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On p-embedding problems in characteristic p

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ABSTRACT

Let K be a valued field of characteristic $p > 0$ with non-p-divisible value group. We show that every finite embedding problem for K whose kernel is a p-group is properly solvable.

1. Introduction

In the proof that every finite solvable group occurs as a Galois group over the rationals, Shafarevich studies the solvability of embedding problems with nilpotent kernel and solvable cokernel. The study of the absolute Galois group $\mathrm{Gal}(K)$ of a field K via embedding problems continues to be central in recent papers, e.g. [1,9,10,15,17]. See also the upcoming book [11] and the references therein.

In this work we consider a field K of characteristic $p > 0$ and the finite embedding problems for K whose kernels are p-groups which we call **finite p-embedding problems**. An obvious necessary condition to have a *proper* solution is to have a *weak* solution (see Section 3 for definitions). This latter condition is automatically satisfied in our case, since $\mathrm{cd}_p(\mathrm{Gal}(K)) \le 1$, for a field of characteristic $p > 0$. We obtain a mild sufficient condition on K to have a proper solution of any finite p-embedding problem.

Theorem 1.1. *Let K be a field of characteristic p admitting a non-p-divisible valuation. Then every finite p-embedding problem for K is solvable.*

Some examples of fields satisfying this condition are the following. If R is a Noetherian domain or a Krull domain of characteristic $p > 0$, then its fraction field K satisfies the hypothesis of Theorem 1.1. If R is an arbitrary domain of characteristic $p > 0$, then the fraction fields of the ring $R[x_1, \dots, x_n]$ of polynomials and of the ring of formal Taylor series $R[[x_1, \dots, x_n]]$ satisfy the hypothesis of Theorem 1.1, for any $n \ge 1$.

The proof of Theorem 1.1 is based on the following cohomological criterion of Harbater. A profinite group Π is called **strongly p-dominating** if $H^1(\Pi, P)$ is infinite for every nontrivial finite elementary p-group P on which Π acts.[1]

Theorem 1.2 ([8, Theorem 1b]). *Let Π be a profinite group. Assume that Π is strongly p-dominating and that $\mathrm{cd}_p(\Pi) \le 1$. Then every finite p-embedding problem for Π is properly solvable.*

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[1] All actions, homomorphism, etc., in this work are assumed to be continuous.

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Harbater's motivation for Theorem 1.2 is to show that every finite p-embedding problem for the étale fundamental group $\Pi := \pi_1(X)$ of an affine variety X over an arbitrary field K of characteristic $p > 0$ has a proper solution [7].

We show that the converse of Theorem 1.2 also holds true; see Theorem 4.2. Moreover, to get the assertion of Theorem 1.2, one may suspect that the infinitude of $H^1(\Pi, \mathbb{Z}/p\mathbb{Z})$ suffices, where Π acts trivially on $\mathbb{Z}/p\mathbb{Z}$. This is true if both the kernel and cokernel are p-groups, but in general it fails; see [8].

By Theorem 1.2, to prove Theorem 1.1 it suffices to show that $\mathrm{Gal}(K)$ is strongly p-dominating. This is carried out by using that for every nontrivial finite elementary p-group P on which Π acts we have $H^1(K, P) = K/f(K)$, for some additive polynomial f (Lemma 3.1). Then using the non-p-divisible valuation of K we construct infinitely many $a \in K$ that are distinct modulo f.

We conclude the introduction with an example. Let K_0 be a field of characteristic $p > 0$ and $K = K_0((x))$ the field of formal Laurent series. Then by Theorem 1.1 every finite p-embedding problem is properly solvable. When K_0 is algebraically closed, Harbater proves this in [8, Example 5] using a similar method. However, when K_0 is arbitrary Harbater invokes a theorem of Katz–Gabber in order to complete his proof (see [8, Proposition 6]).

2. Valuation-theoretic lemmas

Let A be a commutative ring. By a **valuation** of A we shall mean a map $v : x \mapsto v(x)$ of A onto a totally ordered commutative group Γ (written additively), together with an extra element ∞, such that

(1) $\alpha + \infty = \infty$ and $\alpha < \infty$ for all $\alpha \in \Gamma$.
(2) $v(x) = \infty$ if and only if $x = 0$.
(3) $v(xy) = v(x) + v(y)$ for all $x, y \in A$.
(4) $v(x + y) \geq \min\{v(x), v(y)\}$.

If A is a ring with a valuation v on A, we shall also say simply that A is a **valued ring**. The group Γ is called the **value group**.

