Localization of material defects using nonlinear resonant ultrasound spectroscopy under asymmetric boundary conditions

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Abstract

The position of microscopic defects in a one-dimensional bar is analyzed by means of the nonlinear resonant ultrasound spectroscopy technique under asymmetric boundary conditions. Experimental results on cementitious material confirm the feasibility of the method to locate nonlinear scatterers. Good agreement between the real damage position and the inferred experimental prediction is obtained based on information about the nonlinear frequency shift and about the amplitude ratio of the third and fifth harmonic for the fundamental resonance mode.

Keywords: defect imaging; nonlinear elasticity; resonant ultrasound spectroscopy

1. Introduction

Studies of acoustical nonlinearity and nonlinear mechanisms in solids have gained substantial attention in the past due to their potential benefit for nondestructive testing (NDT) of defects in solids. Several techniques based on the classical and nonclassical nonlinear acoustical behavior of solids have been proposed to evaluate the nonlinear parameters of the medium and relate them to its state of damage. Examples are harmonic analysis of wave propagation [1], nonlinear wave modulation spectroscopy [2-3] and single mode nonlinear resonant ultrasound spectroscopy [4]. Whereas Resonant Ultrasound Spectroscopy (RUS) is based on the accurate measurement of the resonance frequency peaks [5], Nonlinear RUS or NRUS uses the amplitude dependence of certain resonance frequencies and harmonics to quantify the location of defects. Van Den Abeele extended the single mode nonlinear resonant ultrasound spectroscopy theory for a one-dimensional bar to a multi-mode application by considering combinations of the nonlinear signatures of several induced modes [6]. The formalism dealt with symmetric free-free

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boundary conditions and was unable to differentiate whether the defect was located on the left or right side of the bar. Here, we study the resonance frequency shift and the amplitudes of the higher harmonics induced by a local zone characterized by a nonclassical nonlinear equation of state for a bar under asymmetric boundary conditions with one free boundary and one fixed boundary. As a result, this model creates the advantage to differentiate defects that are located anywhere in the bar. Experimental results on cement rods support the predictions.

2. Nonlinear signatures in Nonlinear Resonance Ultrasound Spectroscopy

2.1. Theoretical set-up and predictions

We consider a one-dimensional bar with length $L$, density $\rho$ and sound velocity $c$, in the presence of a damage feature. The width of the defect is $d$ and its distance from the receiver is $x_d$. For small strain excursions, the stress-strain relation at the defect positions in the bar can be described as follows:

$$\sigma = K(1 + \beta \varepsilon + \delta \varepsilon^2 + \cdots) \varepsilon + K \frac{\alpha}{2} \{\text{sign}(\varepsilon, \varepsilon)((\Delta \varepsilon)^2 - \varepsilon^2) - 2(\Delta \varepsilon)\varepsilon\},$$

where $\varepsilon = du/dx$ is the strain (with $u$ the displacement), $\Delta \varepsilon$ is the strain amplitude. $K$ is the linear stiffness constant. $\alpha$ is the strength of hysteresis and quantifies the opening of the stress-strain loop, $\beta$ and $\delta$ are combinations of third and fourth order elastic constants representing the classical nonlinearity. At and near the damage feature all material parameters ($K, \alpha, \beta, \delta$) can be non-zero. In this study we only consider nonclassical nonlinearity ($\beta = \delta = 0$). For elastic linear parts of the bar only $K$ is nonzero. If $x=0$ represents the free boundary and $x=L$ the fixed boundary, the conditions $H(0,t)=u(L,t)=0$ apply. Analogous to the work of Van Den Abeele [6], careful analysis of the nonlinear resonance equation for sinusoidal excitation near the resonance frequency $\omega_m=(2m+1)(\pi c/2L)$ of mode “$m$” in this case yields the following expressions for the amplitude dependent shift of the resonance frequency ($\Omega$) and for the induced third and fifth strain amplitude:

$$\frac{\Omega(0) - \Omega(\varepsilon_{1,2m+1})}{\Omega(0)} = -\frac{4}{3 \pi Q \sigma} \sin^{\gamma} \left( \frac{(2m+1)\pi x_d}{2L} \right) \varepsilon_{1,2m+1}$$

$$\varepsilon_{1,2m+1} = \frac{8Q}{5\pi} \sigma \left| \sin \left( \frac{(2m+1)\pi x_d}{2L} \right) \sin^{\gamma} \left( \frac{(2m+1)\pi x_d}{2L} \right) \right| \varepsilon_{1,2m+1}$$

$$\varepsilon_{1,2m+1} = \frac{8Q}{21\pi} \sigma \left| \sin \left( \frac{(2m+1)\pi x_d}{2L} \right) \sin^{\gamma} \left( \frac{(2m+1)\pi x_d}{2L} \right) \right| \varepsilon_{1,2m+1}$$

where $\sigma = \alpha d/L$ and $Q$ is the quality factor (inverse attenuation).

