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Parameterizations of the State Feedback Controllers for Linear Multivariable Systems

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Abstract—A general framework is constructed upon which an explicit parametric formula can be derived for state feedback controllers containing all the possible combination of parameters. The relation between the parameters is nonlinear in general, and therefore, many different constraint may be imposed by the designer to obtain desired performance criteria. A couple of illustrative examples are presented. © 2002 Elsevier Science Ltd. All rights reserved.

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1. INTRODUCTION

The parameterizations of state feedback controllers with eigenvalue assignment problem has been the subject of many investigators in the last two decades. Different methods of parametric eigenvalue assignment for multi-input systems have been proposed [1–9]. Karbassi and Bell [4] have introduced a new method for the parameterizations of state feedback controllers. It has been shown that from a simple algorithm based on a vector companion form obtained by elementary similarity operations and the properties of Kronecker invariants, a group of parametric controllers with linear parameters can be generated. The location of parameters can be specified by a state transition graph [5]. In this paper, the method of Karbassi and Bell [2–4] is extended to construct a general framework to obtain parameterized controllers with nonlinear parameters. It is shown that this controller gain matrix is nonlinear in nature and that the set of controllers with linear parameters are a subset of this general parameterized form. A very interesting outcome of this study is that the nonlinear system of equations for eigenvalue assignment for a given pair of system matrices and a given set of eigenvalues is uniquely determined by the structural properties of the system, that is the Kronecker invariants as defined in [2]. Since generically in almost all practical systems the Kronecker invariants are regular, in this paper we assume that the Kronecker invariants of a given system are regular. The general framework for the case of irregular Kronecker invariants [3] is very much similar but each case must be treated individually. A couple of examples which illustrate the method of obtaining the nonlinear parametric form are

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presented and in the first example an extra constraint which makes the inputs proportional to each other, as an important application of the case of nonlinear parameters, is further imposed.

2. PROBLEM STATEMENT

Consider a controllable linear time-invariant system defined by the state equation

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1}$$

or its discrete-time version

$$x(k+1) = Ax(k) + Bu(k), \tag{2}$$

where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$, and the matrices A and B are real constant matrices of dimensions $n \times n$ and $n \times m$, respectively, with $\operatorname{rank}(B) = m$. The aim of eigenvalue assignment is to design a state feedback controller, K, producing a closed-loop system with a satisfactory response by shifting controllable poles from undesirable to desirable locations. Karbassi and Bell [2–4] have introduced an algorithm obtaining an explicit parametric controller matrix K by performing three successive transformations T, S, and R which transforms the controllable pair (B, A) into standard echelon form, primary vector companion form and parametric vector companion form, respectively. This means that K is chosen such that the eigenvalues of the closed-loop system

$$\Gamma = A + BK, \tag{3}$$

lie in the self-conjugate eigenvalue spectrum $\Lambda = \{\lambda_1, \lambda_2, \dots, \lambda_n\}$. Now a similar but rather different approach is presented in this paper to obtain a controller gain matrix K containing all the possible combination of parameters.

3. SYNTHESIS

Consider the state transformation

$$x(t) = T\widetilde{x(t)}, \tag{4}$$

where T can be obtained by elementary similarity operations as described in [4]. In this way, $\tilde{A} = T^{-1}AT$ and $\tilde{B} = T^{-1}B$ are in a compact canonical form known as vector companion form

$$\tilde{A} = \begin{bmatrix} G_0 \\ I_{n-m}, 0_{n-m,m} \end{bmatrix}, \qquad \tilde{B} = \begin{bmatrix} D_0 \\ 0_{n-m,m} \end{bmatrix}, \tag{5}$$

where G_0 is an $m \times n$ matrix and D_0 is an $m \times m$ upper triangular matrix. Note that if the Kronecker invariants of the pair (B, A) are regular, then $\tilde{A}$ and $\tilde{B}$ are always in the above form. In the case of irregular Kronecker invariants, some rows of I_{n-m} in $\tilde{A}$ are displaced. We may also conclude that if the vector companion form of $\tilde{A}$ obtained from similarity operations has the above structure, then the Kronecker invariants associated with the pair (B, A) are regular.

