused by global mantle warming and melting of mantle (Coltice et al., 2007, 2009).

The thickness of the thermal boundary layer at the CMB may be estimated with the onset times of 825 Myr and 334 Myr for temperatures rise of 335 and 1300 K from $z_{\max} \approx 2.83\sqrt{\kappa t_c} = 652$km and 415 km respectively, which are high compared to about 200 km reported in literature (Loper and Lay, 1995; Poirier, 2000; Gubbins, 2001). The plume hypothesis assumes that hotspots generated by mantle plumes at the CMB are no older than 180 Myr, which would imply a thermal boundary layer of thickness 309 km, which is still large.

The number mantle of plumes is predicted to be 57 and 141 for temperatures rise of 335 and 1300 K in the mantle respectively (Table 2 and Fig. 1). Allowing their formation being confined within 30° North and South of equator will only be about 30% of the predicted value, which is 17 plumes, and 41 plumes respectively. At a high temperature rise of 4000 K at the CMB associated with 800 km diameter plumes, the number of plumes will be about 90, which shows that the huge number of 5200 proposed by Malamud and Turcotte (1999) would be very likely to be impossible. The average number of plumes generated by temperatures rise of 335 K and 1300 K in the mantle (Table 2) is thirty, which coincidentally agrees with the number detected by Montelli et al. (2004, 2006). The agreement is fortuitous as plume diameters predicted in this study are much larger than the range of 200 and 800 km reported by Montelli et al. (2006). However, these two studies and modeling of Zhong (2005, 2006) have shown that the number of plumes probably is tens in number, but more detailed features of plumes are required to assess their number and distribution rationally.

The influence of viscosity on the characteristics of plumes is quite strong, the sizes of plumes double and the critical times quadruple as the viscosity increases by an order of magnitude as they are respectively proportional to $\nu^{1/3}$ and $\nu^{2/3}$ as shown by Eqs. (3) and (4) (Table 3). For mantle of viscosity of 7.08×10^{22} Pa·s, a low heat flux of 25 mW/m^2 and a temperature rise of 650 K would generate a plume of 2850 km diameter, which is close to the depth of the D'' layer; we note natural plume formation will require a mantle depth $d \geq 5\lambda_c$. This large plume may not form, instead convection rolls would form over a long period of heating exceeding four times the critical times (Foster, 1969) of 1960 Myr, which would be longer than the life of the earth, and it is likely to be impossible. If the temperature rise is increased to 1300 K and heat flux to 62 mW/m^2, then it would generate a plume of 2260 km diameter, which is still large and may encounter delay in its formation.

Table 3 The influence of viscosity on sizes of mushroom mantle plumes generated by temperature rise of 335 K and 1000 K.

Investigators	Viscosity (Pa·s)	Plume size (km)	Critical time (Myr)	Heat flux (mW/m^2)
650 K				
Schubert et al. (2001)	1.0×10^{21}	688	114	102
Davies (2005)	1.0×10^{22}	1480	531	48
Walzer et al. (2004)	7.08×10^{22}	2850	1960	25
1300 K				
Schubert et al. (2001)	1.0×10^{21}	541	72	258
Davies (2005)	1.0×10^{22}	1173	334	120
Walzer et al. (2004)	7.08×10^{22}	2260	1230	62

Table 4 Mushroom plumes generated by CHF boundary at the CMB for viscosity of 10^{22} Pa·s.

Heat flux (mW/m^2)	Plume size (km)	Number of plumes		Critical times (Myr)	ΔT_s (°C)
		Whole earth	@ 30%		
20	2019	48	14	822	432
30	1824	58	18	671	585
40	1698	67	20	582	726
50	1606	75	23	520	859
60	1534	83	25	475	984
70	1476	89	27	440	1105
80	1428	95	29	411	1222
90	1386	101	30	388	1334
100	1350	107	32	368	1444
110	1318	112	34	351	1551
120	1290	117	35	336	1656
360	980	301	61	194	3774
800	800[a]	301	90	130	6869

[a] The largest plumes detected by Montelli et al. (2006).

