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On-the-fly Estimation Strategy for Uncertainty Propagation in Two-Step Monte Carlo Calculation for Residual Radiation Analysis

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ABSTRACT

In analyzing residual radiation, researchers generally use a two-step Monte Carlo (MC) simulation. The first step (MC1) simulates neutron transport, and the second step (MC2) transports the decay photons emitted from the activated materials. In this process, the stochastic uncertainty estimated by the MC2 appears only as a final result, but it is underestimated because the stochastic error generated in MC1 cannot be directly included in MC2. Hence, estimating the true stochastic uncertainty requires quantifying the propagation degree of the stochastic error in MC1. The brute force technique is a straightforward method to estimate the true uncertainty. However, it is a costly method to obtain reliable results. Another method, called the adjoint-based method, can reduce the computational time needed to evaluate the true uncertainty; however, there are limitations. To address those limitations, we propose a new strategy to estimate uncertainty propagation without any additional calculations in two-step MC simulations. To verify the proposed method, we applied it to activation benchmark problems and compared the results with those of previous methods. The results show that the proposed method increases the applicability and user-friendliness preserving accuracy in quantifying uncertainty propagation. We expect that the proposed strategy will contribute to efficient and accurate two-step MC calculations.

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1.   Introduction

Particle transport analyses are preformed to get responses (i.e., dose rate, flux, criticality, and power distribution) in a system. The Monte Carlo (MC) method, which is stochastic, is accurate. Therefore it is widely used in the particle transport and analysis fields. The MC approach calculates an average and uncertainty of the responses by its stochastic processes. The uncertainty of the response confirms the reliability of the response; thus, analyzers can directly use it to determine design parameters, design limits, and so on when using the MC method as an analysis tool.

Serial MC simulations might be required to analyze the particle transport phenomenon, such as fuel depletion calculations, the source term generation problem, i.e., the standby service water (SSW)–specific safety requirements (SSR) option in MC N-particle (MCNP) [1], and residual radiation analysis from activated materials [2]. The problem in using serial MC simulations is an inability to accurately evaluate the uncertainty. Usually, for such problems, researchers just use the average value of response estimated from the previous MC calculation as the input for the next calculation. As a result, the uncertainty computed in the last MC calculation is underestimated because it does not consider the stochastic uncertainty generated in previous steps. Thus, to obtain reliable results, researchers need to properly quantify the uncertainty propagation caused by input uncertainty that occurs as a result of previous MC calculations.

The brute force technique [3] analyzes uncertainty propagation by repetitive MC calculations using the same input with different random seeds. Statistically analyzing the results produces the sample standard deviation and it is taken to be the true stochastic uncertainty. The method is accurate because its analysis well reflects the stochastic nature of previous MC calculations. However, the computational cost can be extremely high because it requires a huge number of MC calculations to achieve reliable results for a complex problem.

To prevent this inefficiency, the Oak Ridge National Laboratory proposed an adjoint-based method using an error propagation formula [4]. The method derives a relationship between the true stochastic uncertainty and the uncertainty computed from the previous MC calculation.

After estimating the adjoint flux, the method calculates the true uncertainty. It has an advantage in estimation efficiency over the brute force method because it requires only one additional adjoint calculation. However, it has the following limitations and difficulties: (1) it assumes the covariance term in the derived equation to be zero; and (2) it requires an additional calculation to obtain the adjoint flux.

To overcome the limitations of previous methods, we propose a new on-the-fly estimation strategy for the true stochastic uncertainty of the two-step MC calculations to improve both efficiency and accuracy. The main idea of the proposed approach is that it estimates the information required to analyze uncertainty propagation by adopting importance estimation and covariance of source-term estimation in forward MC calculations [5]. In Section 2, we describe the proposed method in detail. In Section 3, we verify the proposed method using activation benchmark problems.

2.   Materials and methods

Here, we briefly introduce the previously published methods to analyze error propagation. In Section 2.3, we describe our proposed strategy to estimate error propagation.

