Prediction of elastic modulus and Poisson’s ratio for unsaturated concrete

Hailong Wang, Qingbin Li *

Department of Hydraulic Engineering, State Key Laboratory of Hydrosience and Engineering, Tsinghua University, Beijing 100084, PR China

Received 3 July 2005; received in revised form 31 May 2006
Available online 23 June 2006

Abstract

Many concrete structures are located in water environment, but the underwater concrete is usually unsaturated even though it has been soaked in the water for a very long time. Some experiments have proven that the mechanical properties of concrete are affected by the saturation degree of fluid and aspect ratio of pores. Several publications discussed the saturated concrete qualitatively, but few gave quantitative analysis especially for the unsaturated concrete. In terms of the microstructure of unsaturated concrete, equivalent medium and inclusion-based theory of composite materials, a model is proposed to explain the changes happened in the wet concrete and to predict the elastic parameters (including elastic modulus and Poisson’s ratio) of unsaturated concrete. The viscosity of water in pores, micro-cracks and the further hydration of cement are taken into account in this paper by means of the definition of saturation concept according to the effect of pore water on the modulus of concrete. In this model, both stiff effect of water and soft effect of cracks on the concrete are introduced to describe the bulk modulus, at the same time the effect of shear rate on the shear modulus is considered. The comparison between the theoretical models and experimental results in the extreme state indicates that the model proposed in this paper is valid to predict the elastic properties of unsaturated concrete.

© 2006 Elsevier Ltd. All rights reserved.

Keywords: Unsaturated concrete; Elastic modulus; Poisson’s ratio; Water; Effective porosity; Effective saturation

1. Introduction

Many concrete structures such as dams, ship locks, abutment piers of bridges, etc., are often located in the water. A lot of experiments have shown that the elastic properties of wet concrete were affected by the saturation degree, and the strength of saturated concrete decreased whereas the static and dynamic moduli of saturated concrete increased (Rossi et al., 1992; Ross et al., 1996; Yaman et al., 2002a,b). For concrete under water, the general assumption is that the concrete is fully saturated by water, which is not consistent to many experimental investigations even it is stored under water for a long time. For example, Chatterji (2004)
observed that the inner section of the 17 in. cement cylinder was dry when the structure had stored under water for 222 days, and the maximum saturation degree in the samples in Persson’s research was 98% when specimens (1 m in diameter and 0.1 m in thickness) were cured in water for 450 days (Persson, 1997). Therefore it needs a very long time to experimentally obtain the elastic moduli of concrete under different saturated degree. Thus there are some practical meanings and necessity to establish theoretical models to predict the properties of wet concrete.

The elastic modulus of concrete is an essential parameter for analysis of a hydraulic concrete structure, but few literatures have quantitatively dealt with it for wet concrete, especially for unsaturated concrete. Only Yaman et al. (2002a,b) found a valid model to predict the elastic modulus of saturated concrete by comparing different methods, including micromechanical methods, experiential and semi-experiential methods, in which the Kuster–Toksoz method was more effective than others, whereas there were biggish deviations between Kuster–Toksoz method and the experimental data, the reason may be that the effect of water on the shear modulus of concrete and the further hydration of cement were not considered.

Accordingly, the objective of this paper is to predict the elastic modulus and Poisson’s ratio of unsaturated concrete by means of micromechanics of composite materials. Based on the two-phase composite theory, a physical model of unsaturated concrete is decomposed into a two-stage equivalent model. The saturation degree of unsaturated concrete is redefined according to the effect of pore water on the modulus of concrete. The water viscosity in pores and the further hydration of cement are both taken into account to establish the model of unsaturated concrete. As known, the properties of fluid in macroscopic state will be changed dramatically when it flows in the micron- and nano-scale pores. The dimension of cracks and pores in concrete is in micron- and nano-scale in a large extent, so the changes caused by the pore water should be taken into account in the model. For predicting of the bulk modulus of unsaturated concrete, the equivalent medium method is employed by embedding the different inclusions to the matrix in different stage. Both the stiff effect of water and the soft effect of cracks are included in this model.

2. Microstructure of unsaturated concrete

The composite structure of unsaturated concrete employed in this paper is described by micro-level. In this level, the concrete can represent the effect of entrained pores (including air pores, capillary and cracks) and moisture. Mortar, coarse aggregate and interface represent the typical solid phases of hardened concrete. And we mainly use the micromechanics of composite materials to discuss the effects of pores and pore water on the properties of unsaturated concrete. So we merge the three typical solid phases into a matrix phase (intrinsic concrete) in the representative volume element, and the inclusion phase is pores and water in unsaturated concrete. As the extreme states, the inclusion in dry concrete is dry pores and the inclusion in saturated concrete is water.

