Analysis and Optimal Design about a kind of Bearing Beam with Sleeve Joint Structure

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Abstract
Bearing beam with sleeve joint structure is widely used in telescopic machinery. The stress state is special with this kind of structure. Analysis about the sleeve joint structure in cantilever state was conducted. And the mathematical model was proposed after summarizing its stress characteristic. And then according to the mathematical model, optimal model, with bending section modulus as objective function, was presented. The optimal result was given after optimal program calculated. Finally a finite element analysis was carried out to check the optimal result. The analysis was shown that the optimal result is available. And besides avoiding the blindness, the exact parameters can be quickly found by using this optimal model.

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Keywords: Sleeve joint; bearing beam; mathematical model; optical model; finite element analysis

1. Introduction

Sleeve joint structure is frequently used in telescopic machinery, such as fire vehicle scaling ladder, lifting crane. It will enlarge the working areas while being operated. And it will be retracted to make structure compact while being not. So it is convenient to install and transport.

The bearing beam with sleeve joint structure is shown in Figure.1.

Figure 1. The structure of bearing beam with sleeve joint structure

As shown in Figure.1, the beam is joined by some sub-beams. Each some sub-beam can slide in succession to complete the telescopic movement. The state stress of joints in each sub-beam is special [1]. And it will be affected by the load parameter, the length of each sleeve joint. The mathematical model will be set up, and then the optimal model, according to the relative parameter, load state and design requirement. So the optimal calculation can be executed by the optimal program. And the optimal solution will be found. An accurate and pointed design is completed.
2. The stress analysis of bearing beam with sleeve joint structure

Assuming the maximum length of bearing beam is $L$ when extending, and the minimum is $l$. when retracting. The length of each sub-beam is $l_i$, and the sleeve joint is $x_i$. One side of bearing beam is fixed to the ship body while installing. And the other side is cantilevered with force $P$ loaded. According to the parameter given above, analyze the stress state of bearing beam. The forced diagram of bearing beam is shown in Figure 2.

![Figure 2. Force diagram of bearing beam with sleeve joint structure](image)

As being shown in Figure 2 above, the bearing beam is composed of four sub-beams of which the length are $l_0$, $l_1$, $l_2$, and $l_3$ respectively. And the lengths of sleeve joint are $x_0$, $x_1$, $x_2$, and $x_3$ respectively. (In fact $x_3$ is not the sleeve joint length, but the distance between two supporting points on sub-beam 4. But its mechanical principle is similar to others.) The self weight of sub-beams are $P_0$, $P_1$, $P_2$ and $P_3$ respectively. Assuming self weight of each sub-beam is uniformly distributed on length. So the point of force locates on the midpoint of each sub-beam. The joint points of sub-beam are named $ij$ ($i=1,2$. It implies points that are in the same sub-beam. The larger number sites on the right side.) ($j=1, 2, 3, 4$. It implies the number of sub-beam. Also the larger number sites on the right side.) According to the distribution, point 13 and 23 are the supporting points. Base on discussion above, separate sub-beam from bearing beam. And then the mathematical model of stress state can be set up after analyzing sub-beams. Finally the mathematical model of the whole bearing beam can be summed up by model of sub-beams.

2.1. Analysis of sub-beam 0

The figure 3 shown below is the forced diagram of sub-beam 0.

![Figure 3. The forced diagram of sub-beam 0](image)

Extract moment on point 10.

$$M_{10} = N_{20} \times x_0.$$  \hspace{1cm} (1)

Meanwhile

$$M_{10} / W_0 = \sigma_0.$$  \hspace{1cm} (2)

($W_0$: the section modulus in bending of sub-beam 0, $\sigma_0$: stress value of point 10.) According to formula(1), (2)

$$\sigma_{10} \times W_0 = x_0 \times N_{20}.$$  \hspace{1cm} (3)

Base on moment balance, $\sum M_{10} = 0$

Then, $$N_{20} \times x_0 = P_0 \left( l_0 / 2 - x_0 \right).$$  \hspace{1cm} (4)

So, $$\sigma_0 = \left( P_0 l_0 / 2 - P_0 x_0 \right) / W_0.$$  \hspace{1cm} (5)

2.2. Analysis of sub-beam 0 and 1.

The forced diagram is shown in Figure 4 below.
Figure 4. The forced diagram of sub-beam 0 and 1

Extract moment on point 11.

\[ M_{11} = N_{21} \cdot x_i. \]  
\[ M_{11} / W_1 = \sigma_1. \]  

(\[ W_1 \]: the section modulus in bending of sub-beam 1, \[ \sigma_1 \]: stress value of point 11)

According to moment balance, \[ \sum M_{11} = 0 \].