The state feedback matrix which assigns all the eigenvalues to zero, for the transformed pair $(\tilde{B}, \tilde{A})$, is then chosen as

$$u = -D_0^{-1} G_0 \tilde{x} = \tilde{F}\tilde{x}, \tag{6}$$

which results in the primary state feedback matrix for the pair (B, A) defined as

$$F_p = \tilde{F}T^{-1}. \tag{7}$$

The transformed closed-loop matrix $\tilde{\Gamma}_0 = \tilde{A} + \tilde{B}\tilde{F}$ assumes a compact Jordan form with zero eigenvalues

$$\tilde{\Gamma}_0 = \begin{bmatrix} 0_{m,n} \\ I_{n-m}, 0_{n-m,m} \end{bmatrix}. \tag{8}$$

THEOREM. *Let $\tilde{A}_\lambda$ be any matrix in vector companion form, i.e.,*

$$\tilde{A}_\lambda = \begin{bmatrix} G_\lambda \\ I_{n-m}, 0_{n-m,m} \end{bmatrix}, \tag{9}$$

with the eigenvalue spectrum $\Lambda = \{\lambda_1, \lambda_2, \dots, \lambda_n\}$, a set of self conjugate eigenvalues. Then

$$\tilde{K} = D_0^{-1}(-G_0 + G_\lambda) \tag{10}$$

is the feedback matrix which assigns the eigenvalue spectrum to the closed-loop matrix $\tilde{\Gamma} = \tilde{A} + \tilde{B}$.

PROOF. Direct substitution yields

$$\tilde{A} + \tilde{B}\tilde{K} = \begin{bmatrix} G_0 \\ I_{n-m}, 0_{n-m,m} \end{bmatrix} + \begin{bmatrix} D_0 \\ 0_{n-m,m} \end{bmatrix} \left[D_0^{-1}(-G_0 + G_\lambda)\right], \tag{11}$$

or

$$\tilde{\Gamma} = \begin{bmatrix} G_0 - D_0 D_0^{-1} G_0 + D_0 D_0^{-1} G_\lambda \\ I_{n-m}, 0_{n-m,m} \end{bmatrix}. \tag{12}$$

Clearly, $\tilde{\Gamma} = \tilde{A}_\lambda$ has the same eigenvalues as $\tilde{A}_\lambda$.

COROLLARY. *If $\tilde{K}$ is the controller matrix which assigns the set of self-conjugate eigenvalues $\Lambda = \{\lambda_1, \lambda_2, \dots, \lambda_n\}$ to the transformed pair $(\tilde{B}, \tilde{A})$, then*

$$K = \tilde{K}T^{-1} = D_0^{-1}(-G_0 + G_\lambda)T^{-1} \tag{13}$$

is the controller matrix which assigns the same set of eigenvalues to the pair (B, A).

The above theorem leads to a general framework for obtaining the parametric controllers in general. Thus, let

$$\det\left(\tilde{A}_\lambda - \lambda I\right) = P_n(\lambda) = 0, \tag{14}$$

where

$$P_n(\lambda) = (-1)^n \left(\lambda^n + c_1\lambda^{n-1} + \cdots + c_{n-1}\lambda + c_n\right), \tag{15}$$

is the characteristic polynomial of the closed-loop system. Since it is required that the zeros of this polynomial lie in the set $\Lambda = \{\lambda_1, \lambda_2, \dots, \lambda_n\}$, it is clear that

$$P_n(\lambda) = (-1)^n(\lambda - \lambda_1)(\lambda - \lambda_2)\dots(\lambda - \lambda_n). \tag{16}$$