It should be noted that the viscosity at the CMB employed by Walzer et al. (2004) is a realistic value derived from the seismic model PREM. However, large temperature rise may induce very large viscosity contrast in the thermal boundary layer, so that the mantle viscosity is likely to be less than 10^{23} Pa·s. Generally an average viscosity is used in the calculation of Rayleigh number as the stability analysis requires constant physical properties other than a small change in density caused by a small change in temperature. The great uncertainty in the thermal conditions and physical properties of the lower mantle dictates the use of best known values for the time being, and the range of viscosities in Table 3 provides a good glimpse of possible plume sizes.

One may infer from these comprehensive computations that mushroom mantle plumes probably occur at rather high temperature rise and high heat flux at the CMB with viscosity in the order of 10^{23} Pa·s, although they may also form at a lower viscosity of near 10^{21} Pa·s with plume sizes close to those detected by seismic tomography. The only possibility of meeting the requirement of plume size, formation age and temperature rise at a low heat flux of 20 mW/m^2 would be to reduce the viscosity to below 10^{21} Pa·s, which is unlikely to be possible as the viscosity at the CMB is close to 10^{23} Pa·s (Davies, 2005). Therefore, the heat flux at the CMB may still remain high as the age of the oldest continental crust is about half the age of the stable earth, and the heat transport is far from attaining the steady-state upon which the transient instability theory depends.

The rise velocity of a mantle plume can be determined from Eq. (8) for a temperature rise of 1300 K and a critical time of 334 Myr as 0.029 m/yr or 9.2×10^{-10} m/s, which is comparable to the seafloor spreading velocity of 0.01 and 0.05 m/yr. It is an order of magnitude lower than the reported terminal rise velocity of plume of 0.28 m/yr for Reunion plume (Schubert et al., 2005) and 0.4 m/yr for plume rising in a stress-dependent fluid (van Keken, 1997) based on Stoke's law, and the pulse velocities of 0.1–0.25 m/yr estimated by Rudge et al. (2008). More crucial, Peate and Bryan (2008) recently showed that the dynamic pre-volcanic uplift caused by mantle plume is generally lacking in evidence. It will be shown in a separate paper that the rise of a mantle plume formed during transient heat conduction is rather rapid and accelerates continuously, it is far from the steady-state assumed by Stokes's law. It is quite uncertain whether the extreme low velocity of mantle plume of $\sim 10^{-10}$ m/s and its resultant extraordinary low Reynolds number, $\sim 10^{-21}$, is applicable in Stokes's analysis, which really applies to only very small particle without form drag, and not to gigantic mantle plume.

3.1. Constant heat flux boundary

Gubbins (2001) estimated the heat flux at the outer core to be about 25 mW/m^2, while the model of Walzer et al. (2004) for mantle convection is based on 20 mW/m^2. The heat flux from the mantle heat flow of 10 TW is about 66 mW/m^2, while Lay et al. (2006) estimated 85 ± 25 W/m^2 from a heat conductivity of 10 W/(m K) that results in a CMB heat flow of 13 ± 4 TW. Hofmeister's (2008) experiment and model shows a high value of thermal conductivity of 42 W/(m K) ($\pm$ 25%), which may imply a very high heat flux of about 360 mW/m^2. Therefore the steady-state heat fluxes at the CMB and lithosphere are respectively between 20 and 110 mW/m^2 and between 30 and 101 mW/m^2, although unsteady-state heat conduction will predict quite different heat fluxes at both interfaces. The probable sizes of the plumes generated at the lower mantle and upper mantle will be computed for these ranges of heat fluxes.