2.1.   Overview of the brute force method

Fig. 1 illustrates a procedure for the brute force method. First, a seed number is randomly sampled for each simulation. Then, a two-step MC simulation is performed with the random seed numbers until the responses have a reliable distribution. After analyzing the type of response distribution from the MC simulations, the true uncertainty is defined as the sample standard deviation of the responses. This method can analyze uncertainty propagation without any assumptions. However, the calculation efficiency is low because of the repetitive procedure.
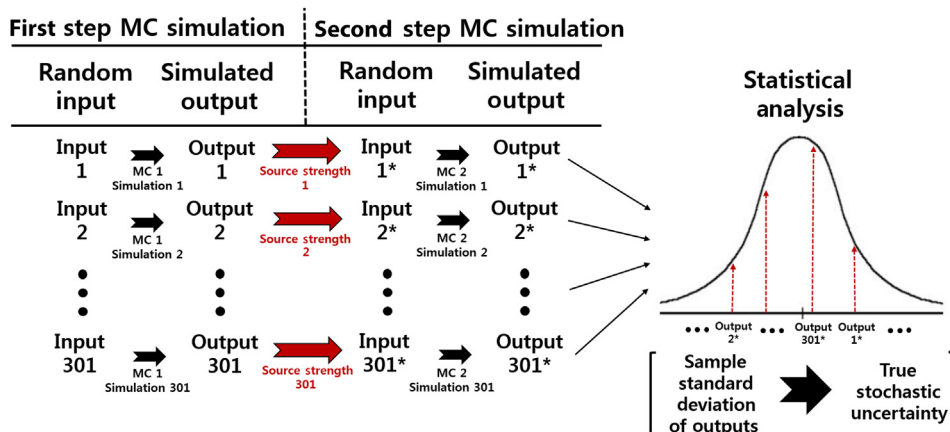

Fig. 1 – Procedure for the brute force technique in a two-step Monte Carlo calculation. MC, Monte Carlo.

2.2. Overview of the adjoint-based method

For a convenient description of the adjoint-based method, we define the propagated uncertainty from the first step MC simulation (MC1) as a hidden uncertainty (σ_h). The uncertainty directly computed in the second step MC simulation (MC2) is defined as an apparent uncertainty (σ_a). The combined uncertainty (σ_c) is defined as the total MC2 uncertainty including the hidden uncertainty. MC1 and MC2 are performed independently; therefore, the relationships among the apparent, hidden, and combined uncertainties can be expressed as Eq. (1).

$$\sigma_c = \sqrt{\sigma_a^2 + \sigma_h^2}. \tag{1}$$

The purpose of error propagation analysis is to estimate the hidden uncertainty caused by the stochastic uncertainty of the MC1. Hence, we express the relationship between the source strength and response in the MC2 as Eq. (2).

$$R = \sum_i S_i C_i, \tag{2}$$

where R is the response in the MC2, S_i is the source strength of cell i computed from the MC1, and C_i is the response contribution of a particle with a unit source strength from cell i to R in the MC2. Using an error propagation formula, we can derive the equation for the standard deviation (STD) of R from Eq. (2) as shown in Eq. (3).

on Eq. (4), Oak Ridge National Laboratory proposed its adjoint-based method for error propagation analysis [4]. They proposed a lower bound concept of the combined uncertainty as an approximation method. Generally, changes in the responses in the MC1 proportionally affect the changes of the other responses in neighboring cells. Therefore, the covariance of the source strengths in neighboring cells will be positive [$\text{cov}(S_i, S_j) > 0$]. Due to the weak relationships of cells separate from each other, the covariance can be approximated to zero [$\text{cov}(S_i, S_j) \cong 0$]. Thus, the second term on the right side of Eq. (4) is positive, and the first term on the right side of Eq. (4) can be defined as the lower bound of the hidden uncertainty and expressed as Eq. (5).

$$\min(\sigma_h) = \sqrt{\sum_i C_i^2 \sigma_{S_i}^2}, \tag{5}$$

where C_i is a discrete value in a unit cell i; therefore, it can be further expanded by considering the energy spectrum of the source term in the MC2, as given in Eq. (6).

$$C_i = \int C_i(E) f_i(E) dE, \tag{6}$$

where $C_i(E)$ is the energy spectrum of the response contribution of particles with a unit source strength from cell i to R, and $f_i(E)$ is a normalized source energy spectrum in the MC2. Because the physical meaning of adjoint flux is equal to the response contribution, it can be replaced with $\varnothing_i^\dagger(E)$. Therefore, Eq. (5) can be expressed as Eq. (7).

$$\sigma_R = \sqrt{\sum_i \left(\frac{\partial R}{\partial S_i}\right)^2 \sigma_{C_i}^2 + 2\sum_{i \neq j} \left(\frac{\partial R}{\partial S_i}\right)\left(\frac{\partial R}{\partial S_j}\right)\text{cov}(S_i, S_j) + \sum_i \left(\frac{\partial R}{\partial C_i}\right)^2 \sigma_{C_i}^2 + 2\sum_{i \neq j} \left(\frac{\partial R}{\partial C_i}\right)\left(\frac{\partial R}{\partial C_j}\right)\text{cov}(C_i, C_j)}$$
$$= \sqrt{\sum_i C_i^2 \sigma_{S_i}^2 + 2\sum_{i \neq j} C_i C_j \text{cov}(S_i, S_j) + \sum_i S_i^2 \sigma_{C_i}^2 + 2\sum_{i \neq j} S_i S_j \text{cov}(C_i, C_j)}, \tag{3}$$

where σ_R is the uncertainty in the response of the MC2 considering the uncertainty of the source, σ_{S_i} is the STD of the S_i, S_j is the source strength of cell j computed from the MC1, $\text{cov}(S_i, S_j)$ is the covariance between S_i and S_j, σ_{C_i} is the STD of the C_j, C_j is the response contribution of particles with a unit source strength from cell j to R in the MC2, and $\text{cov}(C_i, C_j)$ is the covariance between C_i and C_j. In this derivation process, the covariance between the response contribution and source strength is zero because they are independent of each other. The first and second terms on the right side of Eq. (3) are induced from the stochastic uncertainty in the MC1, and the other terms stem from the uncertainty in the response contribution of the MC2. Therefore, the hidden uncertainty in Eq. (1) can be derived by using the terms originated from the stochastic uncertainty in MC1 as Eq. (4).