By equating these two equations the coefficients c_i, $(i = 1, 2, \dots, n)$ can be obtained as follows [2]:

$$\begin{aligned} c_1 &= -\sum_{i=1}^{n}(\lambda_i) = -\operatorname{trace}\left(\tilde{A}_\lambda\right) \\ c_2 &= \sum_{i,j=1,i\neq j}^{n}(\lambda_i\lambda_j) = -\frac{\left(c_1 \operatorname{trace}\left(\tilde{A}_\lambda\right) + \operatorname{trace}\left(\tilde{A}_\lambda^2\right)\right)}{2} \\ &\;\vdots \\ c_n &= (-1)^n\prod_{i=1}^{n}(\lambda_i) = -\frac{\left(c_{n-1}\operatorname{trace}\left(\tilde{A}_\lambda\right) + c_{n-2}\operatorname{trace}\left(\tilde{A}_\lambda^2\right) + \dots + c_1\operatorname{trace}\left(\tilde{A}_\lambda^{n-1}\right) + \operatorname{trace}\left(\tilde{A}_\lambda^n\right)\right)}{n}. \end{aligned} \tag{17}$$

It should be noted that when λ_i, $(i = 1, 2, \dots, n)$ are specified, then c_1 can be calculated easily, while for large n the above recursive formula will facilitate the computation of $c_2, c_3, \dots, c_n$, using the fact that [10]

$$\operatorname{trace}\left(\tilde{A}_\lambda^r\right) = \sum_{i=1}^{n}({\lambda_i}^r). \tag{18}$$

Now by direct computation of $\det(\tilde{A}_\lambda - \lambda I)$ in parametric form and equating the coefficients of the characteristic polynomial with (17), the following nonlinear system of equations is obtained:

$$\begin{aligned} f_1(g_{11}, g_{12}, \dots, g_{1n}, g_{21}, g_{22}, \dots, g_{2n}, \dots, g_{m1}, g_{m2}, \dots, g_{mn}) &= c_1, \\ f_2(g_{11}, g_{12}, \dots, g_{1n}, g_{21}, g_{22}, \dots, g_{2n}, \dots, g_{m1}, g_{m2}, \dots, g_{mn}) &= c_2, \\ &\vdots \\ f_n(g_{11}, g_{12}, \dots, g_{1n}, g_{21}, g_{22}, \dots, g_{2n}, \dots, g_{m1}, g_{m2}, \dots, g_{mn}) &= c_n, \end{aligned} \tag{19}$$

where g_{ij}, $(i = 1, \dots, m,\ j = 1, \dots, n)$, are the elements of G_λ:

$$G_\lambda = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1n} \\ \dots & \dots & \dots & \dots \\ g_{m1} & g_{m2} & \cdots & g_{mn} \end{bmatrix}. \tag{20}$$

In this way, a nonlinear system of n equations with $n \times m$ unknowns is obtained. By choosing $N = n(m-1)$ unknowns arbitrarily it is then possible to solve the system. Thus, different selections can be made to obtain different solutions. Thus, different selections can be made to obtain different solutions. Indeed, the Newton's iterative method, however, if the method does not converge in a reasonable number of iterations, the initial values may be updated randomly. It is interesting to note that with the total number of free parameters is $N = nm - n$ and the number of different combinations of the parametric state feedback controllers is

$$P = \left(\frac{(nm-n)!}{(n)!(nm-2n)!} \right). \tag{21}$$

Clearly, some of these choices lead to linear parameters. Therefore, the set of linear parametric controllers obtained in [2–5] is the subset of the general nonlinear controller K.

It should be noted that since the form of $\tilde{A}_\lambda$ in (9) is only dependent on the Kronecker invariants of the pair (B, A) and is unique, then for a specified set of eigenvalues the coefficients of the characteristic polynomial, c_i, $(i = 1, \dots, n)$, are uniquely determined. Intuitively, the nonlinear system of equations described in (19) are universal for a prescribed set of eigenvalues and known Kronecker invariants and its solutions are independent of the numerical values of the pair (B, A). The controller gain matrix for the original pair (B, A) can then be obtained by (13), in which D_0^{-1}, G_0, and T^{-1} have a crucial effect. In other words, the nonlinear system of equations (19) is uniquely defined for any given pair (B, A) of fixed dimensions and regular Kronecker invariants.

In deriving the nonlinear system of equations, direct computation of $\det(\tilde{A}_\lambda - \lambda I)$ with the parameters is rather cumbersome for large n and m. However, elementary column operations on this matrix lead to the computation of the determinant of an $m \times m$ matrix rather than an $n \times n$ matrix [11]. The following illustrative examples demonstrate these points.

