Available online at www.sciencedirect.com
ScienceDirect
Procedia Engineering 127 (2015) 271 – 278
Procedia Engineering
www.elsevier.com/locate/procedia

International Conference on Computational Heat and Mass Transfer-2015

Dispersion Effects on Mixed Convection over a Vertical Wavy Surface in a Porous Medium with Variable Properties

D. Srinivasacharya[a], R. Bhuvanavijaya[b,*], B. Mallikarjuna[b,c]

[a]Department of Mathematics, National Institute of Technology, Warangal, Telangana, India
[b]Department of Mathematics, JNTUA College of Engineering Anantapur, Andhrapradesh, India
[c]Department of Mathematics, BMS College of Engineering, Bangalore, Karnataka, India

Abstract

In this paper, the dispersion effects and variable properties on mixed convection over vertical wavy surface immersed in a fluid saturated Darcy porous medium are considered. The governing equations are transformed into a set of ordinary differential equations using the similarity transformation and then solved numerically. The numerical results obtained in the present method compared with previously published results and are found to be in good agreement. The variation of the velocity, temperature and concentration as well as rate of heat and mass transfers are presented graphically for various values of pertinent physical parameters.

Peer-review under responsibility of the organizing committee of ICCHMT – 2015

Keywords: Double dispersion; Variable Properties; Vertical Wavy Surface; Mixed Convection;.

1. Introduction

The flow, heat and mass transport in a fluid saturated porous medium are often occurred in many engineering processes, natural environments and geophysical applications such as seepage of water through river bed, migration of pollutants into the soil and aquifers and flow of moisture through porous industrial materials etc. A good review of convective heat and mass transfer flows in a fluid saturated Darcy porous medium is given by Nield and Bejan [1]. In recent years, a great deal of interest has been generated in the study of convective boundary layer flow over a vertical wavy surface because of its enormous applications in engineering and industry like grain storage container where walls are buckled, condensation process, heat transfer devices such as flat plate condensers and flat plate solar collectors in refrigerators and so on. Several researchers have reported the characteristics of heat and mass transfer over a wavy surface in a fluid saturated porous medium. Maria [2] studied natural convective flow over a vertical wavy surface in non-Darcy porous medium with heat and muss flux conditions. Narayana et.al [3] studied cross-diffusion effects on heat and mass transfer flow past a horizontal wavy surface in a porous medium. Ahmed and Aziz [4] investigated steady and unsteady effects on Darcy free convection of a nanofluid over a vertical wavy surface.

* Corresponding author. Tel.: +0-855-422-0990 ; fax: +0-000-000-0000.
E-mail address: bhuvanarachamalla@gmail.com

Peer-review under responsibility of the organizing committee of ICCHMT – 2015
doi:10.1016/j.proeng.2015.11.367

Nomenclature

$\bar{a}$	amplitude of the wavy surface
d	pore diameter
l	characteristic length of the wavy surface
$\overline{u}$	velocity component in the x direction
$\overline{v}$	velocity component in the y direction
T_w	wall temperature
T_∞	ambient fluid temperature
C_w	wall concentration
C_∞	ambient fluid concentration
K	permeability of the porous medium
g	acceleration due to gravity
D	molecular diffusivity
k	mean absorption coefficient
Ra	Darcy - Rayleigh number
Ra_d	pore diameter dependent Rayleigh number
Pe	Peclet number
N	Buoyancy ratio
Le	Lewis number
U_∞	Uniform free stream velocity
$\overline{x}, \overline{y}$	coordinate system

Greek Symbols

α	thermal conductivity
α_o	thermal diffusivity
β	thermal conductivity parameter
β_t	coefficient of thermal expansion
β_c	coefficient of mass expansion
γ	coefficient of thermal dispersion
δ	thermal property of the fluid
ζ	coefficient of solutal dispersion
ξ	stream wise coordinate
$\overline{\sigma}$	surface geometry function
μ	dynamic viscosity of the fluid
ν	kinematic viscosity of the fluid
ρ	fluid density
$\overline{\psi}$	stream function
θ	non-dimensional temperature
ϕ	non-dimensional concentration