$$\sigma_h = \sqrt{\sum_i C_i^2 \sigma_{S_i}^2 + 2\sum_{i \neq j} C_i C_j \text{cov}(S_i, S_j)}. \tag{4}$$

With Eq. (4), Eq. (1) can efficiently estimate the combined uncertainty in the two-step MC calculation if the response contribution and covariance information are obtained. Based

$$\min(\sigma_h) = \sqrt{\sum_i \left\{ \int \varnothing_i^\dagger(E) f_i(E) dE \right\}^2 \sigma_{S_i}^2}. \tag{7}$$

Eq. (7) can estimate the minimum value of the hidden STD if $\varnothing_i^\dagger(E)$ is estimated using an additional adjoint-transport calculation. However, this method has the limitation of requiring an additional adjoint calculation. Also, adjoint fluxes estimated by other adjoint-transport calculators, such as deterministic methods, can cause inaccuracy because of the methodological differences. In addition, to more accurately estimate the combined uncertainty, the covariance between the source strengths should be properly estimated and applied in the error propagation analysis.

2.3. Proposed strategy for error propagation analysis

To solve the problems caused by the adjoint-based method, we here propose an on-the-fly estimation strategy based on the forward-adjoint method [5] and union tally. First, to estimate the minimum hidden uncertainty, we calculate adjoint

fluxes in the forward MC simulation. We modify Eq. (4) into Eq. (8) by multiplying and dividing by S_i and S_j.

$$\sigma_h = \sqrt{\sum_i (S_i C_i)^2 \left(\frac{\sigma_{S_i}}{S_i}\right)^2 + 2\sum_{i \neq j} (S_i C_i)(S_j C_j)\frac{\mathrm{cov}(S_i, S_j)}{S_i S_j}}. \qquad (8)$$

Because the source strength in MC2 is proportional to the response calculated from MC1 as shown in Eq. (9), σ_{S_i} and $\mathrm{cov}(S_i, S_j)$ can be expressed as Eqs. (10) and (11).

$$S_i = P R_i^{\mathrm{MC1}} \qquad (9)$$

$$\sigma_{S_i} = P \sigma_{R_i^{\mathrm{MC1}}} \qquad (10)$$

$$\mathrm{cov}(S_i, S_j) = P^2 \mathrm{cov}\left(R_i^{\mathrm{MC1}}, R_j^{\mathrm{MC1}}\right) \qquad (11)$$

where R_i^{MC1} is the response of cell i in the MC1, $\sigma_{R_i^{\mathrm{MC1}}}$ is the STD of R_i^{MC1}, $\mathrm{cov}(R_i^{\mathrm{MC1}}, R_j^{\mathrm{MC1}})$ is the covariance between R_i^{MC1} and R_j^{MC1}, and P is a proportional constant that is the source strength in the MC2 divided by the response of cell i in the MC1.

Using Eqs. (9–11), Eq. (8) can be rewritten in terms of R_i^{MC1} and R_j^{MC1}, as given in Eq. (12).

$$\sigma_h = \sqrt{\sum_i (S_i C_i)^2 \left(\frac{\sigma_{R_i^{\mathrm{MC1}}}}{R_i^{\mathrm{MC1}}}\right)^2 + 2\sum_{i \neq j} (S_i C_i)(S_j C_j)\frac{\mathrm{cov}\left(R_i^{\mathrm{MC1}}, R_j^{\mathrm{MC1}}\right)}{R_i^{\mathrm{MC1}}, R_j^{\mathrm{MC1}}}}. \qquad (12)$$

Eq. (12) converts the response contribution term to $S_i C_i$, which is defined as the response due to the source of cell i in the MC2. Based on that definition, we can obtain $S_i C_i$ directly during the MC2 [5]. First, the source particles generated in cell i are flagged, and then each response originating from each cell i can be scored. Through the procedure, $S_i C_i$ can be estimated without additional calculation.

