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GRIZZLY BEARS IN YELLOWSTONE NATIONAL PARK

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Abstract—The problem we solved is based on the population of grizzly bears at Yellowstone National Park. Since this population is currently declining, our specific problem centered around a seed group of 52 grizzlies, transported from Yellowstone to another area in the Northwestern United States, similar in climate and availability of proper food. The total land area available for the bears is 1.5 million acres, enough land for 100 bears to thrive. Our problem was to find a harvesting policy to sustain the maximum number of grizzlies on this land.

Using the matrix equation $Lx_i = x_{i+1}$ we determined the this seed population would exceed the level the land area can maintain after 14 years. At this point we began to implement our harvesting procedures. Assuming the bears would be harvested at random, producing a uniform harvesting rate for each age group, we used the matrix equation $Lx - HLx = x$ to solve for a total of 3 percent harvesting yearly after the fourteenth year. Test results confirmed the accuracy of our matrix values.

HARVESTING A GRIZZLY BEAR POPULATION

One naturally-occurring animal population found in an environment with resource constraints is that of the grizzly bear in Yellowstone National Park. However, the Park's resource constraints have become so great that recently the grizzly population has been declining. For our problem we decided to form a hypothetical seed population from members of the Yellowstone community, and then transplant this seed population in order to give the American grizzly a better chance of survival. The seed population would be taken to a large wilderness area in the Northwestern United States with a climate similar to that of Yellowstone Park. The amount of suitable land would be proportionate to Yellowstone and the amount of berries, nuts, forbs, graminales, and animal prey would also be proportionate. Based on the theory that overpopulation creates stress for the animal, thus causing harmful population decreases[1], the size of the seed-generated population would need to be controlled. Controlling the population would entail harvesting it. By harvest we mean the systematic removal of bears from the population. The population will be controlled even if the selection of bears to be harvested is random. The seed population would be transplanted to an area of 1.5 million acres, an area approximately large enough for 100 bears to live healthy lives. This parameter is based on the theory that the 300 to 350 grizzlies in Yellowstone Park need 4.4 million acres to thrive[2]. This 1.5 million acre assumption was made to clarify the problem.

Using population statistics gathered from a healthy and growing grizzly population in Yellowstone Park (the study was conducted from 1957–69 by J. J. Craighead *et al.*[3]), we determined a Leslie Matrix to generate the growth of our seed population. The survivorship rates (b_i's) were determined from a study from Yellowstone National Park, found in *Wild Mammals of North America*. The reproductive rate for each age class (a_i's) was determined by using the average reproductive rate of the entire female population as a guideline. See Table 1 and Table 2 for the contents of the Leslie Matrix.

Table 1.

Age Class	Ai	Bi
[0, 1]	0.0	0.6296
[1, 2]	0.0	.9529
[2, 3]	0.0	.6790
[3, 4]	0.0	.9091
[4, 5]	0.1850	.8200
[5, 6]	.2240	.9756
[6, 7]	.2583	.9500
[7, 8]	.2836	.9737
[8, 9]	.3550	.9730
[9, 10]	.5123	.9444
[10, 11]	.7790	.9706
[11, 12]	.7790	.9344
[12, 13]	.7430	.9032
[13, 14]	.5040	.8571
[14, 15]	.4636	.7917
[15, 16]	.3200	.7368
[16, 17]	.2500	.8571
[17, 18]	.2302	.7500
[18, 19]	.1962	.8889
[19, 20]	.1730	.7500
[20, 21]	.0988	.6667
[21, 22]	.0850	.7500
[22, 23]	.0880	.6667
[23, 24]	.0510	.5000
[24, 25]	.0327	.5000
[25, 26]	0.0	0.0

The model we used to develop the harvesting process was based on the Leslie Matrix. A Leslie Matrix is designed to generate the population of any future period by multiplying the previous period's population by the reproductive and survivorship rates of each female age class. We divided our population into one year classes, and so, by the laws of the Leslie Matrix, the period of growth is also one year, so,

$$Lx_i = x_{i+1}$$

for any year i, a population of x_i, and the Leslie Matrix L.