For heat fluxes between 20 and 120 mW/m^2 the sizes of mushroom plumes generated at the CMB are predicted by Eq. (7) to be 2019 km and 1290 km respectively, Table 4 and Fig. 2. As expected for long heating time of few hundred million years, they

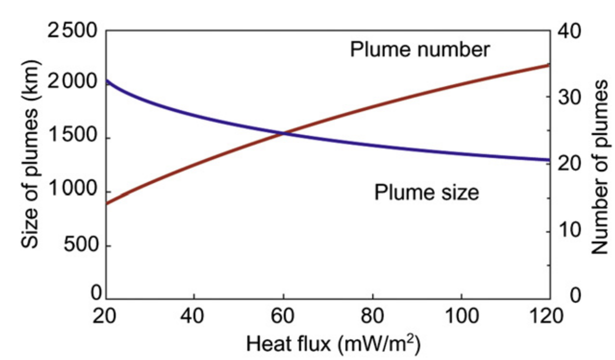

Figure 2 Sizes and number of mushroom plumes generated by CHF boundary at the CMB.

are very close to those predicted by the FST boundary for the same heat flux, which are 1842 km and 1173 km respectively as shown in Table 2. These sizes of plumes are very much larger than those maximum of 800 km detected by Montelli et al. (2006) and 500 km detected by Bijwaard and Spakman (1999) with seismic tomography for Iceland. The expected temperature rise as calculated from the heat conduction equation, $q^o = k\Delta T_s\sqrt{\pi/4\kappa t_c}$ for 20 mW/m^2 is 500 °C, which is still within the range of 200–600 °C proposed by the plumes hypothesis, while that for 120 W/m^2 is an anomalously high of 1656 °C, Table 4. Adding this temperature rise to an assumed initial temperature of the transition zone of 2000 ± 250 K will just reach the temperature of the CMB of 4000 ± 600 K. Thus, the heat flux of 120 W/m^2 may represent the upper limit of the CMB. Hence, the heat flux of 85 mW/m^2 estimated by Lay et al. (2006) that predicts a plume of 1300 km diameter and a temperature rise of 1470 °C is still within the range of plausibility.

For the plume diameter of 800 km detected by seismic tomography, Eq. (7) will estimate an anomalously high heat flux of 800 mW/m^2 and an anomalously high temperature rise of 6900 °C. The latter is at least 3000 °C higher than the acceptable CMB temperature of 4000 ± 600 K (Schubert et al. (2001), page 205), hence, it is unlikely to be possible. A high heat flux of 360 mW/m^2 would predict a plume of 980 km diameter with an anomalously high temperature rise of 3774 °C. These calculations show that there exists a moderate heat flux that will generate the rational plume size within a time frame without causing excessive temperature rise.

The CHF model is particularly useful in assessing the heat loads brought to the hotspots by mantle plumes, or conversely the estimated heat loads reported in literature may be used to estimate the heat flux at the CMB. The heat conveyed to the hotspot by the plume conduit is provided by the interfacial heat flux within an area the size of the mushroom plume, that is, $Q = q^o A_p$. The heat flux of 80 mW/m^2 and a plume diameter of 1428 km (Table 3) will yield a heat rate of 128 GW, which is slightly smaller than the 138 GW estimated by Sleep (1990) using steady-state analysis for Easter, MacDonald, Marquesas, Pitcairn and Tahiti hotspots, while the low heat flux of 20 mW/m^2 and a plume diameter of 2019 km will yield a heat rate of 64 GW that is close to those of Afar and Iceland hotspots. However, these predictions do not take account of the large heat loss by the long column of conduit, which is considerable. The heat transfer by laminar flow in a conduit of diameter d_f is given by $Q \approx (\pi d_f L)(4k\Delta T/d_f) = 4\pi L k \Delta T$, which is independent of the conduit diameter, and temperature excesses of 200 K and 1000 K will yield heat rates between 58 GW and 291 GW respectively for an average heat conductivity of 8 W/(m K) for the mantle and a 2900-km long column. Therefore the heat loss by the plume conduit cannot be neglected in the heat balance for heat transfer in the mantle. Indeed the successful emergence of the mantle plume at the hotspot would require an amount of heat substantially more than that conveyed in the plume conduit, without which the plume conduit will lose its flow and detach from the head of the mushroom. The plume conduit first appears as a protuberance at the unstable boundary layer, then it rises in the mantle as an issuing jet until it encounters sufficient hydrostatic force to fold back as vortex sheet in the shape of a mushroom plume. The mushroom head is essentially powered by the heat flux at the CMB via the heat flow in the plume conduit.