Also, to evaluate covariance $\mathrm{cov}(R_i^{\mathrm{MC1}}, R_j^{\mathrm{MC1}})$ in Eq. (12), we introduce a union tally strategy with on-the-fly scoring. For the estimation, we define a union region that combines two subcells and score it during the MC simulation. Using an error propagation formula, we can express the error relationship between the union region and two subcells as Eq. (13). Then, by rearranging the equation, we can estimate the covariance between R_i^{MC1} and R_j^{MC1} using Eq. (14).

$$\sigma_U^2 = \sigma_{R_i^{\mathrm{MC1}}}^2 + \sigma_{R_j^{\mathrm{MC1}}}^2 + 2\mathrm{cov}\left(R_i^{\mathrm{MC1}}, R_j^{\mathrm{MC1}}\right), \qquad (13)$$

$$\mathrm{cov}\left(R_i^{\mathrm{MC1}}, R_j^{\mathrm{MC1}}\right) = \frac{\sigma_U^2 - \sigma_{R_i^{\mathrm{MC1}}}^2 - \sigma_{R_j^{\mathrm{MC1}}}^2}{2}, \qquad (14)$$

where U is the response in the union region for cells i and j, and σ_U is the STD of the union region. By substituting Eq. (14) into Eq. (12), we can finally estimate the hidden uncertainty using Eq. (15).

$$\sigma_h = \sqrt{\sum_i (S_i C_i)^2 \left(\frac{\sigma_{R_i^{\mathrm{MC1}}}}{R_i^{\mathrm{MC1}}}\right)^2 + \sum_{i \neq j} (S_i C_i)(S_j C_j)\frac{\sigma_U^2 - \sigma_{R_i^{\mathrm{MC1}}}^2 - \sigma_{R_j^{\mathrm{MC1}}}^2}{R_i^{\mathrm{MC1}} R_j^{\mathrm{MC1}}}}. \qquad (15)$$

3.   Results

In order to verify the proposed strategy, we assumed residual radiation analysis problems. By applying the proposed method, we estimated the combined uncertainties. After that, the results were compared with those of brute force method and adjoint-based method. In Section 3.1, we describe the results and analysis of the simple activation benchmark problem. In Section 3.2, analysis of a more realistic benchmark problem and concrete activation in an accelerator facility is presented.

3.1.   Results and verification of simple activation problems

We assumed a simple activation benchmark problem, as shown in Fig. 2. In the MC1, we used a 1-MeV neutron source
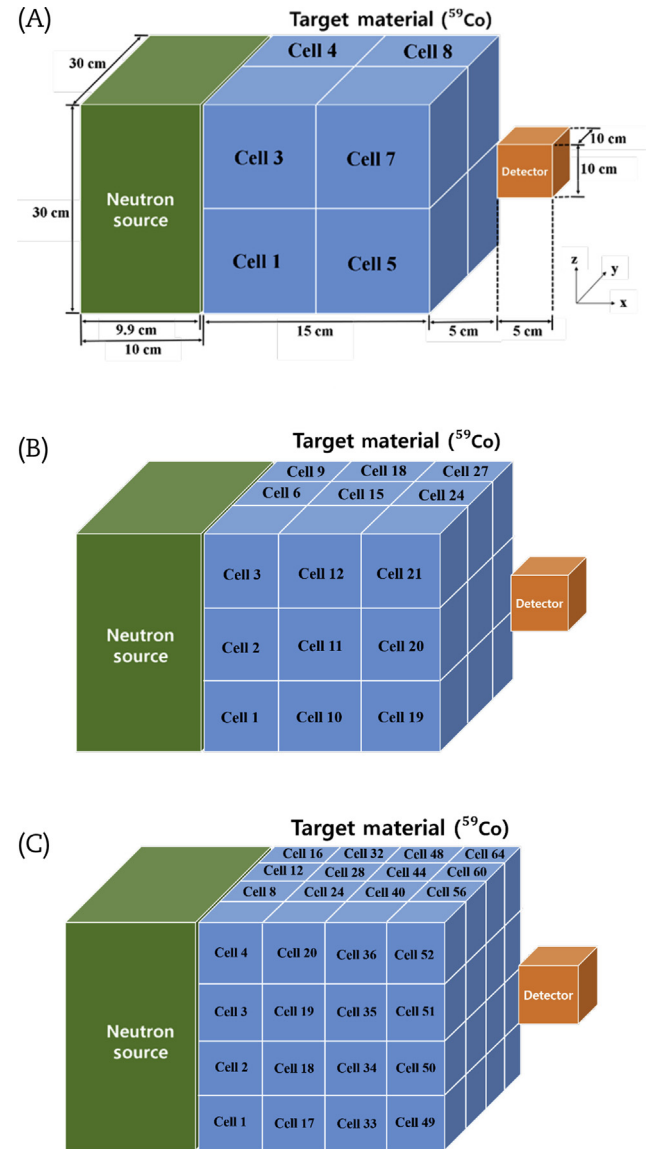