The average mechanical properties of concrete in Yaman et al.’s (2002a,b) experiment are provided in Table 1. The porosity and saturation degree are redefined in this paper, considering the contribution of water and water content in the pores to the modulus of concrete.

3. Effective porosity and saturation of unsaturated concrete

3.1. Effective porosity

The porosity in dry concrete is the summation of that for air pores, capillary and cracks. But for unsaturated concrete, the porosity changes due to the effect of further hydration and other factors. When immature

<table>
<thead>
<tr>
<th></th>
<th>Intrinsic concrete (Yaman et al., 2002a,b)</th>
<th>Water</th>
<th>Dry pores</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus/GPa</td>
<td>45.35</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Bulk modulus/GPa</td>
<td>27.91</td>
<td>2.25</td>
<td>0</td>
</tr>
<tr>
<td>Shear modulus/GPa</td>
<td>18.45</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.229</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>
concrete specimens are stored in water, the ambient water will ingress into the dry concrete and consequent hydration will take place. Thus, the porosity of concrete will be reduced and the effective pore radius will be lessened (Delmi et al., 2006). So we adopt the effective porosity to analyze the properties of unsaturated concrete, when the dry specimens are soaked into water. In order to make the comparison among different states easily and be able to reveal the effect of pore water on the elastic modulus of concrete, the porosity \( \phi \) refers to the porosity of dry concrete. So the effective porosity of unsaturated concrete can be expressed as

\[
\phi_{\text{eff}} = f(k_p, t, \mu) \phi
\]

(1)

where \( f \) is a function of seepage rate \( k_p \), seepage time \( t \) and effective viscosity of water \( \mu \), and \( f < 1 \) for wet concrete.

3.2. Effective saturation

Generally, saturation degree is the volume ratio of water to pores. But this definition is not suitable for the investigation of unsaturated concrete, which can be divided into three zones: saturated zone, unsaturated zone and dry zone. Saturated zone refers to the outermost zone in which the pores are fully filled with water, dry zone to the innermost zone in which the concrete is dry, and the unsaturated zone to those between. In the unsaturated zone, the water gets into pores by membrane forces or surface tension of water (Pichler et al., 2006), so the pores are filled with air and water (see Fig. 1b). As we know, the compressibility of air in the home state is too high to make the pores stiff. Therefore, only the saturated pores have the effects on the properties of concrete comparing with the dry sample. So the dry pores and unsaturated pores can be considered as dry pores, and the effective saturation degree refers to the percentage of saturated pores in all pores of unsaturated concrete. The water in pores of unsaturated concrete is shown in Fig. 1. The effective saturation degree is defined as

\[
S_{\text{eff}} = \frac{V_{p\text{-sat}}}{V_p} = h(k_p, t) S_w
\]

(2)

where \( V_{p\text{-sat}} \) is the volume of pores which are fully filled with water or the volume of water in saturated zone; \( V_p \) is the total volume of pores in concrete (including the pores filled with water); \( S_w \) is the general saturation degree. \( h \) is less than 1.

4. Elastic modulus of unsaturated concrete

The elastic modulus of unsaturated concrete is dependent on the contained water, pore geometry and matrix elasticity. Poro-elastic theory and inclusion-based theory can be used to predict the elastic modulus of unsaturated concrete. Inclusion-based theory can take the effects of pores’ geometry into account by viewing concrete as a solid matrix and embedded different pores as inclusion, so it is more accurate than the poro-elastic theory in application (Wang and Li, 2005). Based on the equivalent medium and inclusion-based theory (Suquet, 1997; Huang, 2004), the physical model of unsaturated concrete in this paper is made up of three phases, and more rational assumptions are needed to decouple the equations to obtain the solution of multiphase composite structure. The decoupling of equations is a complex mathematical manipulation, so another method is adopted to evade the establishment of more assumptions and much more complex mathematical process.

Employed the decomposition idea from modified differential method in micromechanics, the physical model of unsaturated concrete is decomposed into two stages shown in Fig. 2. In the first stage, the saturated pores are embedded into the matrix of concrete, and the elastic modulus of saturated concrete can be obtained.

Fig. 1. Water in pores (b and c are summed up to dry pores due to the highly compressibility of air in the home state): (a) saturated pore, (b) unsaturated pore and (c) dry pore.
by micromechanics of two-phase composite materials. In the second stage, the saturated concrete is homogenized to another kind of equivalent medium whose modulus equals to that for saturated concrete, and the dry pores are embedded into this equivalent medium subsequently. The elastic modulus obtained in the second stage is deemed to that for unsaturated concrete, and the specific analysis will be carried out in the following sections.