Starting with the seed population, the Leslie Matrix generated succeeding populations of increasing size. See Table 3 and Fig. 1. This implied that the grizzly population would eventually attain and surpass a limit matrix, called X_m. X_m is any matrix such that the summation of the values in the column vector X_m equals the size of the maximum healthy population which the environment could sustain. After the population exceeds X_m, $Lx_i = x_{i+1} \geq X_m$, then we would want to begin harvesting the grizzly to prevent over-population. The equation we used to represent harvesting to maintain constant population was

$$Lx - HLx = x$$

where L is the Leslie Matrix and H is the harvesting matrix. Each h in H, is the amount of each class removed from the population. Lx is the growth for one year; HLx is the amount of this growing population removed, and x becomes the maximum population, or $x = X_m$.

Table 2. Leslie Matrix

0	0	0	0	.185	.224	.2583	.2836	.355	.5123	.779	.779	.743	.504	.4636	.32	.25	.2302	.1962	.173	.0988	.085	.088	.051	.0327	0
.6296	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	.9529	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	.679	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	.9091	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	.82	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	.9756	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	.95	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	.9737	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	.973	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	.9444	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	.9706	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	.9344	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	.9032	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	.8571	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	.7917	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.7368	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.8571	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.75	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.8889	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.75	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.6667	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.75	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.6667	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.5	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.5	0

Table 3. Twenty generations generated by the Leslie Matrix, without harvesting.

Female pop.	Total pop.	Year
26	52	0
31.4772	62.9544	1
35.06585812	70.13171624	2
38.55309342814	77.10618685627	3
40.4178632828	80.8357265656	4
41.65177528744	83.30355057488	5
42.42875314236	84.85750628471	6
43.06005059941	86.12010119882	7
43.54340970991	87.08681941982	8
43.79455068739	87.58910137478	9
44.10615480056	88.21230960112	10
44.65898376734	89.31796753468	11
45.98606470045	91.9721294009	12
47.92605089839	95.85210179678	13
50.34491045951	100.689820919	14*
52.67350776302	105.3470155261	15
54.81979699157	109.6395939831	16
56.57915357924	113.1583071585	17
58.15722741936	116.3144548387	18
59.60465086233	119.2093017247	19
61.05808789868	122.1161757974	20