Fig. 2 − Overview of the simple activation benchmark problems. (A) Eight-cell problem. (B) Twenty-seven-cell problem. (C) Sixty-four-cell problem.
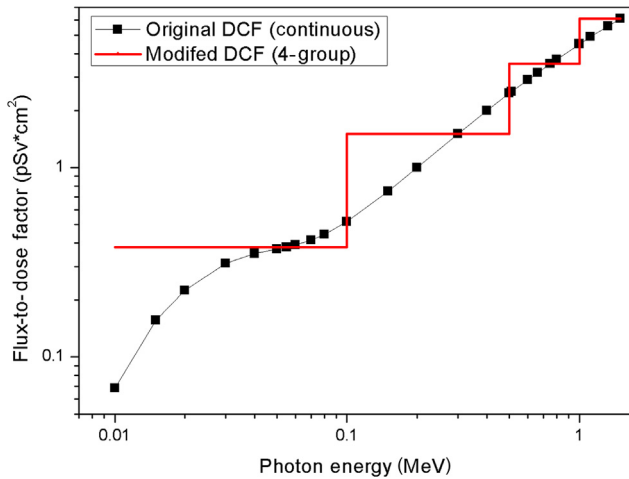
Fig. 3 – Comparison of the original flux-to-dose factor (International Commission on Radiological Protection 116 *anteroposterior* direction) to the modified four-group flux-to-dose factors. DCF, dose conversion factor.

Table 1 – Four-group flux-to-dose factors used for the verification.

Group	Lower energy group boundaries (MeV)	Upper energy group boundaries (MeV)	Flux-to-dose conversion factors (pSv cm^2)
1	1	2	6.12
2	0.5	1	3.53
3	0.1	0.5	1.51
4	0.01	0.1	0.380

uniformly distributed in a rectangle ($9.9 \times 30 \times 30$ cm^3) with 1×10^{10} #/s of source strength. Also, we assumed that a rectangular-type ($15 \times 30 \times 30$ cm^3) target material entirely composed of ^{59}Co was located at the right side of the neutron source. Cobalt-60 isotopes are produced by the (n, γ) reaction in the target material after irradiation from the neutron source. We assumed that the activity of ^{60}Co was equal to its production rate.

Cobalt 60 emits both 1.17-MeV and 1.33-MeV gamma rays for each decay process. Hence, in the MC2, the residual gamma radiation emitted from ^{60}Co is used as the source term. The residual gamma dose rate is detected 5 cm apart from the activated material using a rectangular-type ($5 \times 10 \times 10$ cm^3) detector. The center points of the volumetric source, target material, and detector are on the x-axis. For the activation analyses, we assumed that the benchmark problems are evenly divided as eight cells (2 cells × 2 cells × 2 cells), 27 cells (3 cells × 3 cells × 3 cells), and 64 cells (4 cells × 4 cells × 4 cells).

We used general two-step MC simulations to test the proposed scheme. For the transport analysis, we used MCNP extended 2.7.0 code [1]. For the neutrons, we used the JENDL/HE-2007 [6] cross-section library. Also, we used the MGXSNP photon cross-section library [7] in the MC2 to perform adjoint- and forward-photon transport calculation. In the MC1, we calculated the production rates of ^{60}Co in the target material for each cell using the F8 FT RES tally option in MCNP. We used the F4 tally to detect the residual gamma flux in the MC2 and converted it to the dose rate in the unit of microsievert per hour (μSv/h) by applying the International Commission on Radiological Protection 116 anteroposterior direction flux-to-dose conversion factor (DCF) [8]. For the uncertainty evaluation based on adjoint fluxes, we generated a four-group DCF, as shown in Fig. 3 and Table 1. The compositions and densities follow the National Institute of Standards and Technology database. We applied the particle histories to 4×10^5 and 2×10^7 for the MC1 and MC2, respectively.

To get the adjoint flux in the forward MC calculation, we used an SCX card, which has the function of scoring the particles originated from the certain source distribution in MCNP, in the MC2. Union tally method was realized by getting the additional union responses for couple of cells in MC1. Using the obtained information, covariance of the union responses, $\mathrm{cov}(R_i^{MC1}, R_j^{MC1})$ was estimated by Eq. (14). In this benchmark problem, all combinations of union responses were estimated.

After analyzing the responses and uncertainties, we calculated the combined STD with the proposed method. For comparison, we also used the brute force technique [3] and adjoint-based method [4]. To perform the analysis with the brute force technique, we evaluated 301 responses with changing random seed numbers. To analyze the uncertainty with the adjoint-based method, we performed multi-group

Table 2 – Comparison of the combined standard deviation estimated using each method in a simple activation benchmark problem.