4.1. Bulk and shear modulus of unsaturated concrete

In terms of inclusion-based theory and average stress method (Du and Wang, 1998), the exact expression for effective elastic modulus tensor \( D = (3K, 2G) \) of two-phase composite can be obtained as

\[
D = D_0 + \phi \left[ (D_I - D_0)H \right] \left[ (1 - \phi)I + \phi H \right]^{-1}
\]

where \( D_0 \) is the elastic modulus of matrix phase, \( D_I \) the elastic modulus of inclusions phase, \( I \) the fourth-order isotropic identity tensor, \( \phi \) the volume fraction of the inclusion, \( H \) the tensor that relates to the average strain in each phase, and

\[
D = \frac{1}{3} (3K - 2G) \delta \delta + 2GI
\]

where \( K \) and \( G \) are bulk and shear moduli, and \( D_0D_I = (9K_0K_I, 4G_0G_I) \).

Based on the two-phase composite model of Benveniste (1987), for elliptically-shaped inclusions, \( H \) can be expressed as

\[
H = [I + SD_0^{-1}(D_I - D_0)]^{-1}
\]

where \( S \) is Eshelby’s tensor depending upon \( D_0 \) and the aspect ratio of inclusions.

In the micromechanics, pores are divided into two categories, “stiff” pores and “soft” pores, respectively. The former are geometrically sphere, and the latter are geometrically ellipse or disk. The disk or ellipse pores make the stiffness of the concrete decrease (Stora et al., 2006), but the water in fully filled pores does the stiffness of dry concrete increase, and the pressures inside the pores are independent of orientations because of symmetry properties. So the water-fully-filled pores are taken as stiff pores. And the unsaturated or dry pores are taken as soft pores, the properties of which can be evaluated by an equivalent aspect ratio that is a statistical value of different shape pores.

Now, let us discuss the elastic modulus of saturated concrete in the first stage. Based on the two-phase composite model and the assumption that the saturated concrete is isotropic, the elastic bulk and shear moduli of saturated concrete \( K_1 \) and \( G_1 \) can be obtained from the following equations:
\[ K_1 = K_M - \frac{S_{eff} \phi_{eff} K_M (K_M - K_w)}{S_{eff} \phi_{eff} K_M + (1 - S_{eff} \phi_{eff}) \pi \beta} \]  
\[ G'_1 = G_M - \frac{\phi_{eff} \eta G_M}{1 - \phi_{eff} + \phi_{eff} \eta} \]

The relevant expressions for \( \beta \) and \( \eta \) can be obtained from Berryman (1980) as
\[ \beta = \frac{G_M (3K_M + G_M)}{3K_M + 4G_M} \]  
\[ \eta = \frac{1}{5} \left( 1 + \frac{8G_M}{\pi (G_M + 2\beta)} + \frac{4G_M}{3\pi \beta} \right) \]

where \( K_M \) and \( G_M \) are the elastic bulk and shear moduli of matrix, respectively, \( K_w \) is the elastic bulk modulus of embedded water equaling to 2.25 GPa. From Eq. (7) it can be seen that the shear modulus increases a little due to the further hydration of cement in the concrete.

In fact, the effect of water viscosity in pores is not taken into account in Eq. (7), which disagrees with the experiments on rocks in recent investigations (Mavko and Jizba, 1991; Berryman et al., 2002; Xi et al., 1999). Although some improvement in shear modulus was found in their studies, a rational explain was absent especially for the quantitative analysis. Concrete has the similar properties to rocks in some extent, so the water viscosity should affect the properties of concrete. The viscosity of fluid relates not only with the essential property of the fluid, but also with the effective aperture of pores and cracks. Generally speaking, the viscosity of water in the aperture below 1000 Å is greater than the common state contributed to the surface force of the solid (Mehta and Nonteiro, 1997; Maekawa et al., 1999), whereas the pores with apertures less than 1000 Å in plain concrete are principal fraction. The influence of water, especially those in micro-scale pores should be taken into account, because the viscosity of water in these pores increases dramatically.