* pop. > X_m

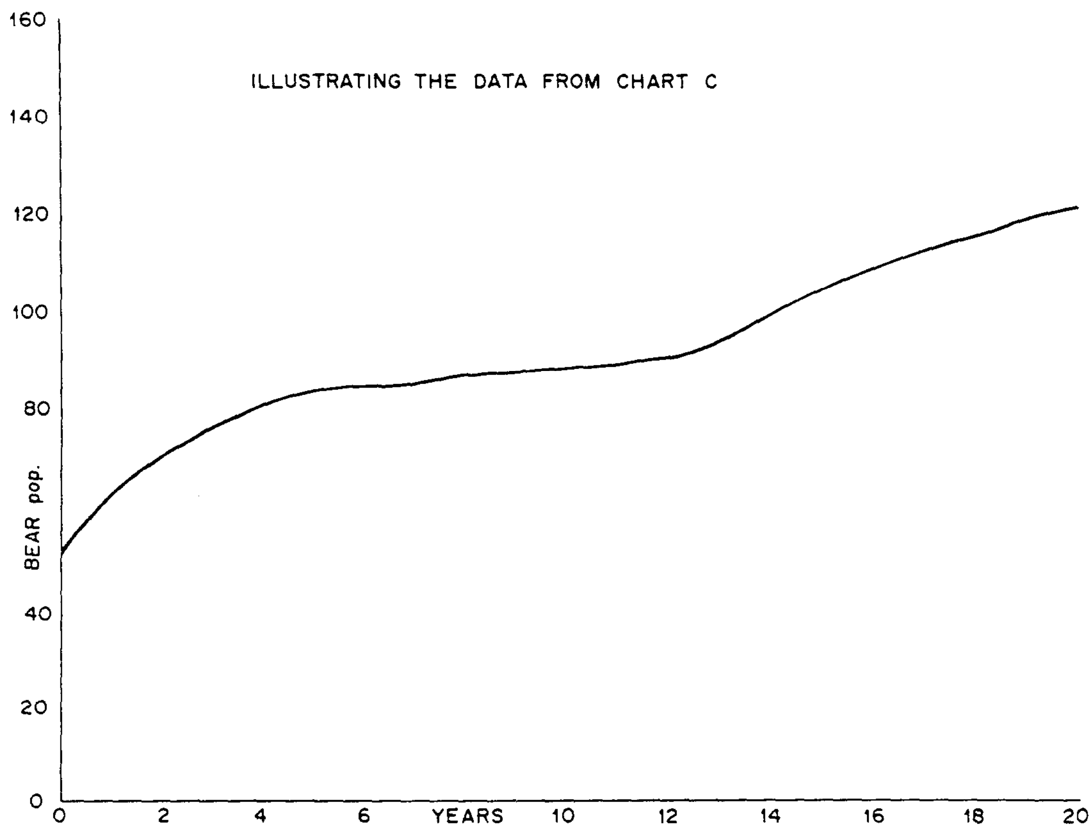

Fig. 1

When we used the Leslie Matrix it was restricted to the female population. Based on data gathered at Yellowstone, we assumed that the total population would grow in proportion to the female population. The ratio of male to female bears in the original seed group and the growing seed population would thus be 1:1. Even though the number of males and females in each age group at Yellowstone was not even, the totals were very close. After inspecting the data on the total population of the seed group, it was obvious that the bears would reach X_m, or 100 bears, after the fourteenth year. The problem then became solving for the amount of bears harvested. Because the ratio of males to females was 1:1, we assumed that any harvesting would have an equal effect on both sexes and that by harvesting either sex the population would decrease by the same amount. Because this model is designed for harvesting wild bears, we felt that the animals were likely to be caught at random and that the amount of bears taken from any age class would be the same, or $h_1 = h_2 = \cdots = h_{26}$. This is our basic model equation for H, and reflects our proposed policy of uniform harvesting.

$$LX_m - HLX_m = X_m \text{ reduces to } LX_m = (1/(1 - h))X_m.$$

Hence $1/(1 - h)$ would be an eigenvalue for L. Using the characteristic equation $\det(\lambda L - I) = 0$, we solved for λ. $\lambda = 1/(1 - h)$ implies that $h = 1 - (1/\lambda)$. Using the characteristic polynomial equation for $\det(\lambda I - L)$:

$$\begin{aligned} p(\lambda) = {} & (\lambda^{26}) - .068511773(\lambda^{21}) - .068022932(\lambda^{20}) - .076525034\,(\lambda^{19}) \\ & - .079819491(\lambda^{18}) - .097287323(\lambda^{17}) - .131267839(\lambda^{16}) \\ & - .1889126(\lambda^{15}) - .190403296(\lambda^{14}) - .170598957(\lambda^{13}) \\ & - .104520631(\lambda^{12}) - .082403643(\lambda^{11}) - .045031209(\lambda^{10}) \\ & - .02592109(\lambda^{9}) - .020457382(\lambda^{8}) - .013076906(\lambda^{7}) \\ & - .010249555(\lambda^{6}) - .004390127(\lambda^{5}) - .002458831(\lambda^{4}) \\ & - .001955215(\lambda^{3}) - .000755462(\lambda^{2}) - .000242192(\lambda) = 0, \end{aligned}$$