Most of the studies reported in the literature have assumed the fluid properties are constant. The variable viscosity and thermal conductivity effects on flow characteristics through porous media have been more important in industrial and engineering fields such as ground water pollution, crude oil extraction and geothermal systems, etc. Yasir et.al [5] studied the effects of variable properties on the flow characteristics in liquid thin film on horizontal shrinking/stretching sheet. He used homotopy perturbation technique to solve the governing equations. Pal and Mondal [6] investigated variable viscosity on MHD mixed convective flow over stretching sheet in a non-Darcy porous medium with non-uniform heat source and soret effect. Vajravelu et.al [7] investigated the effects of fluid properties on unsteady convective boundary layer flow at a vertical stretching sheet with thermal radiation. Rashad [8] studied variable viscosity effect on unsteady MHD flow of a rotating fluid over a stretching surface in a porous medium with thermal radiation. Recently, Srinivasacharya et.al [9] studied variable properties and cross diffusion effects on mixed convective flow over a vertical wavy surface in a fluid saturated porous medium.

In all the above mentioned work, double dispersion effects on convective heat and mass transfer flow along a vertical wavy surface is not analyzed. Hence, the aim of this paper is to investigate double dispersion effects on mixed convective boundary layer flow from a vertical wavy surface embedded in a fluid saturated porous medium with variable properties.

2. Formulation of the problem

Consider the steady, two dimensional, laminar, viscous incompressible fluid over a vertical wavy plate embedded in a saturated porous medium. Assume that the wavy surface is given by

$$\bar{y} = \bar{\sigma}(\bar{x}) = \bar{a} sin\left(\frac{\pi\bar{x}}{l}\right) \tag{1}$$

where $\bar{a}$ represents amplitude of the wavy surface and l represents the characteristics of wavy length. The plate is maintained with constant temperature T_w and concentration C_w, which are higher than the ambient fluid temperature T_∞ and concentration C_∞. The Darcy law is used to describe the porous medium. In addition, we consider thermal and solutal dispersion effects on double diffusive mixed convection. In view of the above assumptions and Boussinesq approximation, the governing equations for the conservation of mass, momentum, energy and concentrations are:

$$\frac{\partial\bar{u}}{\partial\bar{x}} + \frac{\partial\bar{v}}{\partial\bar{y}} = 0 \tag{2}$$

$$\frac{\partial}{\partial\bar{y}}\left(\frac{\mu}{K}\bar{u}\right) = \frac{\partial}{\partial\bar{x}}\left(\frac{\mu}{K}\bar{v}\right) \pm \rho g\left(\beta_t\frac{\partial T}{\partial\bar{y}} + \beta_c\frac{\partial C}{\partial\bar{y}}\right) \tag{3}$$

$$\bar{u}\frac{\partial T}{\partial\bar{x}} + \bar{v}\frac{\partial T}{\partial\bar{y}} = \frac{\partial}{\partial\bar{x}}\left((\alpha + \gamma d\bar{v})\frac{\partial T}{\partial\bar{x}}\right) + \frac{\partial}{\partial\bar{y}}\left((\alpha + \gamma d\bar{u})\frac{\partial T}{\partial\bar{y}}\right) \tag{4}$$

$$\bar{u}\frac{\partial C}{\partial\bar{x}} + \bar{v}\frac{\partial C}{\partial\bar{y}} = \frac{\partial}{\partial\bar{x}}\left((D + \zeta d\bar{v})\frac{\partial T}{\partial\bar{x}}\right) + \frac{\partial}{\partial\bar{y}}\left((D + \zeta d\bar{u})\frac{\partial T}{\partial\bar{y}}\right) \tag{5}$$