As we know, the water in two most-closed plates acts not as lubricated layer but as resistance force for the relative movement between them. Considering this classic case shown in Fig. 3, the distance between two plates is very small and \( h \) aparts with water filling the space between. The lower plate is stationary, while the upper one moves parallel to it with a velocity \( U \) due to a force \( F \) corresponding to some area \( A \) of the moving plate. According to the Newton’s equation of viscosity, the shear stress \( \tau \) can be expressed by (Finnemore and Franzini, 2002; Spurk, 1997; Streeter et al., 1998)
\[ \tau = \frac{F}{A} = \mu \frac{U}{h} = \mu \frac{du}{dy} = \mu \dot{\gamma} \]  
\[ \text{where } \mu \text{ is the coefficient of water viscosity, } \frac{du}{dy} \text{ is the velocity gradient, and } \dot{\gamma} \text{ is shear rate and equals to } \frac{du}{dy}. \]  
\[ \text{The resistance force or the shear force of the plate, because of the movement and viscosity, can be derived from Eq. (10). From this equation, the force } F \text{ is proportional to the distance } h \text{ inversely and to the changing rate of the displacement of the solid normally. For dry concrete in pure shear state, the elastic relationship between shear stress and shear strain is} \]
\[ \tau = G_{dry} \dot{\gamma} \]  
\[ \text{where } G_{dry} \text{ is the shear modulus of dry concrete, and } \gamma \text{ is the shear strain.} \]
Based on the theory of mean stress, the shear stress of saturated concrete is

$$\tau_{\text{sat}} = (1 - q)\tau_s + q\tau_w$$

$$\tau_{\text{dry}} = (1 - q)\tau_s = G_{\text{dry}}\gamma$$

(12)

(13)

where \((1 - q)\) is the fraction of solid area in the shear section, and \(q\) is the fraction of pore area in the shear section.

$$\tau_{\text{sat}} = G_{\text{dry}}\gamma + q\mu\gamma = \left( G_{\text{dry}} + q\mu \gamma \right)\gamma$$

(14)

$$G_{\text{sat}} = (1 + mj)G_{\text{dry}}$$

(15)

with \(m = \mu \gamma / j\).

The shear modulus of saturated concrete is affected by the porosity, viscosity of water and shear rate. \(\tau_w\) cannot be neglected because of the increasing viscosity of pore water. And the viscosity of water should adopt the effective viscosity that takes the increase of viscosity in the microapertures into account. An important influencing factor besides the inertia for the increasing mechanism of plain concrete elastic modulus under high loading rate can be found by the former analysis.

Based on the aforementioned study, the shear modulus of saturated concrete follows the quadratic function of \(S_{\text{eff}}\) considering the influence of the water viscosity in pores.

$$G_1 = [1 + f_1(S_{\text{eff}}\phi)^2 + f_2S_{\text{eff}}\phi]G_1$$

(16)

where \(f_1\) and \(f_2\) are constants and can be obtained from regression analysis of experiments. \(f_1\) and \(f_2\) equals to 0.18 and 0.2 accordingly in this investigation. \(G_1\) is the shear modulus of saturated concrete. \(f_1\) and \(f_2\) will change under high loading rate.

Next, the saturated concrete is homogenized to an equivalent medium whose bulk and shear modulus equals to the saturated concrete and can be obtained from Eqs. (6), (7) and (16). The dry pores are embedded into this equivalent medium consequently, and the bulk and shear moduli of unsaturated concrete are derived accordingly based on the two-phase composite model. The soft influence of cracks on the modulus of concrete is taken into account in this stage.

$$K_{\text{unsat}} = K_1 - \frac{(1 - S_{\text{eff}})\phi K_1^2}{(1 - S_{\text{eff}})\phi K_1 + (1 - \phi + S_{\text{eff}}\phi)\pi\alpha\beta_1}$$

(17)

$$G_{\text{unsat}} = G_1 - \frac{(1 - S_{\text{eff}})\phi\eta_1 G_1}{1 - (1 - S_{\text{eff}})(1 - \eta_1)\phi}$$

(18)

The relevant expressions for \(\beta_1\) and \(\eta_1\) are given as

$$\beta_1 = \frac{G_1(3K_1 + G_1)}{3K_1 + 4G_1}$$

(19)

$$\eta_1 = \frac{1}{5} \left( 1 + \frac{8G_1}{\pi\alpha(G_1 + 2\beta_1)} + \frac{4G_1}{3\pi\beta_1} \right)$$

(20)

where \(K_{\text{unsat}}\) and \(G_{\text{unsat}}\) are the elastic bulk and shear moduli of unsaturated concrete, respectively, \(\alpha\) is the equivalent aspect ratio reflecting the oblation of spherical inclusion and given as

$$\alpha = \frac{1}{N} \sum_{i=1}^{N} \frac{a_i}{b_i}$$

(21)

where \(a_i\) and \(b_i\) are the lengths of minor and major axes of different pores, \(N\) is the total number of different pores. The impact of equivalent aspect ratio on elastic modulus of dry concrete is discussed in Fig. 4, and the basic numerical data are listed in Table 1. From this figure, it can be seen that penny shape cracks or disks soften Young’s modulus very much.
When concrete is dry ($S_{\text{eff}} = 0$) in the home state, Eq. (17) can be developed into Eq. (22), which is used to calculate the effective bulk modulus of dry concrete. The expression of Eq. (22) is similar with the formula of Benveniste (1987).