5. ILLUSTRATIVE EXAMPLES

Consider the system [4]

$$B = \begin{bmatrix} 3 & 3 \\ 0 & 1 \\ 5 & 5 \end{bmatrix}, \qquad A = \begin{bmatrix} 16 & -12 & 11 \\ 5 & 1 & 3 \\ -26 & 20 & -18 \end{bmatrix}.$$

It is desired to obtain parametric state feedback controllers which assign the eigenvalues $\Lambda = \{-1, -2, -3\}$ to the closed-loop system. The transformed pair $(\tilde{B}, \tilde{A})$ in vector companion from and the corresponding transformation matrix are

$$\tilde{B} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad \tilde{A} = \begin{bmatrix} -2 & -4 & 2 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix},$$

$$T^{-1} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 5 & 0 & 3 \end{bmatrix}.$$

Clearly,

$$D_0^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

and

$$G_0 = \begin{bmatrix} -2 & -4 & 2 \\ 0 & 1 & 1 \end{bmatrix}.$$

Now, let us consider

$$\tilde{A}_\lambda = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ 1 & 0 & 0 \end{bmatrix}$$

with the same canonical structure as $\tilde{A}$. Here

$$G_\lambda = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \end{bmatrix}$$

is the parametric controller matrix in the transformed space. Our aim is to obtain the set of equations relating these parameters such that the eigenvalues of $\tilde{A}_\lambda$ are in the set $\Lambda = \{-1, -2, -3\}$. Now

$$\det\left(\tilde{A}_\lambda - \lambda I\right) = -\left(\lambda^3 - (g_{11} + g_{22})\lambda^2 + (g_{11}g_{22} - g_{12}g_{21} - g_{13})\lambda + (g_{22}g_{13} - g_{12}g_{23})\right), \quad (22)$$

while

$$P_3(\lambda) = (-1)^3\left(\lambda^3 + 6\lambda^2 + 11\lambda + 6\right). \quad (23)$$

By equating the coefficients of these two equations, we obtain the nonlinear system of equations

$$-(g_{11} + g_{22}) = 6, \quad (24)$$

$$g_{11}g_{22} - g_{12}g_{21} - g_{13} = 11, \quad (25)$$

$$g_{22}g_{13} - g_{12}g_{23} = 6, \quad (26)$$

which is universal, that is for any given controllable A and B of dimension $n = 3$ and $m = 2$ with regular Kronecker invariants and the prescribed eigenvalues as above is unique.

Here, there are three equations with six unknowns. The first equation is linear in parameters while the other two equations are nonlinear. To obtain explicit solutions, three of these unknowns may be selected arbitrarily. For example, suppose $g_{11} = -2$ and $g_{22} = -4$ then from (25) and (26) we obtain

$$g_{13} = -3 - g_{12}g_{21} \quad (27)$$

and

$$g_{23} = \frac{(6 + 4g_{12}g_{21})}{g_{12}}. \quad (28)$$

Therefore,

$$G_\lambda = \begin{bmatrix} -2 & g_{12} & -3 - g_{12}g_{21} \\ g_{21} & -4 & \dfrac{(6 + 4g_{12}g_{21})}{g_{12}} \end{bmatrix}. \quad (29)$$

The nonlinear parametric controller matrix K for the pair (B, A) is then

$$K = D_0^{-1}(-G_0 + G_\lambda)T^{-1}$$
$$= \begin{bmatrix} -20 - 22g_{21} - 5g_{12}g_{21} - \dfrac{30}{g_{12}} & 9 + g_{12} & -12 - 13g_{21} - 3g_{12}g_{21} - \dfrac{18}{g_{12}} \\ -5 + 22g_{21} + \dfrac{30}{g_{12}} & -5 & -3 + 13g_{21} + \dfrac{18}{g_{12}} \end{bmatrix}. \quad (30)$$

Here K is expressed explicitly in terms of two parameters which result in nonlinear terms.

