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# A Comparative Study On The Derivation Of Unit Hydrograph For Bharathapuzha River Basin

Abhinanda Roy[a], Reeba Thomas[b]*

[a]*M-Tech student, Civil Engineering Department,Government Engineering College, Thrissur-680009 ,India*
[b]*Associate Professor, Civil Engineering Department,Government Engineering College, Thrissur-680009 ,India*

**Abstract**

Several techniques are available for the development of the unit hydrograph. But most of these traditional methods require manual fitting of the unit hydrograph through few points, which does not guarantee the area under the unit hydrograph to be unity. More over most of the stations are ungauged, due to which it becomes difficult to develop the unit hydrograph. So in order to overcome these problems, two methods have been considered in this study for the development of the unit hydrograph for Bharathapuzha river basin. They are the "two parameter Gamma distribution" and "three parameter Beta distribution", both of which are based on Probability Distribution Functions (pdfs). The unit hydrograph developed by the two parameter Gamma distribution match well with the one developed by CWC method, but the unit hydrograph developed by the three parameter Beta distribution does not match well with the one developed by the CWC method. From the unit hydrograph, runoff hydrograph is convoluted for the year 2008. For this the hourly rainfall are generated from daily rainfall values by disaggregation. But on plotting, the simulated discharge hydrograph is found to be greater than the observed discharge. This may be due to non incorporation of the inflow outflow processes of many hydraulic structures such as dams, irrigation schemes etc, existing in the basin in the model study. The data related to these structures could not be obtained due to certain restriction in acquiring the data from authorized agencies.

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*Keywords:* unit hydrograph; Gamma distribution; Beta distribution; CWC method; discharge hydrograph.

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* Corresponding author. Tel.: 9809854862; fax: +0-000-000-0000 .
 *E-mail address:*abhinandaroy123@gmail.com, reeba@gectcr.ac.in

## 1. Introduction

The beginning of rainfall runoff modeling for solving engineering problems dates back to the second half of the nineteenth century. Even today it is very important in activities such as design of hydraulic structures in watershed, flood control and management etc. But in the absence of runoff data, as in case of ungauged stations, it becomes necessary to obtain runoff data with the help of set up models. Historically researchers have relied on conventional techniques; however these techniques do not ensure the area under the graph to be unity. Hence to develop a runoff data for a rainfall event, L. K. Sherman introduced the concept of "unit hydrograph" in 1932. This was the first attempt to predict a unit hydrograph instead of just peak flow rate and time to peak. Later many researchers attempted to improve the unit hydrograph shape by incorporating more complexities in the model. Thus unit hydrograph is the most widely accepted tool for hydrological analysis and synthesis.

However as most of the sites are ungauged, the idea of Synthetic Unit hydrograph (SUH) came into existence. The term 'synthetic' in SUH denotes that the unit hydrograph has been derived from watershed characteristics and not from the rainfall runoff data. Moreover the traditional methods for developing SUH required manual fitting of the points which is subjective and do not assure the area under the graph to be unity. The empirical equations used in the traditional methods also involve certain constants which vary over wide range. Due to the similarity in shape of statistical distribution and a conventional unit hydrograph, several attempts have been made to use pdfs for the development of unit hydrograph. The Gamma distribution and Beta distribution are such pdfs and they not only ensure the area under the graph to be unity but also provide a smooth shape for the unit hydrograph.

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**Nomenclature**

| | |
|---|---|
| $A$ | area of the watershed in sq km |
| $b$ | The scale parameter (in hours) in Beta distribution |
| $B(a,b)$ | The Beta function of $a$ and $b$, where $a$ and $b$ are parameters |
| CWC | Central Water Commission |
| IUH | Instantaneous Unit Hydrograph |
| $K$ | Equal storage coefficient |
| $n$ | Number of linear reservoirs |
| $p$ | Non dimensional shape parameters in Beta distribution |
| pdf | probability distribution function |
| $q$ | depth of runoff per unit of time per unit of effective rainfall |
| $q_p$ | Peak discharge of unit hydrograph per unit area in cumecs per sq km |
| $Q_p$ | Peak discharge in cumecs |
| $r$ | Non dimensional shape parameters in Beta distribution |
| $R_i$ | Rainfall event |
| SUH | Synthetic unit hydrograph |
| $t$ | time in hours |
| $T_B$ | Base width of the unit hydrograph in hours |
| $t_p$ | Time to peak discharge in hours |
| $W_{50}$ | Width of the unit hydrograph measured at 50% peak discharge ordinate in hours |
| $W_{75}$ | Width of the unit hydrograph measured at 75% peak discharge ordinate in hours |
| $W_{R50}$ | Width of the rising side of the unit hydrograph measured at 50% of peak discharge ordinate in hours |
| $W_{R75}$ | Width of the rising side of unit hydrograph measured at 75% of peak discharge in hours |
| $\alpha$ | Dimensionless parameter in Beta distribution |
| $\beta$ | Dimensionless parameter in Gamma and Beta distribution |
| $\gamma$ | Dimensionless parameter in Beta distribution |
| $\Gamma$ | Gamma function |