$$K_{\text{dry}} = K_M - \frac{\phi K_M^2}{\phi K_M + (1 - \phi)\pi\alpha\beta}$$

When concrete is saturated, Eq. (17) can be developed into Eq. (23), which is used to calculate the effective bulk modulus of saturated concrete without drain in the home state.

$$K_{\text{sat}} = K_M - \frac{\phi_{\text{eff}} K_M (K_M - K_w)}{\phi_{\text{eff}} K_M + (1 - \phi_{\text{eff}})\pi\beta}$$

The results of shear modulus of unsaturated, saturated and dry concrete are compared in Fig. 5. From this figure, it can be seen that the shear modulus is improved a little, and the increment becomes large with the increasing of the effective saturation degree.

![Fig. 4. Effect of aspect ratio $\alpha$ on Young's modulus.](image)

![Fig. 5. Shear modulus of unsaturated concrete.](image)
4.2. Young’s modulus of unsaturated concrete

When $K_{\text{unsat}}$ and $G_{\text{unsat}}$ are known, the Young’s modulus of unsaturated concrete $E_{\text{unsat}}$ can be obtained easily based on the theorem of elastic mechanics.

$$E_{\text{unsat}} = \frac{9K_{\text{unsat}}G_{\text{unsat}}}{3K_{\text{unsat}} + G_{\text{unsat}}}$$  

(24)

The comparison between Eq. (24) and experiments of Yaman et al. (2002a,b) is illustrated in Fig. 6. The equivalent aspect ratio $\alpha$ equals to 0.2 in the model. From Fig. 6, it can be seen that the model in this paper fits well with the experiments in dry and saturated concretes when the pores’ aspect is taken into account. Young’s modulus of wet concrete is improved with the increasing of saturation degree. Water in saturated pores limits the deformation and enlarges the stiffness of concrete.

![Fig. 6. Young’s modulus of unsaturated concrete.](image)

![Fig. 7. Poisson’s ratio of unsaturated concrete.](image)
5. Poisson’s ratio of unsaturated concrete

Poisson’s ratio of unsaturated concrete $\nu_{\text{unsat}}$ can also be obtained by the elastic mechanics and given as

$$
\nu_{\text{unsat}} = \frac{1}{2} \left( 1 - \frac{1}{1/3 + K_{\text{unsat}} / G_{\text{unsat}}} \right)
$$

(25)

The comparison between Eq. (25) and experiments of Yaman et al. (2002a,b) is plotted in Fig. 7, and a good agreement is shown between experimental results and theoretical model for saturated concrete. The water in saturated pores limits the concrete deforming into pores, so Poisson’s ratio of wet concrete improves with the increasing of saturation degree. From the formula evaluated above, we can see that $\nu_{\text{unsat}}$ is sensitive to the aspect ratio and water content.

6. Conclusions

The elastic properties of unsaturated concrete are dependent on the contained water, pore geometry and the matrix concrete elasticity. From the theoretical models and experiment, we can see that the elliptic pores and cracks soften the stiffness of concrete, but the water in the fully filled pores restricts the deformation of matrix spreading into the pores and increases the stiffness of wet concrete without drain in the home state. The water and aspect ratio of pores are coupled in developing the model of unsaturated concrete. From the analysis of this paper, the elastic modulus and Poisson’s ratio of wet concrete increase comparing with dry samples, which agrees with the conclusion of Mehta and Nonteiro (1997). And the elastic modulus and Poisson’s ratio of unsaturated concrete are improved with the increasing of effective saturation degree.

The model includes the effects of viscosity of water, share rate and hydration of cement on the moduli of concrete. The viscosity of water in micro-cracks increases dramatically which supplies baffling effect on the deformation of concrete. The shear and Young’s moduli increase with the enhancement of share rate. And the consequent hydration of cement reduces the porosity of concrete and lessens the radius of pores, which increases the moduli of unsaturated concrete.

From Figs. 6 and 7, we can see that the models in this paper fit well with the results of experiments. And the model proposed in this paper is valid to predict the elastic parameters of wet concrete.

Acknowledgements

This work was partly supported by the National Key Basic Research and Development Program (973 Program, No. 2002CB412709), by the National Science Foundation (No. 50225927, 90210010) and by MOE National Key Basic Research Program (No. 2004002).

References