Benchmark problem	Estimated standard deviation (μSv/h)					
	Apparent STD	Reference	Method 1	Method 2	Method 3	Method 4
		Brute force technique (95% confidence interval)	Adjoint[a]	Forward adjoint[a]	Adjoint + union	Proposed scheme
8-cell	0.16029	0.41221 (0.38170, 0.44806)	0.33439	0.33610	0.38999	0.39216
27-cell	0.15216	0.43134 (0.39941, 0.46885)	0.34295	0.34076	0.43634	0.43409
64-cell	0.14818	0.39842 (0.36893, 0.43307)	0.30898	0.30947	0.42746	0.42814

STD, standard deviation.
[a] We performed the adjoint-based calculation using the method of Peplow et al [4] using adjoint fluxes estimated by adjoint Monte Carlo and forward Monte Carlo calculations.
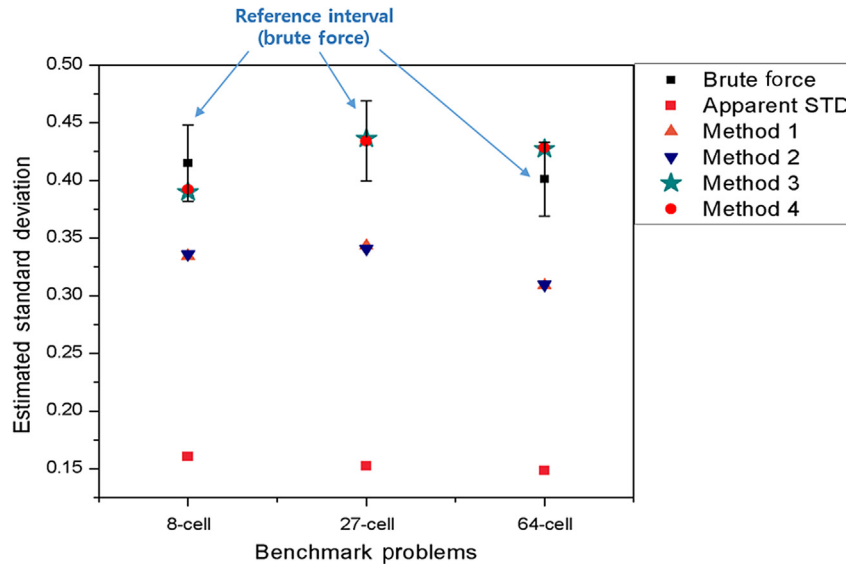
Fig. 4 — Comparison of the combined standard deviation results (σ_c) estimated from each method in the simple activation benchmark problem. STD, standard deviation.

adjoint-transport calculation using the MCNP code [7]. We performed the adjoint calculation using the grouping table merge option parameter to interchange the phase spaces of the source and the response.

To compare the results, we estimated the combined STD using the brute force method. It was regarded as the reference value. Then we carried out the other calculations using four estimation methods: (1) the adjoint-based method (Method 1); (2) forward-adjoint method (Method 2); (3) coupled adjoint-based method with union tally (Method 3); and (4) coupled forward-adjoint method with union tally (Method 4, proposed method), as shown in Table 2 and Fig. 4.

We found the apparent STD was highly underestimated; it was less than half of the minimum value when using the brute force technique, which is why the propagation degree of uncertainty from the MC1 should be estimated in two-step MC simulations. As shown in Fig. 4, Methods 1 and 2, the adjoint-based and forward-adjoint methods, respectively, produced quite similar results. However, the forward-adjoint method used in this study is more efficient and easier to apply because it does not require an additional adjoint calculation. Methods 1 and 2 continue to underestimate the combined uncertainty in comparison to the other methods because the covariance term in Eq. (8) does not properly estimate the combined uncertainty. The results from Methods 3 and 4, which consider source-term covariance, are within reference interval (Fig. 4). The analysis shows that: (1) the adjoint-based method considering a covariance term is accurate and (2) adjoint fluxes estimated by forward MC calculation can also accurately evaluate uncertainty.

3.2.   Results and verification of concrete activation problem in accelerator facility

To prove applicability of the proposed strategy, we assumed the concrete activation problem in β-nuclear magnetic resonance experimental facility in RAON accelerator [9] as described in Fig. 5. In the MC1, the secondary neutrons emitted from the bombardment of 70 MeV proton beam and targets of β-nuclear magnetic resonance experimental facility were used as the source term. It is assumed to be point source with angular and energy dependency and source strength of 3.81111×10^{13} neutrons/s. Also, we assumed that a rectangular-type ($200 \times 75 \times 250$ cm^3) target material composed of the concrete including small amount of impurities (^{59}Co, ^{151}Eu, and ^{153}Eu) was located at the left side of the neutron source. Cobalt 60, ^{152}Eu, and ^{154}Eu isotopes are produced by the (n, γ) reaction in the target material after irradiation from the neutron source and they are the dominant sources of residual gamma radiation emitted from the activated concrete [10]. In the MC2, the residual gamma radiations emitted from ^{60}Co, ^{152}Eu, and ^{154}Eu isotopes were used as the source term. The residual gamma dose rate was measured 35 cm apart from the activated material using a rectangular-type ($30 \times 30 \times 30$ cm^3) detector for each radioisotope source. For the activation analyses, it was assumed that the activated materials are evenly divided as 240 voxels (8 voxels × 3 voxels × 10 voxels).