Instead of specifying three of the parameters, we may impose some constraint on the system performance. For example, suppose it is required that the inputs be proportional to each other, this means that the rows of the transformed controller matrix $G = -G_0 + G_\lambda$ should be made proportional to each other. For this example, we have

$$G = \begin{bmatrix} 2 + g_{11} & 4 + g_{12} & -2 + g_{13} \\ g_{21} & -1 + g_{22} & -1 + g_{23} \end{bmatrix}. \quad (31)$$

If we wish to have $u_2 = cu_1$, where c is a constant, then the second row of G must be proportional to its first row. Applying this constraint, we obtain three more equations

$$g_{21} = c(2 + g_{11}), \quad (32)$$
$$g_{22} - 1 = c(4 + g_{12}), \quad (33)$$
$$g_{23} - 1 = c(-2 + g_{13}). \quad (34)$$

If we choose $c = 2$ and solve the nonlinear system of six equations thus obtained ((24)–(26), and (32)–(34)), we will then obtain

$$G = \begin{bmatrix} 0.5385 & -2.76925 & 0.9235 \\ 1.0770 & -5.5385 & 1.8470 \end{bmatrix}, \quad (35)$$

while for the original system K is found to be:

$$K = \begin{bmatrix} -5.6945 & 2.7692 & -3.3090 \\ 11.3890 & -5.5385 & 6.6180 \end{bmatrix}. \quad (36)$$

It is evident that if this controller gain matrix is applied to the given system, then $u_2 = 2u_1$.

EXAMPLE 2. Consider the system [6]

$$B = \begin{bmatrix} 0 & 0 \\ 5.6790 & 0 \\ 1.1360 & -3.1460 \\ 1.1360 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 1.3800 & -0.2077 & 6.7150 & -5.6760 \\ -0.5814 & -4.2900 & 0 & 0.6750 \\ 1.0670 & 4.2730 & -6.6540 & 5.8930 \\ 0.0480 & 4.27306 & 1.3430 & -2.1040 \end{bmatrix}.$$

It is desired to obtain explicit parametric state feedback controllers which assign the set of eigenvalues $\Lambda = \{-0.2, -0.5, -5.0566, -8.6659\}$ to the closed-loop system. The transformed pair $(\tilde{B}, \tilde{A})$ in vector companion form and the corresponding transformation matrix are

$$\tilde{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \tilde{A} = \begin{bmatrix} -5.2588 & 0.2498 & -1.2439 & 2.6983 \\ -1.3832 & -6.4092 & -11.0617 & 19.5403 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$
$$T^{-1} = \begin{bmatrix} -0.0040 & 0.1839 & 0 & -0.0393 \\ -0.0653 & 0.0098 & -0.3179 & 0.2687 \\ -0.0071 & -0.0071 & 0 & 0.0356 \\ -0.0473 & 0 & 0 & 0 \end{bmatrix}.$$

Clearly,

$$D_0^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and

$$G_0 = \begin{bmatrix} -5.2588 & 0.2498 & -1.2439 & 2.6983 \\ -1.3832 & -6.4092 & -11.0617 & 19.5403 \end{bmatrix}.$$

Now let us consider

$$\tilde{A}_\lambda = \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

with the same conanical structure as $\tilde{A}$. Here

$$G_\lambda = \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \end{bmatrix}$$

is the parametric controller matrix in the transformed space. Our aim is to obtain the set of equations relating these parameters such that the eigenvalues of $\tilde{A}_\lambda$ are in the set $\Lambda = \{-0.2, -0.5, -5.0566, -8.6659\}$. Now it can be easily verified that

$$\begin{aligned} \det\left(\tilde{A}_\lambda - \lambda I\right) &= \lambda^4 - (g_{11} + g_{22})\lambda^3 + (g_{11}g_{22} - g_{12}g_{21} - g_{13} - g_{24})\lambda^2 \\ &+ (g_{22}g_{13} - g_{12}g_{23} + g_{11}g_{24} - g_{14}g_{21})\lambda + (g_{13}g_{24} - g_{14}g_{23}), \end{aligned} \tag{37}$$

while

$$P_4(\lambda) = (-1)^4 \left(\lambda^4 + 14.4225\lambda^3 + 53.5257\lambda^2 + 32.0462\lambda + 4.3820\right). \tag{38}$$