we determined the Leslie Matrix's unique positive eigenvalue. We used the quotient-difference method to obtain an approximate λ value of $\lambda = 0.9728648613429$, which was not very accurate. Using this as a starting point, we substituted various values into $p[\lambda] = 0$ until we had ($\lambda = 1.0296$) with an error of 0.0004. Using $\lambda = 1.0296$ we derived $h = 1 - (1/\lambda)$, or $h = 0.0288$, or approximately 3 percent. We felt a 3 percent harvest would be reasonable in a natural setting. After solving for λ, we were able to determine the harvest rate for each age class and determine an eigenvector X_1, see Table 4. This eigenvector generates the proportion of bears in each class after the harvest if the first age class is proportionately equal to 1.

By multiplying X_1 by the number of females in the first age class when the population equalled X, we were able to determine if the proportions were the same before and after the harvest at X_m. This proved to be off by a certain amount. We accounted for this error by the fact that X_1 is a limit value, see Fig. 2. The solution to maintaining the population at around 100 bears after year 14 would be to randomly remove 2.875 percent $\cong$ 3 percent of the total population which we would assume is about 3 percent uniformly from each age class.

One way we tested the harvesting model was to test $LX_1 - HLX_1 = X_1$ to see if it was true, therefore testing the validity of our eigenvalue. We found that values were very close, and therefore assumed that the values of λ and h were reasonable. Another way we tested our model was to chart the natural growth of our seed population, see Table 5, for the program used. When the total population exceeded the limit of 100, we began harvesting using the equation $Lx - HLx = x$, where x represents the current numbers

Table 4. Proportions of age classes with respect to the first age class

Class	Year 0	Year 15	Year 30	Year 45	Eigenvector X1
1	1.0	1.0	1.0	1.0	1.0
2	1.0	.6024	.6141	.6115	.6115
3	1.0	.5296	.5696	.5662	.5659
4	1.0	.3263	.3749	.3737	.3732
5	1.0	.2715	.3285	.3301	.3295
6	1.0	.2175	.2585	.2628	.2625
7	1.0	.2150	.2415	.2487	.2487
8	1.0	.2106	.2204	.2289	.2295
9	1.0	.2088	.2078	.2157	.2170
10	1.0	.2069	.1975	.2030	.2051
11	1.0	.1963	.1831	.1856	.1881
12	1.0	.1994	.1755	.1747	.1773
13	1.0	.1904	.1627	.1595	.1609
14	0.0	.1683	.1457	.1402	.1411
15	0.0	.1362	.1230	.1172	.1175
16	0.0	.0262	.0944	.0905	.0903
17	0.0	.0306	.0665	.0651	.0647
18	0.0	.0276	.0524	.0543	.0538
19	0.0	.0304	.0358	.0395	.0465
20	0.0	.0298	.0291	.0338	.0338
21	0.0	.0272	.0213	.0243	.0247
22	0.0	.0186	.0144	.0155	.0160
23	0.0	.0147	.0112	.0112	.0116
24	0.0	.0101	.0076	.0072	.0075
25	0.0	.0052	.0039	.0035	.0037
26	0.0	.0027	.0019	.0017	.0018

Note: figures reflect natural growth rate for years 0, 15, 30, 45 as they approach the EIGENVECTOR proportions.

of each female age class. From year fourteen, when the population first exceedeed 100, to year fifty, the population went from 100.7 to 101.6. The highest it every reached was approximately 105. See Table 6. These results led us to believe that the values for the matrix H were quite accurate.