The number of mantle of plumes predicted by the constant heat flux model is not very different from those of similar average heat fluxes predicted by FST boundary (Table 3 and Fig. 2). For instance, a heat flux of 20 mW/m^2 generates 14 and 17 plumes respectively for the FST and CHF boundaries. The thermal condition at the CMB may be moderately conducting, so that the CHF boundary is still applicable. Moreover, the CHF boundary can provide a glimpse of change of temperature at the CMB induced by any heat flux.

Overall this study of plumes generated by heating at the CMB shows that the heat flux is very likely to be higher than 20 mW/m^2, but not exceeding 200 mW/m^2, so that the number of plumes shall not exceed 50 in mantle of viscosity of about 10^{22} Pa·s. Mantle viscosity exceeding 10^{22} Pa·s will cause long delay in the generation of plumes, which may be larger than the depth of the D" layer.

3.2. Plunging mushroom plumes generated by TOP cooling

CGD simulations of Kiefer and Kellogg (1998) have shown predominant plunging plumes descending to the CMB caused by cooling of the lithosphere, thus, hindering the development and rise of hot mantle plumes. The earth surface is well insulated by the atmosphere with air of low thermal diffusivity of 0.03 W/(m·K), hence the appropriate boundary condition is one of CHF. The mean heat fluxes estimated by Pollack et al. (1993) were 65 mW/m^2 for the continent and 101 mW/m^2 for the ocean, while the basal heating rates for the lithosphere of the continent and ocean are respectively 31 ± 11 mW/m^2 and 36 ± 8 mW/m^2 (Malamud and Turcotte, 1999). More recent estimations of Hofmeister and Criss (2005) and Hamza et al. (2008) show that the earth's mean surface heat flux is about 63 mW/m^2. The mushroom plumes generated by cooling for heat fluxes of 30 and 100 mW/m^2 at the upper mantle may be predicted with Eq. (7) $\lambda_c = 31(\kappa k \nu/(g\alpha q^o))^{1/4}$ to yield 585 km and 433 km while the critical times may be predicted with Eq. (6) $t_c = 12.5(k\nu/(g\alpha q^o \kappa))^{1/2}$ to give 139 and 76.4 Myr respectively, Table 4. The smaller plume is close to Iceland plume of 400 km diameter detected by Shen et al. (1998) at 660 km depth, although they assumed a hot rising plume from the lower mantle. The sizes of plume may be reduced by half if the viscosity drops by an order of magnitude (Table 5). These estimations may change with the assumption of a solid lithosphere with a critical Rayleigh number

Table 5 Plunging mushroom plumes generated by top cooling below lithosphere.

Heat flux (mW/m^2)	Plume size (km)	Number of plumes at 30%	Critical time (Myr)	ΔT_s (°C)
Viscosity 10^{21} Pa·s				
30	585	569	139	562
100	433	1039	76.4	1387
Viscosity 10^{20} Pa·s				
30	329	1799	44.1	316
100	243	3298	24.2	780

