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Complexified Starobinsky inflation in supergravity in the light of recent BICEP2 result

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ABSTRACT

Motivated by the recent observation of the B-mode signal in the cosmic microwave background by BICEP2, we study the Starobinsky-type inflation model in the framework of old-minimal supergravity, where the inflaton field in the original (non-supersymmetric) Starobinsky inflation model is promoted to a complex field. We study how the inflaton evolves on the two-dimensional field space, varying the initial conditions. We show that (i) one of the scalar fields has a very steep potential once the trajectory is off from that of the original Starobinsky inflation, and that (ii) the B-mode signal observed by BICEP2 is too large to be consistent with the prediction of the model irrespective of the initial conditions. Thus, the BICEP2 result strongly disfavors the complexified Starobinsky inflation in supergravity.

Recently, B-mode polarization of cosmic microwave background (CMB) has been observed by BICEP2, which indicated a large tensor-to-scalar ratio of [1]

$$r^{(\mathrm{BICEP2})} = 0.2^{+0.07}_{-0.05}. \tag{1}$$

The observation of BICEP2 provides a significant constraint on inflationary models because the value of r is directly related to the scale of inflation (i.e., the expansion rate during inflation). In particular, the BICEP2 result strongly disfavors one of the interesting possibilities, i.e., Starobinsky inflation model [2,3] which utilizes a scalar degree of freedom in the gravitational sector as an inflaton. This is because the Starobinsky inflation predicts r of the order of 10^{-3}, which is significantly smaller than the BICEP2 result.

If one extends the model, this conclusion may change. The extension we consider in the present study is to supersymmetrize the model because supersymmetry is a prominent candidate of the physics beyond the Standard Model. In such a model, the inflation can still be realized solely by the gravitational sector, while new scalar degrees of freedom are automatically introduced, which may affect the dynamics of inflation.

The Starobinsky model is based on a modified theory of gravity, so we need to consider a modified theory of supergravity. There are two minimal off-shell formulations of supergravity: the old-minimal [4–6] and the new-minimal [7] supergravity. Supergravity embedding of Starobinsky model has been studied both in the old-minimal [8–10] and the new-minimal [11,12] supergravities. These studies share the original philosophy of the Starobinsky model in the sense that the supergravity generalizations of the model rely solely on (super)geometrical or (super)gravitational quantities.[1] The old-minimal realization of Starobinsky model is possible with generic "Kähler potential" and "superpotential" of scalar curvature supermultiplet with extra propagating scalar degrees of freedom other than the inflaton (also called scalaron) [8–10].[2] On the contrary, the new-minimal realization has a Higgsed (massive) vector field as well as the inflaton [11,12]. Thus, we consider the old-minimal supergravity because it automatically introduces new scalar degrees of freedom.

In this letter, we study the Starobinsky-type inflation model in the framework of old-minimal supergravity. We pay particular attention to the fact that there exist two scalar degrees of freedom originating from the gravity multiplet in such a model.

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[1] In this respect, see Ref. [13] for the inflationary scenario induced by gravitino condensation. Closely related works to Refs. [8–12] include Refs. [14,15] in the old-minimal formulation and Refs. [16] (see also Refs. [17]) in both formulations. See also other recent related works [18,19] in supergravity. These can reproduce the scalar potential of the dual theory of the Starobinsky model [20], but do not necessarily have pure (super)geometrical or (super)gravitational interpretation. Generalization of the duality [8] between higher-curvature supergravity and standard matter-coupled supergravity has recently been discussed in Ref. [21] which provides the higher-curvature supergravity representation of the attractor model [22].

[2] Imposing a constraint $\mathcal{R}^2 = 0$, one can construct the old-minimal higher-curvature supergravity with only one (pseudo)scalar in addition to the scalaron [23]. Even in this case, the discussion after Eq. (7) holds.