Then, \[ N_{21} \cdot x_i = P_l ((l_1 / 2 - x_i) + P_0 l_1 - x_1 + l_0 / 2 - x_0). \]  

Integrate formula (6), (7), and (8)

\[ \sigma_1 = \left[ \left( P_0 l_1 + P_1 l_1 \right) / 2 + P_0 (l_1 - x_0) - \left( P_0 + P_1 \right) x_1 \right] / W_1. \]  

2.3. Analysis of sub-beam 0, 1 and 2.

The forced diagram is shown in Figure 5 below.

Figure 5. The forced diagram of sub-beam 0, 1 and 2

Extract moment on point 12.

\[ M_{12} = N_{22} \cdot x_2. \]  
\[ M_{12} / W_2 = \sigma_2. \]  

(\[ W_2 \]: the section modulus in bending of sub-beam 2, \[ \sigma_2 \]: stress value of point 12)

According to moment balance, \[ \sum M_{12} = 0 \].

\[ N_{22} \cdot x_2 = P_2 ((l_2 / 2 - x_2) + P_1 l_2 - x_2) \]
\[ + P_0 l_0 / 2 - x_0 + l_1 - x_1 + l_2 - x_2. \]  

Integrate formula (10), (11), and (12)

\[ \sigma_2 = \left[ \left( P_0 l_0 + P_1 l_1 + P_2 l_2 \right) / 2 + \left( P_0 + P_1 \right) x_1 \left( P_0 l_1 + P_1 l_1 \right) / 2 + \left( P_0 + P_1 + P_2 \right) x_1 \right] / W_2. \]  

2.4. Analysis of the whole bearing beam

\[ \sigma_3 = \left[ \left( P_0 l_0 + P_1 l_1 + P_2 l_2 \right) / 2 + \left( P_0 + P_1 + P_2 \right) x_1 \right] \]
\[ + \left( P_1 l_1 - x_1 \right) + \left( P_2 l_2 - x_2 \right) \]  
\[ \left( P_0 + P_1 + P_2 \right) x_1 \right] / W_2. \]  

(14)
Integrate formula (5), (9), (13) and (14), if there are several sub-beams, the stress value of sub-beam n can be described as follow:

\[
\sigma_n = \frac{\sum P_l \left( 2 + \sum_{j=0}^{n-1} P_j + \sum_{k=0}^{n-2} \sum_{l=0}^{k} P_l - \sum_{x=0}^{n} P_x \right)}{W_n}.
\]

(15)

If \( a_n = \sum_{i=0}^{n} P_i l_i / 2 \),

\[
b_n = (l_n - x_{n-1}) \sum_{j=0}^{n-1} P_j + (l_{n-1} - x_{n-2}) \sum_{k=0}^{n-2} P_k + \cdots + (l_1 - x_0) P_0,
\]

\[
c_n = x_n \sum_{s=0}^{n} P_s.
\]

Then \( \sigma_n = \frac{a_n + b_n - c_n}{W_n} \).

(16)

3. The optimization model of bearing beam with sleeve joint structure

According to formula (15), parameters that affect stress state at bearing beam are \( P, I, x \) and \( W \). \( P \) is determined by self weight and external load. And \( I \) is determined by design requirement, \( x \) and \( W \) are determined by particle situation.

But more attention is paid to the optimal section modulus in bending in the case of permissible stress. In other words, the task of optimization is to find the proper sub-beam. So \( W \) is defined as the objective function. \( x \) and \( W \) are defined as optimized parameters. \( \sigma \) is within the range of permissible stirs. So the optimal model of bearing beam with sleeve joint structure can be described as follow:

\[
W_n = \frac{a_n + b_n - c_n}{\sigma_n}.
\]

(17)

And \( a_n = \sum_{i=0}^{n} P_i l_i / 2 \).

\[
b_n = (l_n - x_{n-1}) \sum_{j=0}^{n-1} P_j + (l_{n-1} - x_{n-2}) \sum_{k=0}^{n-2} P_k + \cdots + (l_1 - x_0) P_0,
\]

\[
c_n = x_n \sum_{s=0}^{n} P_s.
\]

The constraint condition is shown in follow formulas:

\[
x_{\text{min}} \leq x_n \leq x_{\text{max}}, \quad \sigma_{\text{min}} \leq \sigma_n \leq \sigma_{\text{max}}.
\]

(18)

4. Optimal calculate to bearing beam of telescopic ladder.

Figure 6 below is three dimension model of bearing beam used in a telescopic ladder, which is applied in rescue vessel. And the sleeve joint structure is adopted to the beam.
As shown above, the bearing beam consists of five sub-beams. And there are two basis beam and one thin steel plate in a sub-beam. Channel steel, which has been modified, is the original model of basis beam. The thin steel plate, of which areas is cut to reduce self weight and wind resistance [2,4], is welded between two basis beams. Besides, the main forced part is not the thin steel plate but the basis beam. So it can be reduced to analyze one side of basis beam [5].