## 2. Objectives

The main objective of the study is to derive the unit hydrograph for the Bharathapuzha river basin by the Gamma distribution and Beta distribution, and compare them. For this, first it is required to determine the peak discharge ($q_p$), time to peak discharge ($t_p$) and base width of the unit hydrograph ($T_B$) by the CWC method. Then compare the unit hydrograph developed by the gamma and beta distribution. Using this unit hydrograph, the discharge hydrograph is convoluted for the year 2008.

## 3. Study Area and Data Required

The study area considered is Bharathapuzha river basin ($10^o25$' to $11^o15$' N and $75^o50$' to $76^o55$' E), located in the state of Kerala, India. With a length of 209km, it is the second longest river in the state, after Periyar River. The river originates in the Anaimalai hills in the Western Ghats. The watershed has a total area of 6186 km$^2$, with two third of the area (4400 km$^2$) within Kerala and the remaining area in Tamil Nadu (1786 km$^2$). The watershed caters 11 dams and many irrigation projects. Some of the important irrigation projects are Chitturpuzha, Walayar, Malampuzha, Cheerakuzhi, Gayathri, Pothundi, Mangalam, Vazhani etc. In this Chitturpuzha is a major irrigation project while others are medium irrigation projects.

The data required for the study are the toposheet of the study area, SRTM data, satellite imagery, land use map, daily/hourly rainfall and discharge data. Bharathapuzha being an interstate river, sharing water between Kerala and Tamil Nadu there were difficulty in obtaining hydrological data. Hence the daily rainfall data is obtained from the website – "globalweather.tamu.edu." for the year 2008.

## 4. Theoretical Background

### 4.1. Central Water Commission (CWC) Method

The CWC method is used for the determination of $q_p$, $t_p$ and $T_B$. For this, first the equivalent slope is to be computed, for which first the longest stream is identified and then the points are identified where bed level changes considerably. Using these, the equivalent slope is computed by using the formula of CWC method. Then $q_p$, $t_p$ and $T_B$ are computed, followed by the computation of other parameters which include $W_{50}$, $W_{75}$, $W_{R50}$ and $W_{R75}$ which are required for plotting the unit hydrograph. All this are computed by using the formula in the CWC method.

### 4.2. Two Parameter Gamma Distribution Method

On the basis of the concept of n-linear reservoirs having equal storage coefficient K, Nash and Dooge (1959) developed the IUH in the form of gamma function as

$$q = (1/K\Gamma n) * (t/K)^{n-1} * e^{-t/K} \tag{1}$$

where n and K determine the shape of the IUH. Equation (1) is used for the derivation of SUH from parameters n and K. In this, the parameter K is derived from

$$K = t_p / n - 1 \tag{2}$$

The value of n is computed on the basis of the dimensionless parameter β, which is given by (3)

$$\beta = q_p * t_p \tag{3}$$

Then on the basis of β values, the formula for n is given in (4) and (5)

$$n = 5.53\beta^{1.75} + 1.04 \qquad ; \; 0.01 < \beta < 0.35 \tag{4}$$

$$n = 6.29\beta^{1.998} + 1.157 \qquad ; \; \beta > 0.35 \tag{5}$$

Thus the two parameters of the gamma distribution that determine the shape of the unit hydrograph are n and K, which can be computed by using $q_p$ and $t_p$. So by knowing the value of $q_p$ and $t_p$ the hydrograph can be developed by using two parameter gamma distribution.