Lemma 2.1. *Let Γ be a nontrivial totally ordered commutative group*

(1) *For any element γ in Γ, there exists $\beta \in \Gamma$ such that $\beta < \gamma$.*
(2) *Let $\gamma_1, \ldots, \gamma_r$ be elements in Γ and let $n_1, \ldots, n_r$ be positive numbers. Then there exists an element γ_0 in Γ such that for all elements $\gamma < \gamma_0$, $\gamma \in \Gamma$, we have $n_i\gamma < \gamma_i$ for all i.*

Proof. (1) If $\gamma \geq 0$, then let $\beta < 0 \leq \gamma$ (such an element exists since Γ is nontrivial).
If $\gamma < 0$, one can takes $\beta = 2\gamma < \gamma$.
(2) We set

$$\gamma_0 := \min\{\gamma_1, \ldots, \gamma_r, 0\}.$$

Now let γ be an arbitrary element such that $\gamma < \gamma_0$. Since $\gamma < \gamma_i$, $\gamma < 0$, it follows that $n_i\gamma < \gamma_i$, for all i. □

Let A be a commutative ring of characteristic p. We say that a polynomial in one variable $f(T)$ with coefficients in A is p**-polynomial** if $f(T) = \sum_{i=0}^{m} a_i T^{p^i}$, $a_i \in A$. Note that a p-polynomial f induces a homomorphism of the additive group.

Lemma 2.2. *Let A be a valued ring of characteristic $p > 0$ with nontrivial value group Γ. Let $f(T) = b_0T + \cdots + b_mT^{p^m}$ be a p-polynomial in one variable with coefficients in A. Then there exists an element $\gamma_0 \in \Gamma$ such that if $a = f(a_1)$, $a_1 \in A$ and $v(a) < \gamma_0$ then $v(a) = v(b_m) + p^m v(a_1)$.*

Proof. By Lemma 2.1, there exists an element $\alpha \in \Gamma$ such that for all $\gamma < \alpha$ in Γ, we have

$$(p^m - p^i)\gamma < v(b_i) - v(b_m), \quad \forall\, 0 \leq i < m.$$

We set

$$\beta := \min\{v(b_i) + \alpha p^i \mid 0 \leq i \leq m\}.$$

Let γ_0 be any element with $\gamma_0 < \beta$. Now assume that $a = f(a_1)$ such that $v(a) < \gamma_0$ ($a_1 \in A$). Let s be an index such that

$$v(b_s a_1^{p^s}) = \min\{v(b_i a_1^{p^i}) \mid 0 \leq i \leq m\}).$$

Then

$$v(b_s) + p^s\alpha > \gamma_0 > v(f(a_1)) \geq v(b_s) + p^s v(a_1).$$

Thus $0 < p^s(\alpha - v(a_1))$ and hence $v(a_1) < \alpha$. By the choice of α, we have

$$v(b_i a_1^{p^i}) = v(b_i) + p^i v(a_1) > v(b_m) + p^m v(a_1) = v(b_m a_1^{p^m}), \quad \forall i < m.$$

Therefore $v(a) = v(b_m) + p^m v(a_1)$ as required. □

Lemma 2.3. *Let Γ be a non-p-divisible totally ordered commutative group. Let α_0, γ_0 be elements in Γ. Then there exist infinitely many elements $\gamma_i \in \Gamma$ such that*

$$\gamma_0 > \gamma_1 > \cdots > \gamma_i > \cdots$$

and $\gamma_i \notin \alpha_0 + p\Gamma$, for all $i > 0$.

Proof. We first consider the case $\alpha_0 = 0$. Since Γ is not p-divisible, there is an element $a_0 \in \Gamma$ such that $a_0 \notin p\Gamma$. By Lemma 2.1 part (2), there exists an element δ_0 such that for all $\delta < \delta_0$, we have $p\delta < \gamma_0 = a_0$. By Lemma 2.1 part (1), there exists an infinite sequence

$$\delta_0 > \delta_1 > \cdots > \delta_i > \cdots .$$

Set $\gamma_i := a_0 + p\delta_i$, for all $i > 0$. Then $\gamma_i \notin p\Gamma$, for all $i > 0$ and $\gamma_0 > \gamma_1 > \cdots > \gamma_i > \cdots$.