According to design requirement, the ladder has a maximum length of 20 meters while being extended, and a minimum length of 5 meters while being retracted. And the cantilevered side can stand 3 persons’ weight while working. Because the basis beams of each sub-beam are derived from channel steel, self weight can be defined by the type channel steel. So the optimization model can be described as finding the optimal section modulus in bending $W_n$, basing on permissible stress $\sigma$ and sleeve joint length $x_s$. In other words, the optimization program aims at finding proper model number of channel steel. The length of each sub-beam is determined by principle of descending order. the lengths of sub-beams are:

$$l_0=4.4\, \text{m}, \ l_1=4.3\, \text{m}, \ l_2=4.2\, \text{m}, \ l_3=4.1\, \text{m}, \ l_4=4.0\, \text{m}.$$  

An initial value which is the mass per unit length of sub-beam 0 is given to describe the weight of sub-beam 0. This mass per unit length is related to specific model number of stander channel steel. Now 20.174$\, \text{Kg/m}$ is set to the initial value which is equal to number 18a channel steel. So the $P_0$ is defined. The mass per unit length of other sub-beams are determined by principle of increasing order. Meanwhile sub-beam which is far away from fixed side is loaded three person’s weight.

Considering the sub-beam 3 and 4 bearing large stress, the sleeve length should be longer. The sleeves lengths of sub-beams are set as follow:

$$x_0=0.3\, \text{m}, \ x_1=0.3\, \text{m}, \ x_2=0.3\, \text{m}, \ x_3=0.6\, \text{m}, \ x_4=1.8\, \text{m}.$$  

The allowable stress of each sub-beam is $[\sigma]=200\, \text{Mpa}$. 

Execute the optimization program after inputting optimal model and initial conditions discussed above. Calculation result is shown in table 1.

<table>
<thead>
<tr>
<th>Objective function</th>
<th>Optimized parameter [m$^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_0$</td>
<td>42.8E-006</td>
</tr>
<tr>
<td>$W_1$</td>
<td>142.9E-006</td>
</tr>
<tr>
<td>$W_2$</td>
<td>263.3E-006</td>
</tr>
<tr>
<td>$W_3$</td>
<td>389.2E-006</td>
</tr>
<tr>
<td>$W_4$</td>
<td>471.6E-006</td>
</tr>
</tbody>
</table>

The optimized parameters which are seen as continuous value in the table above are the proper section modulus in bending of corresponding sub-beam. But in fact the series of section modulus in bending of channel steel are discrete. So the section modulus in bending of sub-beams should be determined according to the calculation result and stander channel steel specification table. The final model numbers of sub-beams are:

$32a$ ($W=475E-006$), $28c$ ($W=393E-006$), $25a$ ($W=270E-006$), $22$ ($W=234E-006$), $20$ ($W=191E-006$).

Because the larger model number, the larger safety factor, and for the sake of matching, sub-beam 3 and 4 adopt the 22 and 20 model number respectively, instead of number nearby calculation result.

Some necessary feature of sub-beam should be modified to make each beam match correctly after proper model numbers are selected. The final structure of bearing beam is shown in Figure 6.
5. Verifying calculation by finite element analysis

Final beam model should be verified to check the optimization result. For one thing, the self weight of sub-beam is estimated. That means the mass of sub-beams have nothing to with model number selected. For another, some modification that may affect mechanical behavior has been carried out to the final bearing beam model.

Three dimension model of bearing beam was built by UG basing on parameter discussed above. The model was analyzed by ANSYS Workbench after importing the model [6, 7]. Alloy structural steel 40Cr was adopted. Its Young Modulus is 2.06e5MPa, and Poisson Ratio 0.28, density 7.85e3 Kg/m³, tensile strength δb=735MPa, yield strength δc=540MPa.

Analysis result shows that the maximum equivalent stress is 238.18MPa. So the safety factor is ξ= 540 /238.18=2.3. And the maximum equivalent stress locates on point 42 on sub-beam 4. It coincides with optimization result.

The stress and strain clouds are presented in Figure 7.

![Figure 7. Cloud of analysis result](image)

6. Conclusion

The stress mathematical model of bearing beam with sleeve joint structure was summed up according to sub-beams analysis. The optimization model was deduced by stress mathematical model and actual conditions. Then the conclusion was applied to a telescopic ladder used on rescue ship. The optimized parameters were quickly found through optimization program. Finally reasonable result was shown after finite element calculation. But this conclusion just suits for cantilever beam, so some necessary jobs still should be finished if other form of force required.

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References