*4.3. Three Parameter Beta Distribution Method*

Haktanir and Sezen (1990) worked on finding the suitability of three parameter Beta distribution as Synthetic Unit Hydrograph. The probability distribution function of beta distribution is given as

$$f(x) = [1/B(a,b)] * x^{n-1} * (1-x)^{b-1} \tag{6}$$

$$B(a,b) = \int_0^1 t^{a-1} * (1-t)^{b-1} \, dt \tag{7}$$

Equation (6) can be converted into a three parameter distribution as

$$q = [t^{(r-1)} * (b-t)^{(p-r-1)}] \big/ [B(r, p-r) * b^{(p-r)}] \tag{8}$$

where r, p, b, q and t have already been defined in nomenclature. The advantage of the equation is that the area under the curve described by (8) for limit t=0 and t=time base of the hydrograph is one unit. So the three parameters in this case are p, r and b. But for the determination of p, r and b, it is required to define three other non dimensional groups, which are α, β and γ, which are given by the equation

$$\alpha = T_B / t_p \tag{9}$$

$$\beta = q_p * t_p \tag{10}$$

$$\gamma = q_p * T_B \tag{11}$$

The non dimensional parameter β, is to be taken as a form factor which quantifies the hydrograph peakness and influences the hydrograph shape. The non dimensional parameter γ can be related to the hydrograph area and the non dimensional parameter α is similar to the scale parameter since it influences the skewness of the unit hydrograph. Using α, β and γ, the three parameters p, r and b are obtained as

$$p = 2.5355 * (0.0481 + 2.961\beta^{1.875}) * (\alpha^{0.954}) - 10.72777\beta \tag{12}$$

$$r = 1 - [(2-p)/\alpha] \tag{13}$$

$$b = T_B \tag{14}$$

Thus by using known $q_p$, $t_p$ and $T_B$, the parameters of the beta distribution can be estimated, and hence the complete shape of the unit hydrograph cen be derived for any ungauged catchment.

## 5. Methodology

### 5.1. Determination Of $q_p$, $t_p$ And $T_B$ By The CWC Method

In this, first it is required to determine $q_p$, $t_p$ and $T_B$ by the CWC method. For this first the watershed is to be delineated, which is done by processing DEM in the GIS platform, by considering Kuttipuram discharge station as an outlet point. Then the longest stream is identified, followed by the identification of those points where bed level changes occur. The equivalent slope is then computed as explained in section 4. Using this, $q_p$, $t_p$ and $T_B$ is obtained. Further $W_{50}$, $W_{75}$, $W_{R50}$ and $W_{R75}$ are computed and using all these points, unit hydrograph is developed by the CWC method.

### 5.2. Derivation Of The Unit Hydrograph By Gamma Distribution

Using the value of $q_p$ and $t_p$ which is already computed, the parameters of the Gamma distribution, that is, n, K and β are estimated using (2) and (3), (4) and (5), as explained in section 4. But prior to plotting of the unit hydrograph, the value of gamma n is determined using the Sterling's equation as shown below.

$$\Gamma n = (e^{-n}) * n^{(n-0.5)} * 2\Pi^{0.5} * [1 + (1/12n) + (1/288n^2) - (139/5184n^3) - (571/2488320n^4)] \tag{15}$$

Once the parameters of the gamma distribution are estimated, then q is developed by varying the value of 't' in (1). Finally the discharge (Q) in cumecs is computed by (16) and the unit hydrograph is plotted

$$Q = 2.776 * q * A \tag{16}$$

### 5.3. Derivation Of The Unit Hydrograph By Beta Distribution

The steps in beta distribution are same as the gamma distribution, with the only difference being in parameter estimation. The parameters p, r and b of beta distribution are estimated using (12-14) from section 4, where $T_B$, which is already determined by the CWC method is used, in addition to $q_p$ and $t_p$. But prior to plotting of the unit hydrograph, the value of B(r,p-r) is determined by using the formula

$$B(r, p - r) = [\Gamma r * \Gamma(p - r)]/(\Gamma p) \tag{17}$$

where $\Gamma(r)$, $\Gamma(p-r)$ and $\Gamma(p)$ are estimated using Sterlings formula. Then q is developed by varying the value of 't' in (8). Finally the discharge (Q) in cumecs is given by (16) and unit hydrograph is plotted.

### 5.4. Determination Of Average Hourly Rainfall

For the determination of average hourly rainfall, first the average daily rainfall is computed by using 'Theissen polygon' option in arcGIS. For this 14 rain gauge stations are considered and the theissen polygon is drawn. Then this average daily rainfall is disaggregated to average hourly rainfall by using the CWC method.