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We study the evolution of the inflaton on the two-dimensional field space. We will see that the potential of one of the scalar fields becomes very steep once the trajectory is off from that of the original Starobinsky inflation. We also show that the tensor-to-scalar ratio in the supergravity Starobinsky model is too small to be consistent with the BICEP2 result even though the field space is enlarged.

The generic action of the old-minimal supergravity [8,10] is, in chiral curved superspace language,[3]

$$S=\int d^4x d^2\Theta 2\mathscr{E}\left[-\frac{1}{8}(\bar{\mathscr{D}}\bar{\mathscr{D}}-8\mathcal{R})N(\mathcal{R},\bar{\mathcal{R}})+F(\mathcal{R})\right]+\text{H.c.} \quad (2)$$

where $N(\mathcal{R},\bar{\mathcal{R}})$ and $F(\mathcal{R})$ are the hermitian and the holomorphic functions of the scalar curvature chiral superfield $\mathcal{R}$, respectively. The superfield $\mathcal{R}$ contains Ricci scalar curvature R in its $\Theta\Theta$ component and gravitino in its Θ and $\Theta\Theta$ components. It also contains a complex scalar M and real vector b^μ. These are auxiliary fields in the case of the minimal action with $N=-3$ and $F=0$. For generic functions N and F, however, these become dynamical.

The theory is classically equivalent [8,24] to the standard matter-coupled supergravity [25]

$$S=\int d^4x d^2\Theta 2\mathscr{E}\left[\frac{3}{8}(\bar{\mathscr{D}}\bar{\mathscr{D}}-8\mathcal{R})e^{-K/3}+W\right]+\text{H.c.} \quad (3)$$

with the following no-scale type Kähler potential and superpotential:

$$K=-3\ln\left(\frac{T+\bar{T}-N(S,\bar{S})}{3}\right), \quad (4)$$

$$W=2TS+F(S). \quad (5)$$

Linearized analysis of the original picture (higher-curvature supergravity) for a simple function $N(\mathcal{R},\bar{\mathcal{R}})$ has been performed in Ref. [26]. Bosonic Lagrangian of the original picture and comparisons of both pictures are described in Ref. [10]. Note that *any* N and F functions lead to the unique Kähler and superpotentials for T because the origin of T is a Lagrange multiplier. In particular, canonically normalized field $X=\sqrt{3/2}\ln(1+2\,\mathrm{Re}\,T/3)$ along the real axis ($\mathrm{Im}\,T=S=0$) has the Starobinsky potential (cf. Eq. (9)). Roughly speaking, $\mathrm{Re}\,T$, $\mathrm{Im}\,T$, S, and $\bar{S}$ in this picture correspond to R, $\partial_\mu b^\mu$, M, and $\bar{M}$ in the original geometrical picture, respectively. In this letter, we focus on the standard matter-coupled supergravity picture.

Consider a Kähler potential for S,

$$N(S,\bar{S})=-3+\frac{12}{m^2}S\bar{S}-\zeta(S\bar{S})^2. \quad (6)$$

The first term (constant) is needed to reproduce Einstein supergravity. The second term leads to the kinetic term of the new degrees of freedom. However, this term produces the scalar potential unbounded below in the region of large $|S|$. Instability for radial $|S|$ direction is stabilized by the third term proportional to ζ (see *e.g.* Refs. [27,28,14,10] and references therein).

Small ζ makes other local minima near the original minimum ($T=S=0$). Because of these reasons, we take a sufficiently large value of ζ. Note that, for sufficiently large ζ, S is stabilized for any value of T. We also assume $F(S)=0$ so that the potential value at the vacuum is zero. Thus, S is set to the minimum $S=0$, and the resultant effective theory has two fields $\mathrm{Re}\,T$ and $\mathrm{Im}\,T$ with only one parameter m.