For the general case, applying the previous argument for $\gamma_0' := \gamma_0 - \alpha_0$, we get an infinite sequence $\gamma_0' > \gamma_1' > \cdots > \gamma_i' > \cdots$ with $\gamma_i' \in \Gamma$ but $\gamma_i' \notin p\Gamma$. Setting $\gamma_i := \gamma_i' + \alpha_0$, we get a desired sequence of elements. □

Proposition 2.4. *Let A be a valued ring of characteristic $p > 0$ with non-p-divisible value group Γ. Let $f(T) = b_0T + b_1T^p + \cdots + b_mT^{p^m}$ be a p-polynomial in one variable with coefficients in A with $m \geq 1$ and $b_m \neq 0$. Then $A/f(A)$ is infinite.*

Proof. Let γ_0 be as in Lemma 2.2. For any a in A such that $v(a) < \gamma_0$ and $v(a) \notin v(b_m) + p\Gamma$, Lemma 2.2 implies that a is not in $f(A)$. By Lemma 2.3 and noting that the valuation map v is onto, we may choose a sequence $\{a_i\}$ of elements from A such that $v(a_i) \notin v(b_m) + p\Gamma$ for all i, and $\gamma_0 > v(a_1) > v(a_2) > \cdots > v(a_i) > \cdots$. For every $i < j$, one has

$$v(a_i - a_j) = v(a_i) \notin v(b_m) + p\Gamma,$$

so $a_i - a_j \notin f(A)$ and hence a_i, a_j have different images in $A/f(A)$. Therefore, $A/f(A)$ is infinite. □

3. Proof of Theorem 1.1 and a corollary

An **embedding problem** $\mathcal{E}$ for a profinite group Π is a diagram

$$\begin{array}{ccc} & & \Pi \\ & & \downarrow{\scriptstyle \alpha} \\ \Gamma & \xrightarrow{f} & G \end{array}$$

which consists of a pair of profinite groups Γ and G and epimorphisms $\alpha : \Pi \to G, f : \Gamma \to G$.

A **weak solution** of $\mathcal{E}$ is a homomorphism $\beta : \Pi \to \Gamma$ such that $f\beta = \alpha$. If such a β is surjective, then it is called a **proper solution**. We will call $\mathcal{E}$ **weakly** (resp. **properly**) **solvable** if it has a weak (resp. proper) solution.

We call $\mathcal{E}$ a **finite** embedding problem if the group Γ is finite.

The **kernel** of $\mathcal{E}$ is defined to be $N := \ker(f)$. We call $\mathcal{E}$ a ***p*-embedding problem** if N is a p-group.

We say $\mathcal{E}$ is a **split** embedding problem if $f : \Gamma \to G$ has a group theoretical section, i.e., $f' : G \to \Gamma$ such that ff' is the identity map on G.

In this note, by a ***K*-group**, where K is a field, we mean an algebraic affine group scheme which is smooth [19]. This notion is equivalent to the notion of a linear algebraic group defined over K in the sense of [3].

First we need the following lemma.

Lemma 3.1. *Let K be an infinite field of characteristic $p > 0$. Let P be a nontrivial finite commutative K-group which is annihilated by p. Then P is K-isomorphic to a K-subgroup of the additive group $\mathbb{G}_a$, of the form $\{x \mid f(x) = 0\}$, where $f(T) = T + b_1T^p + \cdots + b_mT^{p^m}$ is a p-polynomial with coefficients in K, $m \geq 1$ and $b_m \neq 0$.*

Proof. This is well known; see e.g. [4, Proposition B.1.13] or [14, Chapter V, Proposition 4.1 and Subsection 6.1]. □

We are now ready to prove Theorem 1.1.

Proof of Theorem 1.1. We have $\mathrm{cd}_p(\mathrm{Gal}(K)) \leq 1$ (see, e.g., [18, Chapter II, Proposition 3]). By Theorem 1.2 it suffices to prove that $\mathrm{Gal}(K)$ is strongly p-dominating.

Indeed, let P be a nontrivial elementary p-group on which $\mathrm{Gal}(K)$ acts. Consider P as a finite K-group. Then P is commutative and annihilated by p. Hence by Lemma 3.1, P is K-isomorphic to a subgroup of $\mathbb{G}_a$ defined as the kernel of $f : \mathbb{G}_a \to \mathbb{G}_a$, where $f(T) = T + \cdots + b_mT^{p^m}$ is a p-polynomial in one variable with coefficients in K with $m \geq 1$ and $b_m \neq 0$. We have the following exact sequence of K-groups

$$0 \to P \to \mathbb{G}_a \xrightarrow{f} \mathbb{G}_a \to 0.$$

From this exact sequence we get the following exact sequence of Galois cohomology groups

$$H^0(K, \mathbb{G}_a) \xrightarrow{f} H^0(K, \mathbb{G}_a) \to H^1(K, P) \to H^1(K, \mathbb{G}_a).$$