### 5.5. Convolution

The discharge hydrograph is then obtained by multiplying the ordinates of the unit hydrograph with the rainfall. Rain events are selected and labeled as R1,R2,R3 etc. Runoff from the storm event R1 on area A2 arrives at the outlet at the same time as the rainfall event $R_2$ on $A_1$. Hence

$$Q_t = R_i A_1 + R_{i-1} A_2 + R_{i-2} A_3 + \_\_\_\_ + R_1 A_j \tag{18}$$

## 6. Results and discussions

### 6.1. Delineation Of The Watershed

First the watershed is delineated using Kuttipuram discharge station as the outlet point. The delineated watershed is shown in fig 1.(a) and the longest stream identified is shown in fig 1.(b).
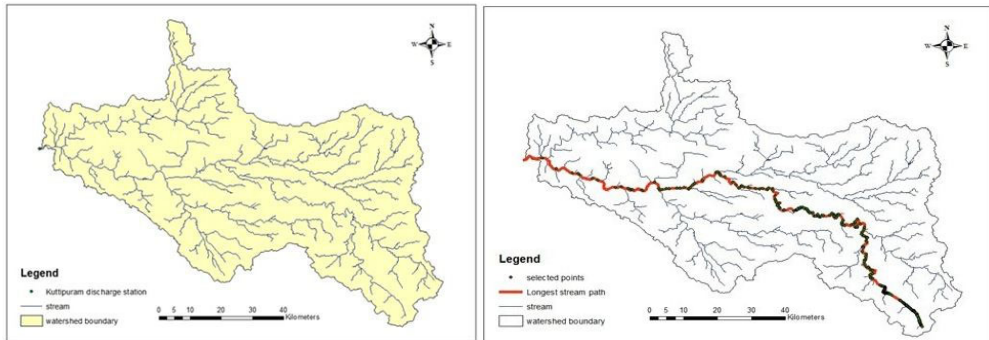
Fig.1. (a) Delineated watershed map of Bharathapuzha river basin; (b) Map showing longest path and points selected for slope determination

Then the peak discharge, time to peak discharge and other parameters which are required for the development of the unit hydrograph by the CWC method are computed as shown in table 1.

Table 1. Parameter estimated by CWC method.

| Parameter | Value obtained |
|---|---|
| Peak discharge ($q_p$) | $0.081 \text{ m}^3/\text{s/km}^2$ |
| Time to peak discharge ($t_p$) | 23.58 hrs |
| Base width of the unit hydrograph ($T_B$) | 75.15 hrs |
| $W_{50}$ | 29.69 hrs |
| $W_{75}$ | 14.02 hrs |
| $W_{R50}$ | 9.33 hrs |
| $W_{R75}$ | 4.89 hrs |

### 6.2. Results Of Gamma and Beta Distribution

After the determination $q_p$, $t_p$ and $T_B$ by the CWC method, the various parameters of the Gamma and Beta distribution are computed as shown in table 2 and table 3 respectively.

Table 2. Parameter estimated by Gamma distribution.

| Parameter | Values obtained |
|---|---|
| β | 0.68 |
| n | 3.816 |
| K | 8.37 hours |
| Gamma n (Γn) | 7.24 |

Table 3. Parameter estimated by Beta distribution.

| Parameter | Values obtained |
|---|---|
| β | 0.68 |
| α | 3.186 |
| p | 4.268 |
| r | 1.711 |
| b | 75.15 hours |
| B(r,p-r) | 0.148 |

Substituting these parameters in (1) for gamma distribution and (8) for beta distribution, the equation for q reduces to the form (19) and (20) respectively

$$q = (0.0184) * (t/8.37)^{2.816} * e^{-t/8.37} \tag{19}$$

$$q = [t^{0.711} * (75.15 - t)^{1.557}] / 200561.33 \tag{20}$$

Then by varying the value of 't', the value of q are obtained. Finally discharge (Q) is obtained by using (16). The unit hydrograph thus developed for gamma and beta distribution is shown in fig 2.(a) and fig 2.(b) respectively.
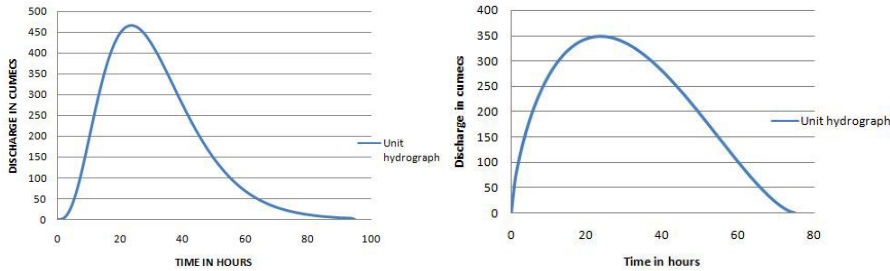