After stabilization of S, the Lagrangian density is given by

$$\mathcal{L}=-\frac{3}{(2\,\mathrm{Re}\,T+3)^2}\left(\partial_\mu\mathrm{Re}\,T\partial^\mu\mathrm{Re}\,T+\partial_\mu\mathrm{Im}\,T\partial^\mu\mathrm{Im}\,T\right)-\frac{3m^2}{(2\,\mathrm{Re}\,T+3)^2}\left(\mathrm{Re}\,T^2+\mathrm{Im}\,T^2\right). \quad (7)$$

Canonical normalization of both fields at the same time is impossible in this case. We find it useful to define the semi-canonical basis that does not have kinetic mixing and realizes canonical normalization at the vacuum ($X=Y=0$)[4]:

$$X=\sqrt{\frac{3}{2}}\ln\left(1+\frac{2}{3}\mathrm{Re}\,T\right),\qquad Y=\sqrt{\frac{2}{3}}\mathrm{Im}\,T. \quad (8)$$

Then, the Lagrangian density becomes

$$\mathcal{L}=-\frac{1}{2}\partial_\mu X\partial^\mu X-\frac{1}{2}e^{-2\sqrt{2/3}X}\partial_\mu Y\partial^\mu Y-\frac{3m^2}{4}\left(1-e^{-\sqrt{2/3}X}\right)^2-\frac{m^2}{2}e^{-2\sqrt{2/3}X}Y^2. \quad (9)$$

The third term is the Starobinsky potential. Looking at the second and fourth terms, one may naively guess that chaotic inflation [29] is possible neglecting the common factor $e^{-2\sqrt{2/3}X}$. However, as we shall see, this exponential factor strongly drives X to the positive direction in the large Y region.

Now let us investigate if the fields X and/or Y play the role of inflaton which are responsible for the present density fluctuations of our universe. For this purpose, we first study the evolution of these fields. The evolution equations for X and Y are given by

$$\ddot{X}+3H\dot{X}+\sqrt{\frac{3}{2}}m^2e^{-\sqrt{2/3}X}\left(1-e^{-\sqrt{2/3}X}\right)-\sqrt{\frac{2}{3}}e^{-2\sqrt{2/3}X}\left(m^2Y^2-(\dot{Y})^2\right)=0, \quad (10)$$

$$\ddot{Y}+3H\dot{Y}-2\sqrt{\frac{2}{3}}\dot{X}\dot{Y}+m^2Y=0, \quad (11)$$

where the "dot" denotes the derivative with respect to time t and $H\equiv\dot{a}/a$ (with a being the scale factor) is the expansion rate of the universe. When the energy density of the universe is dominated by that of T, we obtain

$$H=\sqrt{\frac{\rho_T}{3}} \quad (12)$$

where ρ_T is the total energy density:

$$\rho_T=K_T+V_T, \quad (13)$$

$$K_T=\frac{1}{2}\dot{X}^2+\frac{1}{2}e^{-2\sqrt{2/3}X}\dot{Y}^2, \quad (14)$$

$$V_T=\frac{3m^2}{4}\left(1-e^{-\sqrt{2/3}X}\right)^2+\frac{m^2}{2}e^{-2\sqrt{2/3}X}Y^2. \quad (15)$$

By solving the above equations numerically, we follow the trajectories of X and Y with various initial values.

In Fig. 1, we show the contours of the potential and the evolutions of the fields on (X,Y) plane. As representative initial conditions, we choose $(X(t_{\text{init}}),Y(t_{\text{init}}))=(0,100)$, $(0,80)$, $(-2,100)$

[3] Throughout this letter, we use the Planck unit $M_P=1$, where $M_P\simeq 2.4\times 10^{18}$ GeV is the reduced Planck scale.

[4] Alternatively, one may transform $\mathrm{Im}\,T$ into canonically normalized form by $Z=\sqrt{\frac{2}{3}}e^{-\sqrt{2/3}X}\mathrm{Im}\,T$. Then the potential for Z is also simplified, $V=V^S(X)+\frac{m^2}{2}Z^2$, where $V^S(X)$ is the Starobinsky potential. However, in this basis, X is no more canonically normalized and there is a kinetic mixing between X and Z.