By Hilbert 90 $H^1(K, \mathbb{G}_a) = 0$ (see e.g. [18, Chapter II, Proposition 1]), hence

$$H^1(K, P) \simeq H^0(K, \mathbb{G}_a)/\mathrm{im}(f) = K/f(K).$$

The latter is infinite by Proposition 2.4. So we conclude that $H^1(K, P)$ is infinite, and hence $\mathrm{Gal}(K)$ is strongly p-dominating. □

We recall that a Hilbertian field is a field K having the irreducible specialization property: for every irreducible polynomial $f(T, X) \in k[T, X]$ that is separable in X, there exists $a \in K$ such that $f(a, X)$ is irreducible in $k[X]$ (we refer readers to [6, Chapters 12, 13] for more details about Hilbertian fields). In [5], Dèbes and Deschamps give the following conjecture.

Conjecture 3.2 (*[5, 2.1.2]*). *Let K be a Hilbertian field. Then every finite split embedding problem for* $\mathrm{Gal}(K)$ *has a proper solution.*

An easy consequence of Theorem 1.1 is a simple proof of [12, Theorem 8.3] which asserts that Conjecture 3.2 holds true whenever K is of characteristic $p > 0$ and if the kernel of the embedding problem is a p-group. Namely, we have

Corollary 3.3. *Let K be a Hilbertian field of characteristic $p > 0$. Then every finite p-embedding problem for* $\mathrm{Gal}(K)$ *is properly solvable.*

Proof. Let $\mathcal{E} = (\alpha : \mathrm{Gal}(K) \to A, f : B \to A)$ be a finite p-embedding problem for $\mathrm{Gal}(K)$. Consider the finite p-embedding problem $\mathcal{E}_t := (\alpha \circ pr_t : \mathrm{Gal}(K(t)) \to A, f : B \to A)$ for $\mathrm{Gal}(K(t))$ obtained by composition with the restriction map $\mathrm{Gal}(K(t)) \to \mathrm{Gal}(K)$. Since $K(t)$ has discrete valuations, Theorem 1.1 gives a proper solution of $\mathcal{E}_t$, say $\theta_t : \mathrm{Gal}(K(t)) \to B$. By the irreducible specialization property (applied to a polynomial a root of which generates the solution field of θ_t over $K(t)$) θ_t specializes to a proper solution θ of $\mathcal{E}$ (see [6, Lemma 16.4.2]). □

Remark 3.4. (1) Let G be a finite p-group, K a Hilbertian field of characteristic $p > 0$. By considering the finite (split) p-embedding problem $(\mathrm{Gal}(K) \to \{1\}, G \to \{1\})$, Corollary 3.3 implies that G is realizable over K. In other words, this proposition shows that every finite p-group is realizable over an arbitrary Hilbertian field of characteristic $p > 0$. This last statement is a special case of a theorem of Shafarevich, [6, Theorem 16.4.7].

(2) Corollary 3.3 can also be derived from Ikeda's theorem [6, Proposition 16.4.5]. Here we sketch the proof: one starts with a finite embedding problem for K corresponding to an exact sequence $1 \to P \to B \to A \to 1$, where P is a p-group and $B = \mathrm{Gal}(L/K)$. We use the usual trick of decomposing this embedding problem to a series of embedding problems in order to assume that P is a minimal normal subgroup of B. In particular P is *abelian*. Since $\mathrm{cd}_p(K) \leq 1$ we can replace this embedding problem by a bigger *split* embedding problem with the same kernel by taking the fiber product of B and the image of a weak solution. Now we use Ikeda's result that gives a *regular* solution over K, i.e., a solution over $K(t)$ with the extra condition that the solution field is regular over L. Then one uses Hilbertianity to reduce the solution to a solution over K.

Unfortunately, we do not know whether any finite p-embedding problem over a field of characteristic $p > 0$ has a regular solution.

(3) For recent results concerning Conjecture 3.2, we refer readers to [2,15–17].

4. Embedding problems with *p*-kernel

In this section we show that the converse of Theorem 1.1 also holds true; see Theorem 4.2.

Let

$$\mathcal{E} := \qquad \begin{array}{ccccccccc} & & & & & & \Pi & & \\ & & & & & & \downarrow{\scriptstyle\alpha} & & \\ 1 & \longrightarrow & P & \longrightarrow & \Gamma & \xrightarrow{f} & G & \longrightarrow & 1 \end{array}$$

be an embedding problem for Π with abelian kernel P. Since P is abelian, there is an induced conjugation action of G on P by choosing representatives in Γ. This in turn yields an action of Π on P via $\alpha : \Pi \to G$. Let $H^1(\Pi, P)$ be the corresponding Galois cohomology group.