For the transport calculation, the same transport code and cross-section data were used as those of simple activation benchmark problem in section 3.1. In the MC1, we calculated the production rates of ^{60}Co, ^{152}Eu, and ^{154}Eu in the target material for each cell using the FM card with MT = 102 in MCNP. In the MC2, residual gamma dose rates were estimated in the unit of (μSv/h) for each isotope by the same manner of the simple activation benchmark problem. Irradiation condition was assumed to be 10 years irradiation and 10 years decay. Hence, residual gamma dose rates at the 20 years after the first irradiation were evaluated for each radioisotope. The compositions and densities follow the National Institute of Standards and Technology database and ANSI/ANS-6.4-2006 [11]. We applied the particle histories to 1.5×10^6 and 1×10^8 for the MC1 and MC2, respectively.

For the uncertainty evaluation by adjoint-based method, four-group DCF described in Fig. 3 and Table 1 was used and
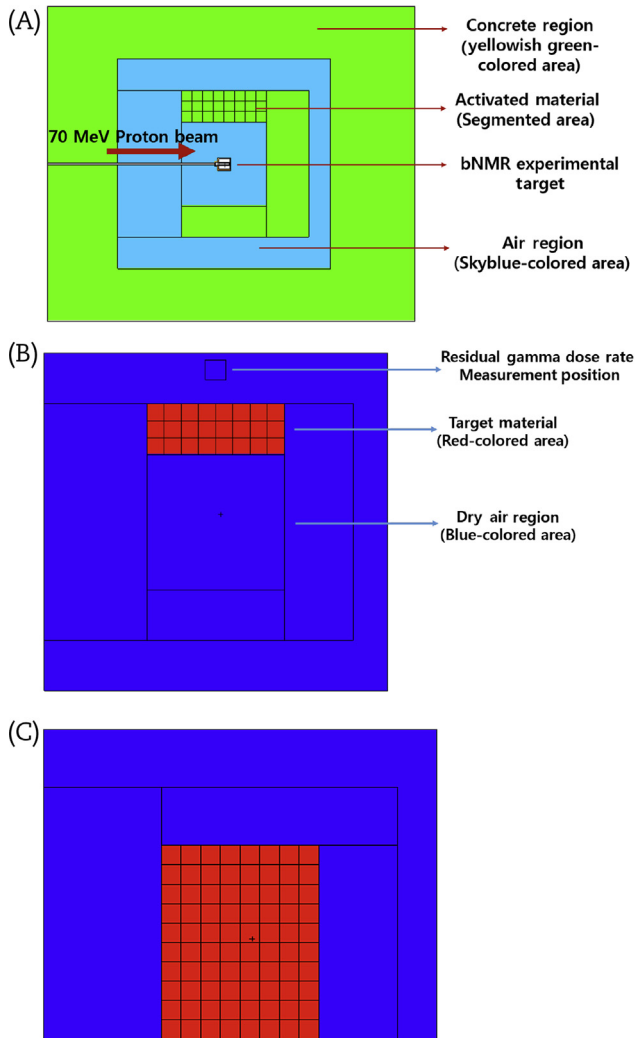
Fig. 5 – Monte Carlo N-Particle eXtended (MCNPX) modeling for the concrete activation in accelerator facility benchmark problem. (A) XY cross-section of MCNPX modeling for β-nuclear magnetic (bNMR) resonance experimental facility in a RAON accelerator [9]. (B) XY cross-section of MCNPX modeling for residual gamma transport calculation. (C) XZ cross-section of MCNPX modeling for the target material.

Fig. 6 – Comparison of the combined standard deviation results (σ_c) estimated from each method in a concrete activation benchmark problem. (A) ^{60}Co source problem. (B) ^{152}Eu source problem. (C) ^{154}Eu source problem. STD, standard deviation.

the grouping table merge option parameter option [7] was applied. SCX card in MCNP was used to estimate the adjoint fluxes in the forward MC calculation. To estimate the source-term covariance, only the neighboring union responses were evaluated in this benchmark problem. To perform the analysis with the brute force technique, we evaluated 101 responses with changing random seed numbers. After analyzing the combined uncertainty by each method, four methods introduced in Section 3.1 were compared in Fig. 6 and Table 3.