Fig.2. (a) Unit hydrograph developed by Gamma distribution ; (b) Unit hydrograph developed by Beta distribution

### 6.3. Comparison Of The Methods

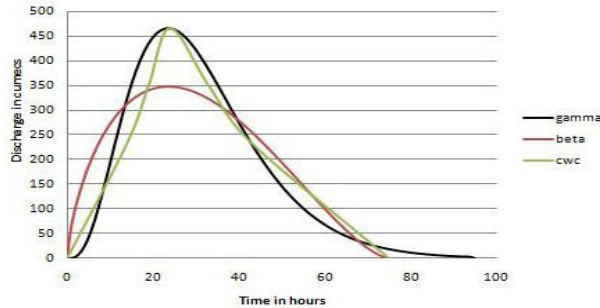The unit hydrograph developed by all the methods are shown in fig 3.



Fig.3. Comparison of unit hydrograph developed by different methods

From this it is seen that the unit hydrograph developed the CWC method and gamma distribution match with each other. However the unit hydrograph developed from beta distribution is different. Even though both give the same value for time to peak discharge, but the peak discharge vary. In case of beta distribution, the peak discharge is under estimated. However the value for base width of the unit hydrograph ($T_B$) is different for all of them. The beta distribution resulted in a hydrograph of flat shape as the value of $T_B$ is taken as a parameter for unit hydrograph derivation. As the unit hydrograph developed by the CWC method and gamma distribution match with each other, they can be considered as good methods for developing unit hydrograph.

### 6.4. Generation Of Discharge Hydrograph

After the generation of unit hydrograph, the discharge hydrograph is simulated for the year 2008 by convolution. A comparison of actual discharge hydrograph with the simulated one for the period July to September 2008 is shown in fig 4. This is done by taking the hourly rainfall derived from daily data by disaggregation.
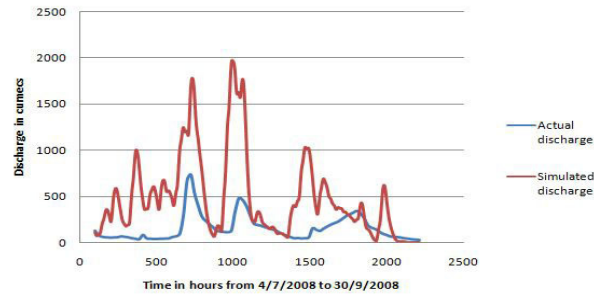
Fig.4. Simulated discharge Vs observed discharge

It is found that the simulated values are much higher than the observed values. Bharathapuzha basin has large areal extent of order of 6186 $km^2$, which caters many dams and irrigation structures. The inflow-outflow processes of these hydraulic structures are not accounted in the model study due to non availability of data. So these may be the reasons for over estimation of simulated discharge.

## 7. Conclusion

The unit hydrograph is developed by the gamma distribution and beta distribution, both of which depend on the watershed characteristics rather than the rainfall-runoff data. Some of the results observed in the study are:

1. The unit hydrograph developed by the gamma distribution is matching with the one developed by the CWC method.
2. The unit hydrograph developed by the beta distribution is flat. This is attributed to the fact that in beta distribution, the base width of the unit hydrograph is taken as a parameter for the derivation of unit hydrograph and any error in the estimation of base width can cause corresponding change in peak so as to adjust the unit runoff volume.
3. The simulated discharge however does not match with the observed discharge. This can be attributed to various physical characteristics pertaining to the study area. Firstly, the Bharathapuzha river basin is spread over an area of 6186 $km^2$ and it caters large number of dams and irrigation projects. However due to interstate water issues and for maintaining confidentiality in related data, various hydrologic details like inflow-outflow parameters couldn't be collected from authorized agencies. Hence due to the unavailability of the data, operation of these structures couldn't be incorporated in the model for discharge computation.

   For the same reason as above, the rainfall data also were unavailable and hence they were downloaded from the website "globalweather.tamu.edu". Due to the lack of hourly data, it was disaggregated using CWC method.

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