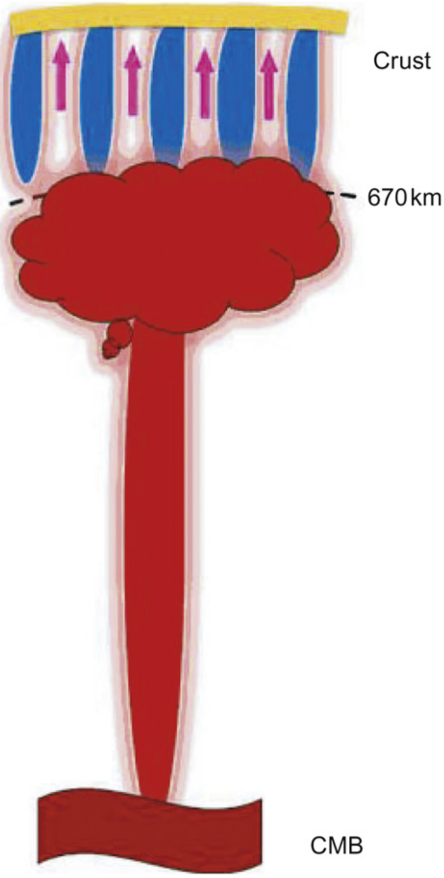

Figure 3 Heat trap caused by large number of plunging plumes.

of 669 and a critical dimensionless wavenumber of 2.09 (Sparrow et al., 1964), the values of plume size and critical time will increase by 19% and 17% respectively.

Thus far, there has been no report on the detection of cold plunging plumes, while hot mushroom plumes detected by Montelli et al. (2006) are all generated at the CMB as they show positive temperature anomalies with respect to the ambient mantle rock. The temperature of the lower lithosphere would have dropped by 562 °C and 1387 °C (from $q^{o} = k\Delta T_{s}\sqrt{\pi/4\kappa t_{c}}$) at the onset of convection caused by cooling fluxes of 30 mW/m^2 and 100 mW/m^2 respectively. The predicted temperature drop of 562 and 1387 °C are excessive when they are compared to paleotemperature over the last 200 Myr (Larsen, 1991), which shows a maximum drop in atmospheric temperature of only about 23 K from the peak in 110 Ma to the present condition. Clearly plunging plumes may not form in volcanic active areas such as the ring of fire in the Pacific rim.

The rising hot plumes are outnumbered by the descending cold plumes by 569 to 18 or by a ratio of 32:1 for a cooling rate of 30 mW/m^2. It is apparent that few rising plumes will survive the onslaught of the raining cold plumes as have been shown by the CGD simulations of Kiefer and Kellogg (1998), van Keken and Yuen (1995) and Tackley et al. (1994), and the laboratory experiments of Lenardic et al. (2005). The frequency of cold plunging plumes and hot rising plumes are inversely proportional to their critical times of formation. For a heat flux of 30 mW/m^2, the ratio of the frequencies of plunging and rising plumes is about 4.8. The laboratory experiments of Schaeffer and Manga (2001) employed a low range of frequencies of 0.3–1, which is quite different from our computed value based on heat fluxes currently known in literature. The vast ensemble of cold down-welling may neutralize the small number of ascending hot plumes near the transition zone, so that the lithosphere and the earth surface does not have to be hot, while few, if any of the hot plumes will rise to the lithosphere (Fig. 3). Indeed the high average cooling rate of 4 °C per Myr arising from a temperature drop of 562 °C over 139 Myr must be balanced with rather rapid heat supply from the lower mantle by convection, apart from internal heating. However, the heating rate at the CMB is rather low, only about 0.87 °C per Myr for a temperature rise of 585 °C over 671 Myr. It may be inferred that the mantle is still undergoing a cycle of unsteady-state and transient convection at the top and bottom. The hotspots are generally cluttered along the faults between major tectonic plates, and they may have nothing to do with the hot plumes as hot materials adjacent to the thermal boundary layer can easily move up by sheer necessity of conservation of mass in the asthenosphere caused by plunging plumes. This may be a vital clue to claim of Foulger et al. (2001) and Foulger (2002) of the shallow origin of Iceland hot spot as there appears to be a total absence of the hot picrite glasses indicative of hot temperature.

More important, the coalescence and mixing of the cold and hot plumes may be the origin of the transition zone at 410 km–660 km, as spinel Mg_2SiO_4 in the transition zone of density 3860 kg/m^3 may be a product of transition of olivine (density 3350 kg/m^3) of the upper mantle and a mixture of perovskite and magnesiowustite (density 4870 kg/m^3) of the lower mantle at 660 km. Brunet and Yuen (2000) and Farnetani and Samuel (2005) have shown in their numerical simulations that plumes originated in the CMB may be trapped in the transition zone as they arrive at the phase boundary between 410 and 660 km. Therefore the thermal history of the earth may be more complicated than a simple conduction and convection model. CGD simulations with the same boundary conditions and heat fluxes in this study will easily verify the density of the transition zone as a result of the mixing of cold plunging and hot rising plumes.