where $\bar{u}$ and $\bar{v}$ are velocity components in $\bar{x}$ and $\bar{y}$ directions respectively. μ is the kinematic viscosity, K is the permeability of the porous medium, U_∞ is the uniform free stream velocity, ρ is the density of the fluid, β_t is the thermal expansion coefficient, β_c is the solutal expansion coefficient, g is the acceleration due to gravity, α is thermal conductivity, D is the molecular diffusivity, γ is the coefficient of thermal dispersion, ζ is the coefficient of solutal dispersion. The associated boundary conditions are

$$\left.\begin{array}{lll} \bar{u} = 0, \quad \bar{v} = 0, \quad T = T_w, \quad C = C_w, & \text{at} & \bar{y} = \bar{\sigma}(\bar{x}) = \bar{a} sin\left(\frac{\pi\bar{x}}{l}\right) \\ \bar{u} \to U_\infty \quad T \to T_\infty \quad C \to C_\infty & \text{as} & \bar{y} \to \infty \end{array}\right\} \tag{6}$$

The fluid properties namely, viscosity and thermal conductivities are assumed to be vary as an inverse linear and linear function of the temperature respectively and these can be written as (see Lai and Kulacki [14], Seddeek and Salem [11] and Slattery [12])

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}(1 + \delta(T - T_\infty)) \qquad \text{or} \qquad \frac{1}{\mu} = b(T - T_r) \quad \text{and} \quad \alpha = \alpha_o(1 + E(T - T_\infty)) \tag{7}$$

where $b = \frac{\delta}{\mu_\infty}$, and $T_r = T_\infty - \frac{1}{\delta}$. Both b and T_r are constants and their values depend on the reference state and the thermal property of the fluid i.e. δ, α_o is the thermal diffusivity at the wavy surface temperature T_w and E is a constant depending on the nature of the fluid. The variable thermal conductivity can be written in the non-dimensional form (see Slattery [12]) as

$$\alpha = \alpha_o(1 + \beta\theta) \tag{8}$$

where $\beta = E(T_w - T_\infty)$ is the thermal conductivity parameter. Introducing the stream function $\bar{\psi}$ through $\bar{u} = \frac{\partial\bar{\psi}}{\partial\bar{y}}$, and$\bar{v} = -\frac{\partial\bar{\psi}}{\partial\bar{x}}$ and the following dimensional variables

$$x = \bar{x}/l, \quad y = \bar{y}/l, \quad a = \bar{a}/l, \quad \sigma = \bar{\sigma}/l \quad \psi^* = \frac{\bar{\psi}}{\alpha_o}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty} \tag{9}$$

in eqns. (3)-(5), we get

$$\frac{1}{\theta - \theta_r}\left(\frac{\partial\theta}{\partial y}\frac{\partial\psi^*}{\partial y} + \frac{\partial\theta}{\partial x}\frac{\partial\psi^*}{\partial x}\right) + \frac{\partial^2\psi^*}{\partial y^2} + \frac{\partial^2\psi^*}{\partial x} = \pm\Delta\left(1 - \frac{\theta}{\theta_r}\right)\left(\frac{\partial\theta}{\partial y} + N\frac{\partial\phi}{\partial y}\right) \tag{10}$$

$$\begin{aligned}\frac{\partial\psi^*}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial\psi^*}{\partial x}\frac{\partial\theta}{\partial y} &= \beta\left(\left(\frac{\partial\theta}{\partial x}\right)^2 + \left(\frac{\partial\theta}{\partial y}\right)^2\right) + (1 + \beta\theta)\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right)\\ &+ \frac{\gamma d}{l}\left[\frac{\partial^2\psi^*}{\partial y^2}\frac{\partial\theta}{\partial y} - \frac{\partial^2\psi^*}{\partial x^2}\frac{\partial\theta}{\partial x} + \frac{\partial\psi^*}{\partial y}\frac{\partial^2\theta}{\partial y^2} - \frac{\partial\psi^*}{\partial x}\frac{\partial^2\theta}{\partial x^2}\right]\end{aligned} \tag{11}$$