Due to the nature of our problem and our data, there are many possible errors. Many of these errors come from our inability to evaluate and analyze the data collected from our sources. The original data on the Yellowstone grizzlies was only to four significant digits, and the methods used to collect this data were unknown to us. We had to create logical a_i's for our Leslie Matrix and had no real way to test their validity. When we solved for λ, we found only a limit value and were able to find λ to three significant digits with an error of 0.0004. Our results were also subject to rounding errors because all data was rounded to four decimal places for calculations. When introducing a seed population, we assumed it would have population statistics similar to those at Yellowstone and that when introducing these bears into a new ecosystem they would not significantly change the ecosystem. The fact that we must limit the seed population of the grizzlies to a maximum of 100 means that the numbers for each age class will be relatively low (see Table 7). Therefore, it will be impossible to realistically harvest 3 percent of each age class. The actual figures will range from 0 percent to as high as 100 percent for the upper age classes! With harvesting we allowed for uniform harvesting in all age classes and that removing both males and females would have the same effect on the population. From our research we know that this is a somewhat inaccurate assumption and would lead to a certain amount of error in figuring the harvesting numbers and the post-harvesting population level. Even though our assumptions and data allow for a certain amount of error, the methods of this model provide a reasonable basis for planning population growth and a population harvest system as demonstrated in our harvest projections.

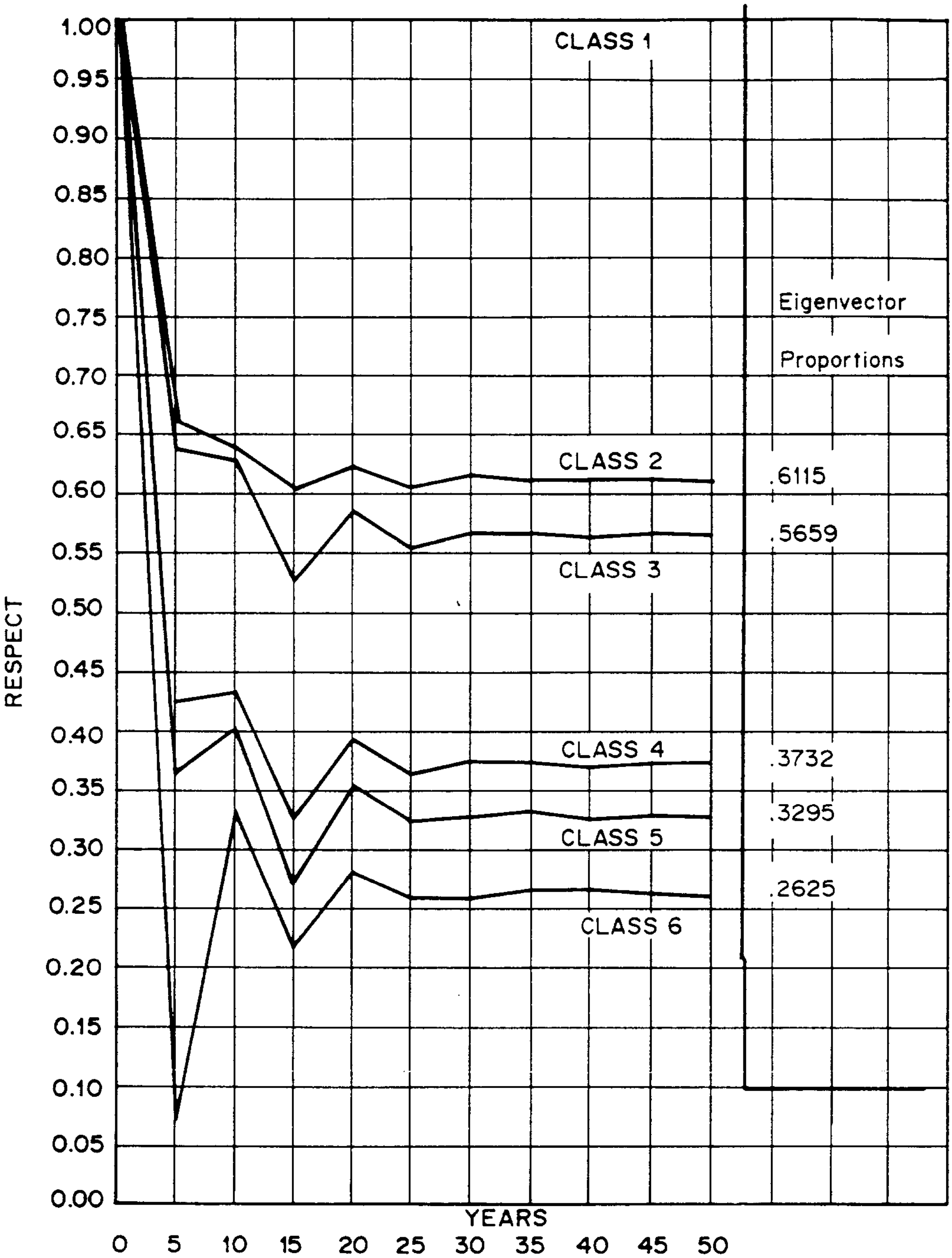
1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0.60
0.55
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
RESPECT
CLASS 1
CLASS 2
CLASS 3
CLASS 4
CLASS 5
CLASS 6
Eigenvector
Proportions
.6115
.5659
.3732
.3295
.2625
YEARS
0 5 10 15 20 25 30 35 40 45 50