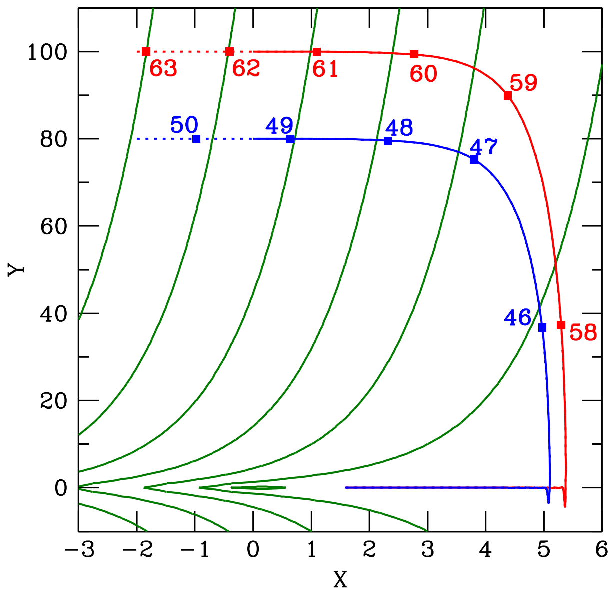

Fig. 1. Evolutions of the fields on the (X, Y) plane. The green closed contour at around the minimum (i.e., $(X, Y) = (0, 0)$) corresponds $V_T(X, Y)/m^2 = 0.1$, while other green lines represent the contours of $V_T(X, Y)/m^2 = 1$, 10, 10^2, 10^3, 10^4 and 10^5 from right to left. The solid red, solid blue, dashed red, and dashed blue lines represent the evolutions of the fields with initial conditions $(X(t_{\rm init}), Y(t_{\rm init})) = (0, 100)$, $(0, 80)$, $(-2, 100)$ and $(-2, 80)$, respectively. Note that dashed lines overlap with the solid lines for $X > 0$. Points with numbers show the e-folding numbers for each trajectory. The trajectories are terminated at the end of inflation (i.e., $\epsilon_H = 1$). (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

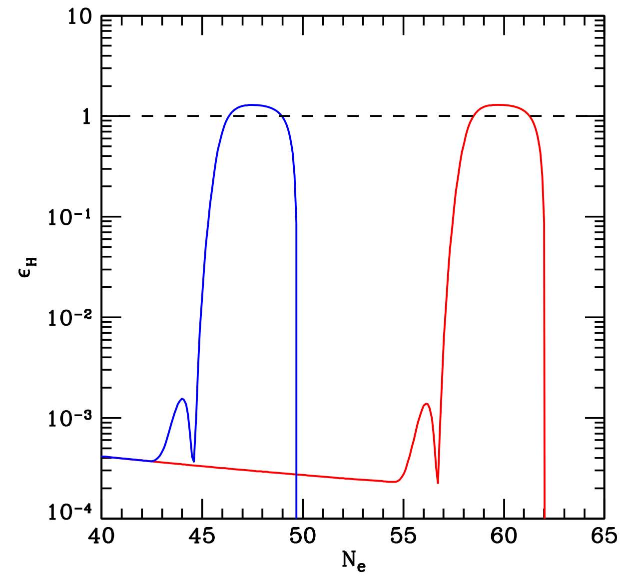

Fig. 2. The slow-roll parameter ϵ_H as a function of the e-folding number N_e, for the initial conditions $(X(t_{\rm init}), Y(t_{\rm init})) = (0, 100)$ (red line) and $(X(t_{\rm init}), Y(t_{\rm init})) = (0, 80)$ (blue line). The dashed line, corresponding to $\epsilon_H = 1$, is drawn to guide the eyes. Note that N_e decreases with time. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

and $(-2, 80)$. (The initial values of $\dot{X}$ and $\dot{Y}$ are taken to be zero.) With such initial conditions, we can see that T starts to move to the X direction first, then it settles to the real axis (i.e., $Y \simeq 0$). After reaching to the real axis, the motion of T is well approximated by the single-field inflation with X; the situation is almost the same as the non-supersymmetric original Starobinsky inflation. As can be seen from the dashed lines, the trajectories are almost unchanged even if X starts from $X < 0$.