By equating the coefficients of these two equations, we obtain the nonlinear system of equations

$$-(g_{11} + g_{22}) = 14.4225, \tag{39}$$

$$g_{11}g_{22} - g_{12}g_{21} - g_{13} - g_{24} = 53.5257, \tag{40}$$

$$g_{22}g_{13} - g_{12}g_{23} + g_{11}g_{24} - g_{14}g_{21} = 32.0462, \tag{41}$$

$$g_{24}g_{13} - g_{14}g_{23} = 4.3820, \tag{42}$$

which is universal, that is for any given controllable A and B of dimension $n = 4$ and $m = 2$ with regular Kronecker invariants and the prescribed eigenvalues as above is unique. Here we have four equations with eight unknowns. If we choose $g_{11} = -10$, $g_{12} = 1$, $g_{21} = -9.3007$, and $g_{13} = -g_{24}$, say, then we will obtain $g_{22} = -4.4225$, and $g_{14} = -(g_{24}^2 + 4.3820)/g_{23}$ where g_{23} is the solution of

$$g_{23}^2 + (5.5875g_{24} + 32.0462)g_{23} + 9.3007\left(g_{24}^2 + 4.3820\right) = 0. \tag{43}$$

Therefore,

$$G_\lambda = \begin{bmatrix} -10 & 1 & -g_{24} & -\dfrac{(g_{24}^2 + 4.3820)}{g_{23}} \\ -9.3007 & -4.4225 & g_{23} & g_{24} \end{bmatrix}. \tag{44}$$

This will lead to an explicit formula with nonlinear parameters for the controller matrix. With $g_{24}^2 = 0$, it is easy to verify that one possible solution is

$$G_\lambda = \begin{bmatrix} -10 & 1 & 0 & 0.1426 \\ -9.3007 & -4.4225 & -30.7195 & 0 \end{bmatrix}, \tag{45}$$

producing

$$K = \begin{bmatrix} 0.0820 & -0.8736 & -0.2385 & 0.4319 \\ 0.9664 & -1.2970 & -0.6315 & 0.1455 \end{bmatrix}. \tag{46}$$

One of the feedback matrices obtained in [6] is

$$K_3 = \begin{bmatrix} 0.1028 & -0.633 & -0.1187 & 0.1463 \\ 0.8361 & 0.5270 & -0.2577 & 0.5427 \end{bmatrix}. \tag{47}$$

Using equation (13) backwards we can obtain G_λ for this controller matrix and it is found that in fact

$$G_\lambda = \begin{bmatrix} -8.8241 & 0.6232 & -3.8866 & 0.7080 \\ 1.9336 & -5.5983 & 1.7338 & -1.4435 \end{bmatrix}, \tag{48}$$

whose elements satisfy the nonlinear system of equations (39)–(42).

6. CONCLUSION

The advance the paper presents over the previous work is the development of a method based on the structural properties of parametric vector companion forms, a general framework for explicit formulas for state feedback controllers with nonlinear parameters in arbitrary eigenvalue assignment was presented. Clearly, it is a simple matter to obtain the linear parametric controllers from this general form.

An interesting outcome of this study is that the nonlinear system of equations obtained for a given pair of (B, A) and a given set of eigenvalues is not only unique, but is universal and that it only depends on the Kronecker invariants of the system as defined in [2] and the prescribed set of eigenvalues. Also, the number of free parameters obtained in this way is much greater than the number obtained previously by Amin and Elabdalla [1] or O'Reilly and Fahmy [7]. The general framework for the case of irregular Kronecker invariants [3] is very much similar, but each case must be considered individually. However, the Kronecker invariants of most practical systems are regular.

The method does not require prior knowledge of the open-loop eigenvalues and the controller does not impose any restriction on the position of the desired eigenvalues or their nature and multiplicity.

The problem of minimizing the condition number of the closed-loop eigenvector matrix and other measures of robustness [12] using the state feedback matrix with nonlinear parameters and extensions to techniques for modifying the locations of the assigned eigenvalues to regions of the complex plane are currently being developed.

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