$$\frac{\partial\psi^*}{\partial y}\frac{\partial\phi}{\partial x} - \frac{\partial\psi^*}{\partial x}\frac{\partial\phi}{\partial y} = \frac{1}{Le}\left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}\right) + \frac{\zeta d}{l}\left[\frac{\partial^2\psi^*}{\partial y^2}\frac{\partial\phi}{\partial y} - \frac{\partial^2\psi^*}{\partial x^2}\frac{\partial\phi}{\partial x} + \frac{\partial\psi^*}{\partial y}\frac{\partial^2\phi}{\partial y^2} - \frac{\partial\psi^*}{\partial x}\frac{\partial^2\phi}{\partial x^2}\right] \tag{12}$$

where $Ra = \frac{g\beta_t K(T_w - T_\infty)l}{\alpha_o \nu}$ is the modified Rayleigh number, $\nu = \frac{\mu_\infty}{\rho}$ is the kinematic viscosity of the fluid, $N = \frac{\beta_c(C_w - C_\infty)}{\beta_t(T_w - T_\infty)}$ is the buoyancy ratio, $Le = \frac{\alpha_o}{D}$ is the Lewis number and $Ra_d = \frac{g\beta_t K(T_w - T_\infty)d}{\alpha_o \nu}$ is the pore diameter dependent Rayleigh number which describes the relative intensity of the buoyancy force, such that d is the pore diameter. $Ds = \gamma Ra_d$ is the thermal dispersion parameter and $Dc = \zeta Ra_d$ is the solutal dispersion parameter.

The associated boundary conditions are given by

$$\psi^* = 0, \quad \theta = 1, \quad \phi = 1 \quad \text{on} \quad y = asin(x), \psi^*_y \to \frac{\alpha_o}{l}U_\infty, \quad \theta \to 0, \quad \phi \to 0 \qquad \text{as} \quad y \to \infty. \tag{13}$$

The effect of the wavy surface can be transferred from the boundary conditions into the governing equations by the coordinate transformation

$$x = \xi, \quad \eta = \frac{y - asin(x)}{\xi^{1/2}(1 + a^2cos^2(\xi))Pe^{-1/2}}, \quad \psi^* = Pe^{1/2}\xi^{1/2}f(\eta), \quad \theta = \theta(\eta) \quad \phi = \phi(\eta) \tag{14}$$

Substituting eqn. (14) into eqns. (10) (13) and letting $Pe \to \infty$ (i.e boundary layer approximation), we obtain the following equations:

$$f'' + \frac{1}{\theta - \theta_r}\theta' f' = \pm\Delta\left(1 - \frac{\theta}{\theta_r}\right)(\theta' + N\phi') \tag{15}$$

$$\beta(\theta')^2 + (1 + \beta\theta)\theta'' + \frac{1}{2}f\theta' + Ds\frac{1 + a^2cos^3(\xi)}{(1 + a^2cos^3(\xi))^2}(f''\theta' + \theta'' f') = 0 \tag{16}$$

$$\frac{1}{Le}\phi'' + \frac{1}{2}f\phi' + Dc\frac{1 + a^2cos^3(\xi)}{(1 + a^2cos^3(\xi))^2}(f''\phi' + \phi'' f') = 0 \tag{17}$$

where prime denotes differentiation with respect to η. From Eqns. (15) and (17), it can be noted that in the convection due to vertical wavy surface in a fluid saturated porous medium, the field variable, heat and mass transfer characteristics are not similar because the -coordinate cannot be eliminated. However, we found the local non-similarity solutions for some convective boundary layer flows dealing with fluid saturated Darcy porous medium, the technique are more complex to extend in this case. Hence, for ease of analysis, it is decided to proceed with evaluating local similarity solutions for the equations, (15)-(17). For that we take $\xi = \frac{x}{l}$ and then vary ξ-location to study the influence of various parameters.