Zhao (2004) has interpreted high velocity anomaly below several patches of low velocity anomaly beneath Iceland as indication of sunken subducted slabs 150 to 200 thick, which may be present in the vicinity above the CMB as have been indicated by Tackley's (2002) study. If this is true, then nearly half of the area of the earth above the transition zone covered with high velocity anomalies as shown in tomographic images of Zhao (2004) and Montelli et al. (2006) may be laden with either cold plunging plumes or slabs. This huge number of slabs or possibly cold plunging plumes distributed mainly in the northern hemisphere may be obstructing the emergence of the hot mantle plumes as hotspots. This could be a speculative reason for the existence of only a third of the hotspots in the northern hemisphere.

These computations show that mantle plumes may be predicted easily with the known heat flux or temperature rise, if the physical properties of the mantle rock are known. However, the uncertainties in the physical properties and thermal conditions at the CMB seem to be the greatest challenge to correctly predict the characteristics of the plumes. Notwithstanding the difficulties in these uncertainties, our equations do provide meaningful understanding of the formation and evolution of the plumes. In future, more CGD simulations can be conducted with the guide of the predictions in this study and with more realistic thermal conditions, thus providing a better and deeper understanding of the possibilities and problems of the plume hypothesis. However, there remain the critical issues of verification of the hypothesis in terms of the true sizes and rise velocities of plumes.

4. Conclusion

Morgan's (1971) original plume hypothesis did not provide the size of plumes and age of hotspots. It is the incorporation of geodynamics later that leads to the concept of mushroom plumes of about 1200 km diameter with temperature anomalies of about 500 °C to 1500 °C. There has been no equation to predict the formation of mantle plumes and theory that substantiates the claim that a particular hotspot is generated by a plume, although the circumstantial evidence is merely fair. There are more fundamental questions of thermal plumes even if we can predict their formation with the theory presented in this paper, they are the high heat fluxes originated from the outer core and the physical properties of the sub-solidus mantle, without which we cannot predict any thing reliably and accurately.

This study shows that the sizes and ages of plumes can be predicted easily with equations developed by Tan and Thorpe (1996, 1999a and b) from transient instability theory, if the thermal conditions and physical properties of the mantle are known. The plumes generated at the CMB and lithosphere rising and falling through the mantle have been calculated for various heat fluxes and viscosities.

The hot rising mushroom plumes of 1842 km and 1173 km diameter generated by 20 and 120 mW/m^2 at the CMB are too large, and they exceed the limit of 1000 km of the plume hypothesis. The temperature rise of 335 °C for low heat flux is within the limit, whereas 1777 °C has exceeded the limit of the plume hypothesis. The number of plumes is between 17 and 41, which is within the range suggested by several researchers. The life spans of these mushroom plumes are estimated to be between 1155 Myr and 468 Myr, which are below the age of continental crust. It appears that the plume diameter of 800 km detected by seismic tomography would require a high temperature rise of 4000 °C and a heat flux of 538 mW/m^2 at the CMB.

The thickness of the thermal boundary layer at the CMB may be estimated with the onset times of 825 Myr and 334 Myr for temperatures rise of 335 and 1300 K as 652 km and 415 km respectively, which are high compared to about 200 km reported in literature (Loper and Lay, 1995; Poirier, 2000; Gubbins, 2001).

It was also found that the heat loss by the plume conduit is substantial and in the order of magnitude as the heat found at the hotspots, since laminar heat flow in the plume conduit is dependent on the height of the 2900-km tall conduit. Consequently the heat flux at the CMB is higher than the low value of 20 mW/m^2 assumed in many simulation studies, in order that sufficient heat is conveyed upward to maintain the attachment of the conduit to the mushroom plume head.