Fig. 2.

Table 5. Program Seedtest

```
1 PRINT
2 PRINT
3 PRINT
4 PRINT
5 PRINT
6 PRINT
7 PRINT
10 DIM B(26, 26)
20 MAT B = ZER
200 B(2, 1) = .6296
201 B(3, 2) = .9529
202 B(4, 3) = .6790
203 B(5, 4) = .9091
204 B(6, 5) = .8200
205 B(7, 6) = .9756
206 B(8, 7) = .9500
207 B(9, 8) = .9737
208 B(10, 9) = .9730
209 B(11, 10) = .9444
210 B(12, 11) = .9706
211 B(13, 12) = .9394
212 B(14, 13) = .9032
213 B(15, 14) = .8571
214 B(16, 15) = .7917
215 B(17, 16) = .7368
216 B(18, 17) = .8571
217 B(19, 18) = .7500
218 B(20, 19) = .8889
219 B(21, 20) = .7500
220 B(22, 21) = .6667
221 B(23, 22) = .7500
222 B(24, 23) = .6667
223 B(25, 24) = .5000
224 B(26, 25) = .5000
349 B(1, 5) = .185
350 B(1, 6) = .224
351 B(1, 7) = .2583
352 B(1, 8) = .2836
353 B(1, 9) = .355
354 B(1, 10) = .5112
355 B(1, 11) = .7790
356 B(1, 12) = .7790
357 B(1, 13) = .743
358 B(1, 14) = .504
359 B(1, 15) = .4636
360 B(1, 16) = .32
361 B(1, 17) = .2500
362 B(1, 18) = .2302
363 B(1, 19) = .1962
364 B(1, 20) = .173
365 B(1, 21) = .0988
366 B(1, 22) = .083
367 B(1, 23) = .088
368 B(1, 24) = .051
369 B(1, 25) = .0327
400 DIM S(26, 1)
410 MAT S = ZER
411 S(1, 1) = 2
412 S(2, 1) = 2
413 S(3, 1) = 2
```