The associated boundary conditions are

$$f(0) = 0, \quad \theta(0) = 1, \quad \phi(0) = 1, \quad f'(\infty) = 1, \quad \theta(\infty) = 0 \quad \phi(\infty) = 0 \tag{18}$$

The engineering design quantities of physical interest include Nusselt number and Sherwood numbers which are defined as

$$Nu_\xi = -\left(1 + Ds\frac{f'(0)}{1 + a^2cos^2(\xi)}\right)\frac{\theta'(0)Pe_\xi^{1/2}}{(1 + a^2cos^2(\xi))^{1/2}} \quad \text{and}$$

$$Sh_\xi = -\left(1 + Dc\frac{f'(0)}{1 + a^2cos^2(\xi)}\right)\frac{\phi'(0)Pe_\xi^{1/2}}{(1 + a^2cos^2(\xi))^{1/2}} \tag{19}$$

3. Results and Discussions

The highly coupled and nonlinear governing boundary layer equations (15) - (17) along with boundary conditions (18) are solved by employing shooting technique that uses fourth order Runge-Kutta method and Newton-Raphson method (See Mallikarjuna et al. [13]). In the absence of the effects of double dispersion and variable properties on convective heat and mass transfer flow over flat surface reduces to those limiting cases of mixed convective flow Lai [14] who studied mixed convective flow over a vertical plate with variable properties. The comparison results found to be a very good agreement with Lai [14] as shown in Table-2.

Table 1. Comparison of Local Nusselt number and the local Sherwood number for $a = 0, \beta = 0$, R=0 and $\theta_r \to \infty$ at $N = 0.5$, $Le = 1$

	$Nu_\xi Pe_\xi^{-1/2}$	$Sh_\xi Pe_\xi^{-1/2}$	$Nu_\xi Pe_\xi^{-1/2}$	$Sh_\xi Pe_\xi^{-1/2}$
Δ	Lai [14]	Lai [14]	Present	Present
0.1	-0.5640	-1.6344	-0.5640	-1.6344
1.0	-1.1060	-1.9599	-1.1058	-1.9600
1.5	-1.3860	-2.3674	-1.3860	-2.3672

Figure 1 represents the non-dimensional fluid velocity, temperature and concentration profiles for different values of variable viscosity θ_r and variable thermal conductivity β with transverse coordinate η . It is observed from fig. 1 that an increase in θ_r results in decrease the velocity profile while increase in temperature and concentration. The present investigation shows that the flow variable are more pronounced for smaller values of θ_r. It is also noticed from fig. 1 that an increase in β enhanced momentum and thermal boundary layer thickness, while depreciation in solutal boundary layer thickness.

The varaition of non-dimensional velocity, temperature and concentration for different values of thermal dispersion (Ds) and solutal dispersion parameter (Dc) is shown in fig. 2. It is clear that the flow variables are favorably influenced by thermal dispersion parameter. For large Ds, in a very small boundary layer region adjacent to the wall, the velocity and temperature profiles are greatly enhanced and as a result momentum and thermal boundary layer thickness is greatly enhanced caused by increasing thermal dispersion parameter (Ds). But the reverse trend is noted in concentration profile for higher values of Ds, i.e., increase in Ds is seen to reduce solutal boundary layer thickness. We also observed from fig. 2 that increases in solutal dispersion Dc results an enhancement in flow velocity and concentration profile while depreciation in temperature profile.

Figure 3 presents the effects of mixed convection parameter Δ and ξ-location on velocity, temperature and concentration profiles across boundary layer respectively. From present numerical analysis, we conclude an interesting aspect that when $\xi = 0$, the flow governing boundary layer equations are free from ξ and it quickly approach the similarity solutions. It is noticed from fig.3 that momentum, thermal and solutal boundary layer thickness increases in the downstream direction. We also conclude from fig.3 that as the mixed convective parameter increase, the fluid velocity increased whereas temperature and concentration profiles are decreased.