The plume hypothesis has been silent on the occurrence and the characteristics of the cold plunging plumes originating beneath the lithosphere. The predictions show that the cold plunging plumes in mantle rock to be of reasonable sizes and onset times, respectively 585 km and 139 Myr, while the number of plumes of 569 is large. It implies that cold plunging plumes are ubiquitous in the asthenosphere and provide substantial cooling to the earth surface. They may also generate currents that drive the process below tectonic plates. More interesting is the implication that the more numerous cold plunging plumes may block the rise of the hot rising plumes and result in the formation of the transition zone.

The transient instability theory shows that transient convection caused by unsteady-state heat conduction occurs at critical Rayleigh number less than 1000, as it is determined by the transient penetration depth $z_{\max} \approx 2.7\sqrt{\kappa t_c}$ of the order of few hundreds km, which is considerably smaller than the large depth of the mantle of several thousands km. The latter tend to inflate the value of the Rayleigh number artificially, whereas the transient convection is a localized phenomenon that does not involve the whole depth of the mantle. Therefore, extremely large Rayleigh number of 10^6 to 10^{11} often quoted in literature (Gurnis and Davies, 1986; Kiefer and Kellogg, 1998; Davies, 1999; Condie, 2001; Montelli et al., 2006) are not appropriate.

Acknowledgments

We thank Thomas Ruedas of Department of Terrestrial Magnetism, Carnegie Institution of Washington for his very useful critical and helpful comments. Tan is especially grateful to Tan Sri Syed Jalaluddin Salim for his intellectual support. This research was supported financially by Loyalport Foundation for which we are most grateful.

Appendix

Physical properties.

Parameters	Lower mantle[a]	Perovskite[b]	Upper mantle[c]	Outer core[d]
κ (m^2/s)	2.05×10^{-6}	1.33×10^{-6}	1.00×10^{-6}	8.60×10^{-6}
α (K^{-1})	1.4×10^{-5}	2.2×10^{-5}	3.0×10^{-5}	1.30×10^{-5}
μ (Pa·s)	1×10^{22}	1−10	1×10^{21}	1.00×10^{-2}
ν (m^2/s)	2.05×10^{18}	2.44×10^{-4}−10^{-3}	2.78×10^{17}	1.0×10^{-6}
ρ (kg/m^3)	4870	4100	3600	10000
c_p (J/(kg K))	1200	733	1100	700
k (W/(m K))	12	1.4−5	4	60

a Schubert et al. (2001), Table 3.1, pp. 69; Table 11.3, pp. 512.
b Schubert et al. (2001), Table 4.11, pp. 190.
c Schubert et al. (2001), pp. 297.
d Gubbins (2001) listed physical properties of outer core based on molten iron-Ni.

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Nomenclature

a: wavenumber, m^{-1}
$\tilde{a}$: dimensionless wavenumber = a/d
c_p: specific heat, J/(kg•K)
d: depth of fluid layer, m
D_p: diameter of plume, m
g: acceleration due to gravity, m/s^2
h: heat transfer coefficient, W/(m^2 •°C)
H: heat generation, W/m^3
k: thermal conductivity of the fluid, W/(m•°C)
K_e: permeability, m^2
q°: constant heat flux, W/m^2
Q: rate of heat transfer, W
t: time, s
t_c: critical time of stable heat conduction before the onset of convection, s
T: temperature, °C
T_0: initial temperature, °C
T_s: surface temperature, °C
z: vertical distance in fluid measured from the interface, m

Greek symbols
α: volumetric coefficient of thermal expansion of the fluid, K^{-1}
κ: thermal diffusivity, m^2/s
λ: wavelength, m
φ: porosity
ρ: density, kg/m^3
ν: kinematic viscosity, m^2/s

Abbreviations

CHF: constant heat flux boundary condition
FST: fixed surface temperature boundary condition

Subscripts
c: critical
g: gas phase
l: liquid phase
0: initial state
s: surface