Table 5. (*Continued*)

```
414 S(4, 1) = 2
415 S(5, 1) = 2
416 S(6, 1) = 2
417 S(7, 1) = 2
418 S(8, 1) = 2
419 S(9, 1) = 2
420 S(10, 1) = 2
421 S(11, 1) = 2
422 S(12, 1) = 2
423 S(13, 1) = 2
800 MAT P = B*S
820 FOR W = 1 TO 26
830 T1 = T1 + S(W, 1)
832 T2 = T2 + P(W, 1)
840 NEXT W
845 IF L = 0 THEN PRINT T1, (2*T1)
850 IF (2*T2) < 100 THEN PRINT T2, (2*T2)
900 IF (2*T2) >= 100 GOTO 1000
905 L = L + 1
906 IF L = 51 GOTO 9990
907 IF L = 31 THEN PRINT
908 IF L = 31 THEN PRINT
909 IF L = 31 THEN PRINT
910 IF L = 31 THEN PRINT
911 IF L = 31 THEN PRINT
912 IF L = THEN INPUT Q
913 IF L = THEN PRINT
914 IF L = 31 THEN PRINT
915 IF L = 31 THEN PRINT
916 IF L = 31 THEN PRINT
917 IF L = 31 THEN PRINT
918 IF L = 31 THEN PRINT
919 IF L = 31 THEN PRINT
920 IF L = 31 THEN PRINT
923 MAT S = P
925 T1 = 0
926 T2 = 0
930 GOTO 800
1000 DIM H(26, 26)
1010 PRINT T2, (2*T2),' > Xm'
1100 MAT H = ZER
1200 FOR J = 1 TO 26
1300 H(J, J) = .028749029
1400 NEXT J
2000 MAT M = P
2100 MAT N = H * M
2200 MAT O = M - N
2350 FOR V = 1 TO 26
2360 T3 = T3 + O(V, 1)
2370 NEXT V
2371 E = (2*T2) - (2*T3)
2372 PRINT T3, (2*T3),'after harvesting', E
2390 MAT P = O
2395 T3 = 0
2400 GOTO 905
9990 PRINT
9991 PRINT
9992 PRINT
9993 PRINT
9999 END
```

Table 6.

$X_m = 100$

If total pop. > X_m then a harvest will take place.

Year	Female pop.	Total pop.		# of bears harvested
0	26	52		
1	31.4772	62.9544		
2	35.06585812	70.13171624		
3	38.55309342814	77.10618685627		
4	40.4178632828	80.8357265656		
5	41.65177528744	83.30355057488		
6	42.42875314236	84.85750628471		
7	43.06005059941	86.1201119882		
8	43.54340970991	87.08681941982		
9	43.79455068739	87.58910137478		
10	44.10615480056	88.21230960112		
11	44.65898376734	89.31796753468		
12	45.98606470045	91.9721294009		
13	47.92605089839	95.85210179678		
14	50.34491045951	100.689820919	> Xm	
	48.8975431687	97.79508633741	after harvesting	2.894734581602
15	51.15919556081	102.3183911216	> Xm	
	49.68841836402	99.37683672803	after harvesting	2.941554393588
16	51.71307405237	103.4261481048	> Xm	
	50.22637338676	100.4527467735	after harvesting	2.973401331223
17	51.83831114907	103.6766222982	> Xm	
	50.34801003854	100.6960200771	after harvesting	2.980602221071
18	51.75228833748	103.504576675	> Xm	
	50.26446029925	100.5289205985	after harvesting	2.97565607646
19	51.51544769005	103.0308953801	lmt Xm	
	50.03442859046	100.0688571809	after harvesting	2.962038199176
20	51.25449934927	102.5089986985	> Xm	
	49.78098226109	99.56196452219	after harvesting	2.947034176345
21	51.00308666916	102.0061733383	> Xm	
	49.53679745142	99.07359490284	after harvesting	2.932578435482
22	50.78168136288	101.5633627258	> Xm	
	49.32175733271	98.64351466542	after harvesting	2.919848060341
23	50.68717440536	101.3743488107	> Xm	
	49.22996735846	98.45993471691	after harvesting	2.914414093814
24	50.7409862052	101.4819724104	> Xm	
	49.2822321213	98.5644642426	after harvesting	2.917508167802
25	50.91797970166	101.8359594033	> Xm	
	49.4541372266	98.90827445319	after harvesting	2.927684950127
26	51.1601385684	102.3202771368	> Xm	
	49.68933426106	99.37866852211	after harvesting	2.94160861469
27	51.38901072881	102.7780214576	> Xm	
	49.91162656908	99.82325313816	after harvesting	2.954768319446
28	51.55814449793	103.1162889959	> Xm	
	50.07589790658	100.1517958132	after harvesting	2.964493182714
29	51.64972014687	103.2994402937	> Xm	
	50.16484084453	100.3296816891	after harvesting	2.969758604689
30	51.66933998387	103.3386799677	> Xm	
	50.18389663026	100.3677932605	after harvesting	2.970886707213
31	51.64406529255	103.2881305851	> Xm	
	50.15934856178	100.3186971236	after harvesting	2.969433461545
32	51.59859032691	103.1971806538	> Xm	
	50.11518095724	100.2303619145	after harvesting	2.966818739337
33	51.55597064498	103.11194129	> Xm	
	50.07378654979	100.1475730996	after harvesting	2.96438190388
34	51.53425530959	103.0685106192	> Xm	
	50.0526955092	100.1053910184	after harvesting	2.963119600778
35	51.54459122734	103.0891824547	> Xm	
	50.06273427935	100.1254685587	after harvesting	2.963713895973
36	51.59071336005	103.1814267201	> Xm	
	50.10753044554	100.2150608911	after harvesting	2.966365829037
37	51.66528062511	103.3305612502	> Xm	