The stream wise variations of Nusselt number and Sherwood number at the walls are shown in fig. 4 for different values of variable viscosity θ_r and variable thermal conductivity β parameters. It is noticed that the non-dimensional

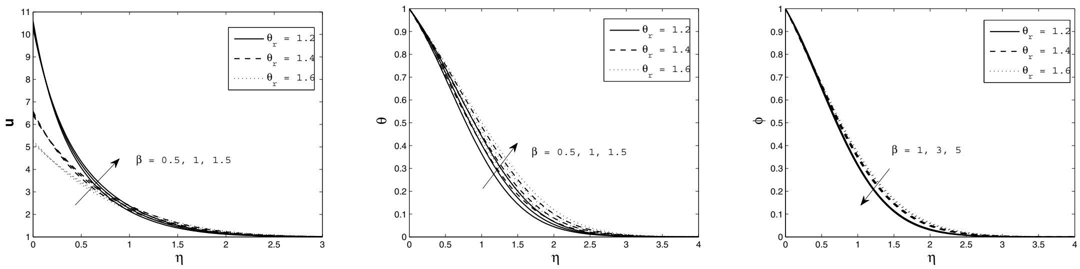

Fig. 1. Velocity, Temperature and Concentration Profiles for different values of θ_r and β for $N = 1, Le = 1, Ds = 0.3, Dc = 0.3, \Delta = 1, a = 0.5$, and $\xi = 1$

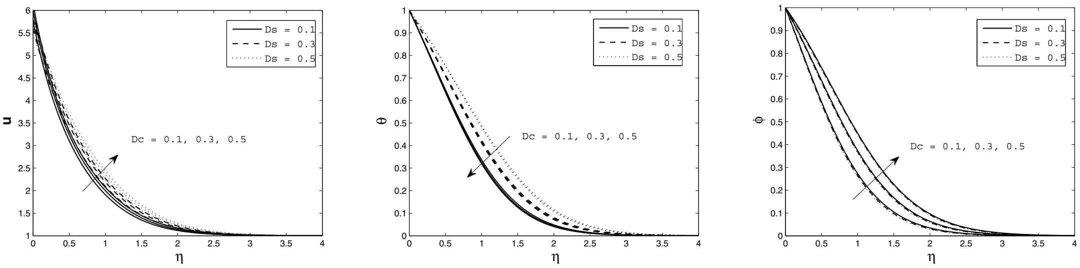

Fig. 2. Velocity, Temperature and Concentration Profiles for different values of Ds and Dc for $N = 1, Le = 1, \beta = 0.5, \theta_r = 1.5, \Delta = 1, a = 0.5$, and $\xi = 1$

Nusselt number and Sherwood number decrease with increasing values of θ_r. It can also be observed from fig. 4 that Nusselt number decrease but Sherwood number increases with increasing values of β. Variation of Nusselt number and Sherwood number against the thermal dispersion (Ds) and solutal dispersion (Dc) are shown in fig. 5. For higher values of Ds, the temperature and concentration gradient are highly influenced and increased at the wall in the boundary layer regime and hence as a result Nusselt and Sherwood number are greatly increased with increasing values of Ds. The influence of these Nusselt number and Sherwood number against solutal dispersion parameter (Dc) is also seen from fig. 5. It is observed from this fig.5 that increases in Dc results an enhancement in Nusselt number and Sherwood number. The non-dimensional Nusselt number and Sherwood number for different values of amplitude of the wavy surface (a) with fixed values of the other parameters is presented in fig. 6. Increasing amplitude of the wavy surface from 0.3 to 0.5 through 0.4, the amplitude of the Nusselt number and Sherwood number increased more significantly. It is an important to note that for $a = 0$ the wavy surface becomes flat plate and the results are true for the model of convective boundary layer flow over vertical flat plate in a porous medium.