The theory thus predicts a relative frequency shift that is proportional to the strain amplitude and a quadratic strain amplitude dependence of the odd harmonics. Note that these proportionality coefficients are strongly dependent on the position of the defect. Some typical examples showing the position dependence of these proportionality coefficients for the relative frequency shift and for the odd harmonics are displayed in Figures 1, 2 and 3 for various resonance modes (we assumed $\sigma=1$, and $Q=100$).
Fig. 1 Dependence of the strain proportionality coefficient of the relative frequency shift on the position $x_d$ in a resonance bar with asymmetric boundary conditions for the case of a nonclassical nonlinear defect, (a) for the excitation near the fundamental mode ($m=0$), (b) for the second bar mode excitation ($m=1$).

Fig. 2 Dependence of the (quadratic) strain proportionality coefficient of the third harmonic on the position $x_d$ in a resonance bar with asymmetric boundary conditions for the case of a nonclassical nonlinear defect, (a) 3rd harmonic proportionality coefficient for the excitation near the fundamental mode ($m=0$), (b) 3rd harmonic proportionality coefficient for the second bar mode excitation ($m=1$).

Fig. 3 Dependence of the (quadratic) strain proportionality coefficient of the fifth harmonic on the position $x_d$ in a resonance bar with asymmetric boundary conditions for the case of a nonclassical nonlinear defect, (a) 5th harmonic proportionality coefficient for the excitation near the fundamental mode ($m=0$), (b) 5th harmonic proportionality coefficient for the second bar mode excitation ($m=1$).
2.2. Localisation potential

Combining Eqs.(3) and (4), we define the ratio $s$ of the harmonic proportionality coefficients for the third and fifth harmonics for an arbitrary mode $m$:

$$
    s = \frac{5 \sin \left( \frac{5(2m+1)\pi}{2L} x_d \right)}{21 \sin \left( \frac{3(2m+1)\pi}{2L} x_d \right)}
$$

(5)

Figure 4 illustrates the absolute value of this ratio as function of the defect position for the fundamental resonance mode.

From the above equation, we can easily deduce the position $x_d$ of the defect:

$$
    x_d = \frac{2L}{(2m+1)\pi} \arcsin \left[ R(s) \right]
$$

(6)

with

$$
    R(s) = \sqrt{(100 - 84s) \pm \sqrt{(100 - 84s)^2 - 4 \times 80 \times (25 - 63s)}}
$$

(7)

3. Experimental study

3.1. Experimental study

For the experimental study we used cubic-shaped cementitious rods with a size of $160 \times 60 \times 60$ mm$^3$. The weight ratio of cement to sand was 1:3. To verify the feasibility of the nonlinear resonance method, we introduced a local nonlinear feature in the rods by embedding a hollow hard plastic pipe in the cement matrix. The length of the pipe is 30 mm and its outer diameter 3 mm. The thickness of the plastic shell is 0.6 mm. The orientation of the pipe is perpendicular to the length of the rod. The pipe introduces a zone of reduced stiffness and surfaces of imperfect contact between plastic and cement, by which frictional effects and asymmetric stress-strain response induce a zone
of locally amplified nonlinearity. The purpose of the joint experimental-theoretical study is to detect this zone using the nonlinear resonance method. A low amplitude sweep over frequency revealed that the natural frequency of the fundamental mode of the rod is near 10KHz. The experimental block diagram can be seen in Figure 5.

A tone-burst signal (10.0KHz central frequency, 50 cycle burst, 100Hz pulse repetition frequency) generated by an arbitrary waveform generator (Agilent 33250) is fed into a power amplifier (ENI A150). The amplifier has a flat gain of 55dB. The output of the amplifier is applied to a 28mm diameter Lead Zirconate Titanate (PZT) transducer (Shantou Ultrasonic Co.) that is used as the transmitting transducer (at \( x=L \)). The acoustic wave arriving at the other end of the rod \((x=0)\) is detected by an accelerometer (PCB Piezotronics) and recorded by a digital oscilloscope (Agilent 54810). The accelerometer’s receiving sensitivity is 10mV/g (1.02mV/(m/s)^2) and its -3dB frequency range is from 5Hz to 100KHz. Without the sample, the amplitude of the second harmonics of the transmitter is 35dB lower than that of the fundamental and the amplitude of the third harmonic is even much lower. In fact, with an acoustic power as low as 1W, the nonlinear effects of the transmitter are so weak that they can be entirely neglected.