Two weak solutions β and $\beta' : \Pi \to \Gamma$ of $\mathcal{E}$ are defined to be equivalent, and denoted by $\beta \sim \beta'$, if there is an element p in P such that $\beta' = \mathrm{inn}(p) \circ \beta$. (Here $\mathrm{inn}(p) \in \mathrm{Aut}(\Gamma)$ denotes left conjugation by p.) One can check that $\sim$ is an equivalence on the set of weak solutions to $\mathcal{E}$. Denote by $\mathrm{WS}(\mathcal{E})$ the set of weak solutions of $\mathcal{E}$ modulo the equivalence relation $\sim$. We have a cohomological description of $\mathrm{WS}(\mathcal{E})$.

Lemma 4.1. *With notation as above, assume that $\mathcal{E}$ is weakly solvable. Then* $\mathrm{WS}(\mathcal{E})$ *is a $H^1(\Pi, P)$-torsor. In particular, any weak solution θ of $\mathcal{E}$ induces a bijection*

$$\mathrm{WS}(\mathcal{E}) \cong H^1(\Pi, P).$$

Proof. See [13, Proposition 9.4.4]. □

Next we prove the converse of Theorem 1.2. For future reference we formulate it as an if and only if theorem.

Theorem 4.2. *Let Π be a profinite group. Then every finite p-embedding problem for Π has a proper solution if and only if $\mathrm{cd}_p(\Pi) \leq 1$ and Π is strongly p-dominating.*

Proof. ($\Leftarrow$): This is Theorem 1.2.
($\Rightarrow$): It suffices to prove that Π is strongly p-dominating. Let P be a nontrivial elementary abelian p-group on which Π acts continuously. We have to show that $H^1(\Pi, P)$ is infinite.

Since the action of Π on P is continuous, it factors via a finite quotient. I.e., there is a map $\alpha : \Pi \to G$ and an action of G on P that induces the action of Π on P. Let Γ be the semidirect product of P and G. We get the following split embedding problem with elementary abelian p-kernel

$$\mathcal{E} := \qquad \begin{array}{ccccccccc} & & & & & & \Pi & & \\ & & & & & & \downarrow{\scriptstyle \alpha} & & \\ 1 & \longrightarrow & P & \longrightarrow & \Gamma & \xrightarrow{f} & G & \longrightarrow & 1 \end{array}$$

For any $n > 0$ let Γ_G^n be the nth fold fiber product of Γ over G, i.e.,

$$\Gamma_G^n = \{(\gamma_1, \dots, \gamma_n), \gamma_i \in \Gamma, \text{ and } f(\gamma_1) = \cdots = f(\gamma_n) \in G\}.$$

We have a map $f_n : \Gamma_G^n \to G$, defined by $f_n((\gamma_i)_{i=1}^n) = f(\gamma_1)$.

We have an embedding problem $\mathcal{E}_n$ for Π corresponding to the exact sequence

$$1 \longrightarrow P^n \longrightarrow \Gamma_G^n \xrightarrow{f_n} G \longrightarrow 1.$$

By assumption, there is a proper solution β to $\mathcal{E}_n$. By composing β with the projections $\mathrm{pr}_i : \Gamma_G^n \to \Gamma$, we get n proper solutions $\beta_1, \dots, \beta_n$.

We show that these β_i are pairwise non-equivalent (and in particular distinct). Indeed, if $\beta_i \sim \beta_j$, for some $1 \leq i < j \leq n$, then there is a element $p \in P$ such that $\beta_i(s) = p\beta_j(s)p^{-1}$, for all $s \in \Pi$. Since P is a nontrivial group, we can take two different elements q, q' from P. Set $x = (1, \dots, q, \dots, q', \dots, 1) \in \Gamma^n$, where q, q' are in ith and jth entry, respectively and 1 is in all other entries. Then $x \in \Gamma_G^n$. Since β is a *proper* solution, there exists s in Π such that $\beta(s) = x$. We then have

$$q = \beta_i(s) = p\beta_j(s)p^{-1} = pq'p^{-1} = q',$$

a contradiction.

Therefore, we get that $\mathrm{WS}(\mathcal{E})$ is infinite, and by Lemma 4.1, $H^1(\Pi, P)$ is infinite, as needed. □

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