4. Conclusions

Similarity solutions for variable properties and double dispersion effect on mixed convective heat and mass transfer over vertical wavy surface in a fluid saturated porous medium is presented. Increasing variable viscosity parameter tends to reduce flow velocity, Nusselt number and Sherwood number while enhance temperature and concentration profiles. The flow velocity, temperature and Sherwood number are increased for higher values of variable thermal conductivity. But the results of concentration and Nusselt number are reversed for larger values of variable thermal conductivity parameter. An increase in Ds is seen to increase the hydrodynamic velocity, temperature, Nusselt number and Sherwood number but decrease in concentration profile. For higher values of Dc, The flow velocity, concentration, Nusselt number and Sherwood number results are increased but the actual temperature profiles is decreased.

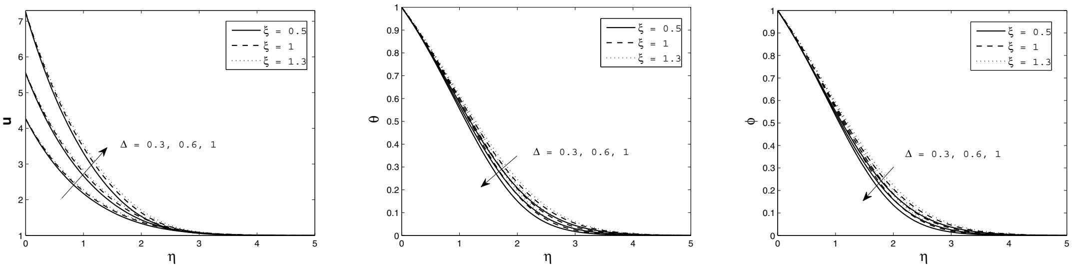

Fig. 3. Velocity, Temperature and Concentration Profiles for different values of ξ and Δ for $N = 1, Le = 1, Ds = 0.3, Dc = 0.3, \beta = 0.5, \theta_r = 1.5, a = 0.5$,

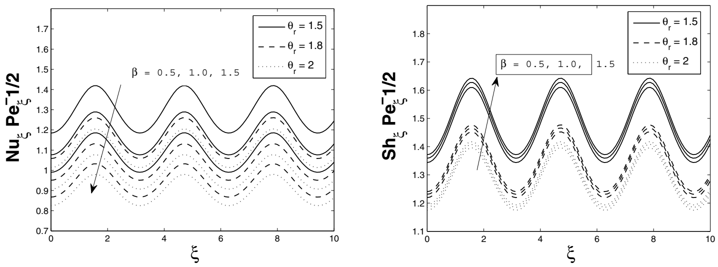

Fig. 4. Axial distribution of the Nusselt number and Sherwood number for different values of θ_r and β for $N = 1, Le = 1, Ds = 0.3, Dc = 0.3, \Delta = 1, a = 0.5$

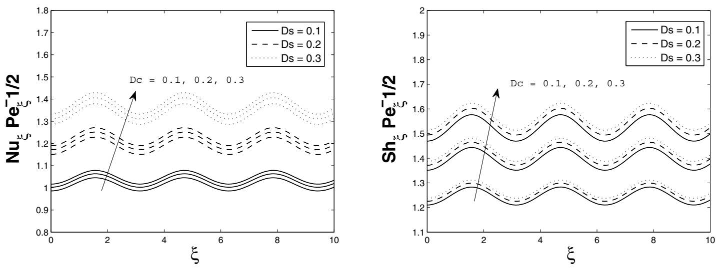

Fig. 5. Axial distribution of the Nusselt number and Sherwood number for different values of Ds and Dc for $N = 1, \theta_r = 1.5, \beta = 0.5, Le = 1, \Delta = 1, a = 0.5$

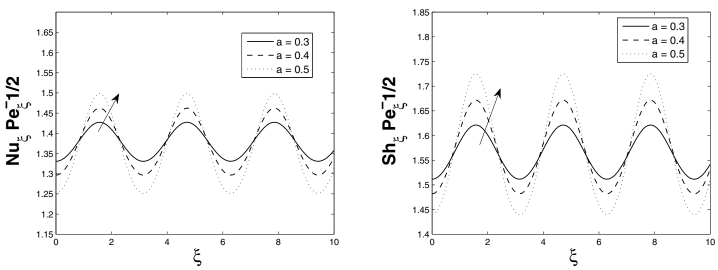

Fig. 6. Axial distribution of the Nusselt number and Sherwood number for different values of a for $N = 1, Ds = 0.3, Dc = 0.3, \theta_r = 1.5, \beta = 0.5, Le = 1, \Delta = 1, a = 0.5$