On each contour, in particular for $Y \neq 0$, we show several points which give rise to some specific values of the e-folding numbers until the end of inflation. Here, the e-folding number is defined as

$$N_e(t) \equiv \int_t^{t_{\rm end}} dt' H(t'), \tag{16}$$

where $t_{\rm end}$ is the time at the end of inflation. In our analysis, we define it by $\epsilon_H(t_{\rm end}) = 1$, where the slow-roll parameter ϵ_H is given by

$$\epsilon_H \equiv -\frac{\dot{H}}{H^2} = 1 - \frac{\ddot{a}}{aH^2} = \frac{3K_T}{\rho_T}. \tag{17}$$

We have used the Einstein equation in the last equality. We can see that the change of the e-folding value in the period of $Y \gg 1$ is small. Therefore, a large value of the e-folding number during inflation, which is necessary to solve the horizon and flatness problems, should be accumulated when T is on the real axis.

For $\epsilon_H < 1$ and $\epsilon_H > 1$, the expansion of the universe is accelerating and decelerating, respectively. Thus, for inflation to happen, $\epsilon_H < 1$ is necessary. To see when the expansion is accelerating, in Fig. 2, we plot ϵ_H as a function of N_e, taking $(X(t_{\rm init}), Y(t_{\rm init})) = (0, 100)$ and $(X(t_{\rm init}), Y(t_{\rm init})) = (0, 80)$. We can see that, just after the start of the motion, ϵ_H significantly increases and soon becomes larger than 1. In this period, the expansion of the universe is decelerating and not inflating. The drop of ϵ_H at $N_e \simeq 57$ (45) in the red (blue) line corresponds to the point at which Y becomes most negative and $\dot{Y} \simeq 0$ (cf. Fig. 1).

Thus, the universe transits from the decelerating epoch to the Starobinsky-type inflation. We call the period in between as "transition period," and the period of the Starobinsky-like expansion as "Starobinsky-inflation period." The important point is that the transition period is very short; during the transition period, N_e changes ~ 3 or so. (For the case of $(X(t_{\rm init}), Y(t_{\rm init})) = (0, 100)$, for example, the transition period corresponds to $55 \lesssim N_e \lesssim 58$.) This is due to the fact that the motion of Y becomes suppressed soon after the condition $\epsilon_H < 1$ is satisfied. If we require that the causal connection be realized for the scale much longer than k_*^{-1} (with k_* being the wavenumber corresponding to the present Hubble scale), the mode with the wavenumber k_* should leave the horizon in the Starobinsky-inflation period. Then, the tensor-to-scalar ratio becomes $O(10^{-3})$ and is too small to be consistent with the value given in Eq. (1). Thus, in the light of the recent BICEP2 result, the Starobinsky inflation is disfavored even if the field space is complexified in the framework of old-minimal supergravity.

One of the possibilities to change this conclusion may be to consider the case where the mode with k_* exits the horizon in the transition period. However, such a solution looks unlikely. Even though the density fluctuations with the wavenumber $\sim k_*$ may be altered, fluctuations with the wavenumber k larger than $\sim 10k_*$ have almost the same property as those in the case of Starobinsky inflation. Consequently, for the angular scale of $\theta \lesssim \pi/l$ with $l \gtrsim O(10)$, the density perturbations behave as those in the Starobinsky model. The BICEP experiment is sensitive to the B-mode signal with $l \sim 50$–150, while the scalar-mode fluctuations for such an angular scale is well studied by using CMB and other observables. Thus, in the present model, it is difficult to enhance the tensor-mode fluctuations without conflicting observations.

Note added: While we are preparing the manuscript, the paper [30] showed up on arXiv, which has some overlap with this letter. See also Refs. [31] and [32] for recent related works.

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