From the results, we found that the proposed scheme (Method 4) still can accurately evaluate the uncertainty propagation in more realistic and complex benchmark problems within 95% confidence interval of the reference. In concl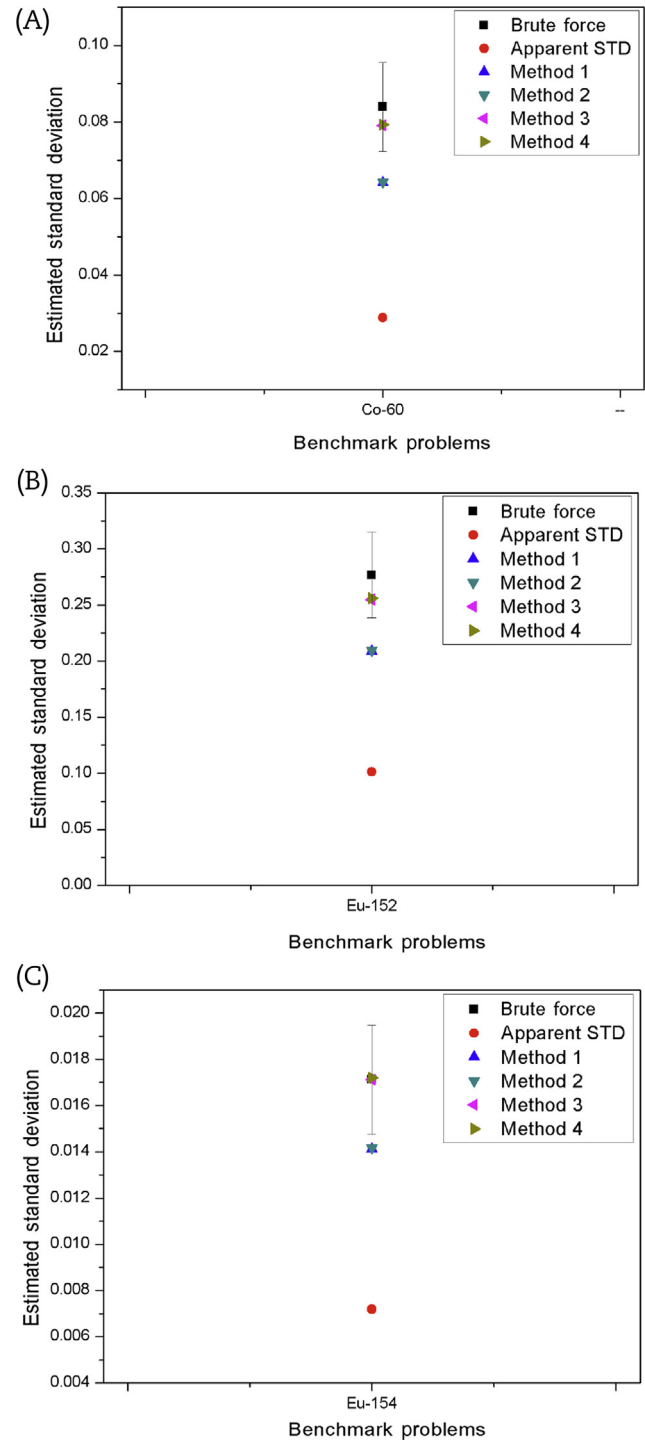usion, our proposed strategy adopting importance estimation and source-term covariance estimation in forward MC calculation has big advantages in applicability and user-friendliness because it does not require additional calculations.

Table 3 – Comparison of the combined standard deviation estimated using each method in a concrete activation benchmark problem.

Benchmarkproblem	Estimated standard deviation (μSv/h)					
	Apparent STD	Reference	Method 1	Method 2	Method 3	Method 4
		Brute force technique (95% confidence interval)	Adjoint	Forward adjoint	Adjoint + union	Proposed scheme
^{60}Co[a]	0.028830	0.082317 (0.072319, 0.095549)	0.064135	0.064302	0.079097	0.079323
^{152}Eu[a]	0.10125	0.27125 (0.23831, 0.31485)	0.20865	0.20944	0.25493	0.25597
^{154}Eu[a]	0.0071851	0.016793 (0.014753, 0.019492)	0.014108	0.014172	0.017123	0.017212

STD, standard deviation.
[a] We performed the residual radiation analyses for each source of radioactive isotope.

4. Discussion

In this study, we proposed an on-the-fly error propagation analysis strategy to efficiently estimate uncertainty in two-step MC calculations. The main focus is obtaining the information to estimate error propagation during the two-step MC simulation itself. We used activation benchmark problems to evaluate the combined uncertainties estimated using the proposed scheme and compared the results with those from the adjoint-based method and the brute force technique. The results show that our proposed method can accurately analyze the error propagation within the confidence interval of the reference results calculated by the brute force technique. Also, our analysis reveals that even though the calculation accuracy of the proposed strategy is similar to that of previous methods, our method is more efficient because it estimates the uncertainty during the two-step MC simulations without additional calculations. As a result, the proposed scheme can be directly inserted into MC code to estimate the combined uncertainties in two-step MC calculations.

Conflicts of interest

All authors have no conflicts of interest to declare.

Acknowledgments

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