Acknowledgements

One of the authors Dr. B. Mallikarjuna wishes to thank to the Department of Science and Technology, New Delhi, India for providing financial support to enable conducting this research work under Inspire Fellowship Program and

B.M.S. College of Engineering, Bangalore-19, for giving support to modify the paper through the TECHNICAL EDUCATION QUALITY IMPROVEMENT PROGRAMME [TEQIP-II] of the MHRD, Government of India.

References

[1] D.A, Nield, D.A. and A, Bejan. Convection in Porous Media, Springer-Verlag, New York, 2012.

[2] N, Maria. Free convective heat and mass transfer induced by a constant heat and mass fluxes vertical wavy wall in a non-Darcy double stratified porous medium, International Journal of Heat and Mass Transfer, 54, (2011), 2310-2318.

[3] M. Narayana, P. Sibanda, S.S. Motsa, and P.G.Siddheshwar, On double-diffusive convection and cross diffusion effects on a horizontal wavy surface in a porous medium, Boundary Value Problems, (2012), 2012:88.

[4] Ahmed, S.E.; and Aziz, M.M.A.E. Effect of local thermal non-equilibrium on unsteady heat transfer by natural convection of a nanofluid over a vertical wavy surface, Meccanica, 48, (2013), 33-43.

[5] Q, Yasir. Khan, Wu, N. Faraz, and A. Yildirim, The effects of variable viscosity and thermal conductivity on a thin film flow over a shrinking/stretching sheet, Computers and Mathematics with Applications, 61, (2011), 3391-3399.

[6] D. Pal and H. Mondal, MHD non-Darcy mixed convective diffusion of species over a stretching sheet embedded in a porous medium with non-uniform heat source/sink, variable viscosity and soret effect, Communication in Nonlinear Science and Numerical Simulations, 17, (2012), 672-684.

[7] K. Vajravelu, K.V. Prasad, and C.O. Ng, Unsteady convective boundary layer flow of a viscous fluid at a vertical surface with variable fluid properties, Nonlinear Analysis: Real World Applications, 14(1), (2013), 455-464.

[8] A.M. Rashad, Effects of radiation and variable viscosity on unsteady MHD flow of a rotating fluid from stretching surface in porous medium, Journal of Egyptian Mathematical Society, 22(1), (2014), 134-142.

[9] D. Srinivasacharya, B. Mallikarjuna and R. Bhuvanavijaya, Soret and Dufour effects on mixed convection along a vertical wavy surface in a porous medium with variable properties, Ain Shams Engineering Journal, 6, (2015), 553-564.

[10] EC.Lai, FA.Kulacki. Effects of variable viscosity on convective heat transfer along a vertical surface in a saturated porous medium. International Journal of Heat and Mass Transfer, 33, (1990), 1189-1194

[11] M.S.Seddeek, A.M.Salem. Laminar mixed convection adjacent to vertical continuously stretching sheets with variable viscosity and variable thermal diffusivity, Heat and Mass Transfer, 41 (2005) 1048-1055.

[12] J.C.Slattery. Momentum, energy and mass transfer in continua, Mc. Graw-Hill, New York, 1972.

[13] B. Mallikarjuna, A.M. Rashad, Ali.J. Chamkha and S. Hariprasad Raju, Chemical reaction effects on MHD convective heat and mass transfer flow past a rotating vertical cone embedded in a variable porosity regime, Afrika Mathematika, 2015, Published online.

[14] F.C. Lai. Coupled heat and mass transfer by mixed convection from a vertical plate in a saturated porous medium, International Communications in Heat and Mass Transfer, 18, 1991, 93-106.