Table 6. (*Continued*)

	50.17995397413	100.3599079483	after harvesting	2.970653301972
38	51.75496160255	103.5099232051	> Xm	
	50.26705671054	100.5341134211	after harvesting	2.97580978401
39	51.84422535008	103.6884507002	> Xm	
	50.353754212	100.707508424	after harvesting	2.980942276142
40	51.9205802413	103.8411604826	> Xm	
	50.42791397424	100.8558279485	after harvesting	2.985332534107
41	51.97818845904	103.9563769181	> Xm	
	50.48386601167	100.9677320233	after harvesting	2.988644894752
42	52.01746145179	104.0349229036	> Xm	
	50.522009944	101.044019888	after harvesting	2.990903015567
43	52.04346320132	104.0869264027	> Xm	
	50.54726416849	101.094528337	after harvesting	2.99239806567
44	52.06318485153	104.1263697031	> Xm	
	50.5664188404	101.1328376808	after harvesting	2.993532022258
45	52.08330099874	104.1666019975	> Xm	
	50.58595666792	101.1719133358	after harvesting	2.994688661656
46	52.10909740724	104.2181948145	> Xm	
	50.61101145471	101.2220229094	after harvesting	2.996171905049
47	52.14353802646	104.2870760529	> Xm	
	50.64446193957	101.2889238792	after harvesting	2.998152173773
48	52.18703648501	104.37407297	> Xm	
	50.68670985968	101.3734197194	after harvesting	3.000653250666
49	52.23775646824	104.4755129365	> Xm	
	50.73597169264	101.4719433853	after harvesting	3.003569551201
50	52.29232406008	104.5846481202	> Xm	
	50.7889705192	101.5779410384	after harvesting	3.00670708176
51	52.34702308237	104.6940461648	> Xm	
	50.84209699772	101.6841939954	after harvesting	3.009852169316

Table 7.

Age class	Number of bears in each age class
0–1	10.28584316328
1–2	5.945030074955
2–3	5.167011995047
3–4	3.210371186269
4–5	2.851561113715
5–6	2.369621117632
6–7	2.383560784069
7–8	2.305685637626
8–9	2.286038072135
9–10	2.234056276132
10–11	2.208965889327
11–12	2.178864616081
12–13	2.002787712922
13–14	1.708041317855
14–15	.3554945760263
15–16	.4470220073698
16–17	.3456457288593
17–18	.436077381522
18–19	.3599502844727
19–20	.3901948876437
20–21	.2999653195293
21–22	.210512503716
22–23	.1621488936911
23–24	.1111044886165
24–25	.05882279151655
25–26	.0303022828748

Note: Total Females Total Bears Year
50.34491045951 100.689820919 14

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