The transmitting transducer is coupled to the prepared samples using vacuum grease as a coupling medium. The back of the transmitting transducer is cemented to a massive metallic loading mass. In addition, the sample at the emitter side was tightly bounded to the loading mass by means of an experimental bracket. This assures that the boundary condition at the emitter side can be considered close to that of an absolutely rigid boundary. An accelerometer is positioned at the other end of the sample with oil chosen as coupling medium. A 0.09Kg cylindrical aluminum plate is added at the accelerometer position to improve the coupling between the accelerometer and the sample. The total weight of the accelerometer and aluminum plate is small enough so that the corresponding sample boundary can be considered as acoustically soft. During the experiment, the relative positions of the transducer, the sample and accelerometer remained unchanged.

3.2. Samples

Two samples of the same basic composition and geometry are considered in the experiment. Sample 1 is a pure cementitious sample without plastic cylinder embedded. We call it the ‘reference’ sample. It is used for comparison with Sample 2 which is the so-called ‘defect’ sample. The defect, in the form of an embedded hollow plastic cylinder or tube, is oriented perpendicular to the length of the sample and to axis of the transducer. The position of the inclusion is 40 mm outside the center of the bar. This allows us to consider two cases for the defect sample by turning the sample 180 degrees in the horizontal plane. Doing so, we consider in case 1 the situation when the defect is at position \( x_d=120\text{mm} \), whereas case 2 corresponds to a defect at 40mm. Note that \( x_d \) is always measured starting from the free surface of the sample.

3.3. Experimental results

The maximum emitting voltage applied to the amplifier was 300mV. Within this range, we observed no noticeable amplitude dependence of the resonance frequency for the reference sample. Figure 6a shows the power spectrum of the received signal at maximum input voltage for the reference sample (sample 1) for an excitation at the natural frequency of the fundamental mode (10.006kHz). The second harmonics induced by the nonlinearity of the electronic system and by the intrinsic classical nonlinearity of the cementitious sample is evident. The third and higher harmonics are very small and embedded in the noise. The output power spectrum at maximum input excitation for the
defect sample in case 1 and 2 are displayed in Figure 6b and 6c respectively. We observe that many higher harmonics are generated in the defect sample compared to the reference sample, with the odd harmonics dominating the preceding even ones.

In addition, Figures 7 and 8 illustrate the dependence of the resonance frequency shift and of the third and fifth harmonics on the fundamental amplitude for both defect cases. The strain proportionality of the resonance frequency shift, the dominance of odd harmonics and the fact that the amplitudes of the odd harmonics are of second order in the fundamental amplitude support the hypothesis that nonclassical nonlinearity in the form of hysteresis (from friction) is most probably the predominant mechanism for the nonlinearity in the defect samples. For the ‘defect’ sample in case 1, when the defect is 120mm away from the free boundary, we observed a shift at maximum excitation of 94Hz (approximately 1%, see (Figure 7a). For case 2 when sample 2 was turned and the defect was 40mm away from the free boundary, the maximum shift of the resonance frequency only amounts to 18Hz (Figure 7b). From the general behavior of Eq.(2) as illustrated in Figure 1a, and the magnitude of both resonance frequency shifts measured on sample 2, we can assure that the defect is located near the fixed boundary of the sample in case 1 and near the free boundary of the sample in case 2. Moreover, applying the fifth to third harmonic ratio expression (Eq. (6) and Figure 4) for a range of excitation amplitudes, we obtain a prediction of the defect positions $x_d=123.1mm$ ($s=X_5/X_3=0.121$) and $x_d=43.8mm$ ($s=X_5/X_3=0.207$) for case 1 and case 2 respectively, which is in excellent agreement with the actual positions of the defect for the two cases.
4. Conclusion

In this study, we extended the nonlinear resonant ultrasound spectroscopy technique to a one-dimensional bar with asymmetric boundary conditions (one free boundary and one fixed boundary). These new calculations substantiate that the method can unambiguously differentiate defects at opposite sides of the center of the bar, thus eliminating the symmetry problem that occurred in a bar with two free boundaries. In addition, we have illustrated that the position of the defects can be estimated from the measured amplitudes of the odd harmonics. The results obtained in this paper ensure that the application of nonclassical nonlinear acoustics of solids for nondestructive evaluation of material defects is feasible.

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